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ALGORITHM FOR DESIGNING THE SAW DIAGRAM FOR VEHICLES

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Abstract *The paper presents an algorithm by which one can draw the speed variation diagram corresponding to the speed gearbox according to the engine rotation in various regimes. The diagram obtained allows to identify the optimal zone for gear shift and the overlay when switching between gearbox. The algorithm is created both for the situation when the last gear is regarded a direct grip (the engine and the transmission's output rotate at the same speed) and overdrive gears. The results obtained can be used to study the vehicles performance related to traction or power balance, traction or power characteristic, the dynamic characteristic, etc. The method can be applied to any type of vehicles.*
Key words: *calculus algorithms, vehicles, vehicle speed, engine speed*

1. INTRODUCTION

The study of the optimal change of gearbox in the case of wheel based vehicles can be done based on the saw diagram. In order to obtain this diagram one must know the extreme vehicle speeds corresponding to the gearbox, which depend on the functioning areas of the vehicle engine (the rotations corresponding to the various regimes of the engine) the way its transmission is done (front based, rear based or four wheel based, gearbox with or without overdrive gears, the wheel size and the optimal pressure in them when running etc.

The algorithm elaborated for this purpose aims to graphically represent the vehicle speed variations according to engine rpm so that one can easily identify the vehicle speeds that the gearbox cover and the moment of passage from one gear to another while vehicle running.

2. NOTATIONS USED IN THE NUMERICAL CALCULUS MODEL

The numerical algorithm that was elaborated in the MathCAD program is based on the minimal and maximal vehicle speed values for each gear, respectively the rpm's corresponding to the various running regimes of the engine.

This enables the graphical representation of the diagram $v = f(n)$ (saw) and at the same time enables the identification of the overlays in the case of gear shifts.

Thus, the main notations are found in table 1.

Table 1

The main notations used in the algorithm

Item	Notation	U.M.
❑ The number of gearbox	k	-
❑ A certain gear (j = 1...k)	j	-
❑ The minimum speed for gear I	v_{min_I}	km/h
❑ The minimum speed for gear j	v_{min_j}	km/h
❑ The maximum speed for gear j	v_{max_j}	km/h
❑ The minimum speed for gear k	v_{max_k}	km/h
❑ The maximum possible speed	v_{max_c}	km/h
❑ Idling speed engine	n_{min}	rpm
❑ The engine speed corresponding to the effective maximum engine torque (M_{max})	n_M	rpm
❑ The engine speed corresponding to the maximum engine power (P_{max})	n_p	rpm
❑ The intervention engine speed of the limitative speed regulator	n_r	rpm
❑ The engine speed specific for the economic regime of the engine	n_{ec}	rpm

Table 1 (Cont)

Item	Notation	U.M.
❑ Maximum engine speed	n_{max}	rpm
❑ Gear span (Gear cover)	A_j	km/h
❑ The total transmission ratio when the geabox is in gear *j*	i_{t_j}	-
❑ The transmission ratio of the main transmission	i_o	-
❑ The transmission ratio when the gearbox is in gear *j*	i_{cv_j}	-
❑ The transmission ratio of the final transmission	i_{tf}	-
❑ The dynamic ray of the driving wheel of the vehicles	r_d	m

This algorithm takes into account that the last gear *k* from the gearbox is regarded as a direct grip *(d.p.)* or overdrive gear. In the case of a gearbox with an overdrive gear in the last gear *k*, where the transmission ratio is subunit, the direct grip *(d.p.)* is considered done in the gear before the last one *(k-1)* [9].

3. EVALUATION OF THE EXTREME SPEEDS CORRESPONDING TO THEIR GEARBOX

The speeds n_{min}, n_M, n_p, n_r *and* n_{max} delineates the running of internal combustion engines [2, 3, 7, 8, 11, 12]. The running area $[n_M, n_{max}]$ of the engine without the speed limitation regulator and the running area $[n_M, n_r]$ of an engine with a limitation regulator is called the stable running area of the engine or the stability area, because once the engine load increases and the engine speed decreases the engine torque produced is increased and it balances the additional resistives torques. The higher the stability area, the better the engine is for propelling the vehicle. In the case of engine speeds lower than n_M, the engine enters in the instable area (non-stable), because when the engine speed decreased because of the increase in the load, the torque produced decreases and this leads to an engine stop. This is one reason why the engines must have an elastic coefficient as low as possible, so that they have a stability zone as wide as possible. The economic run of the engine is characterised by the engine speed corresponding to the effective minimal fuel consumption n_{ec}, in the optimal area of shifting gearbox [9], between n_M and n_p, around the value of 75 % from speed n_p.

The propulsion of the vehicle is done by transmitting the power produced by the engine towards its driving wheel [1, 4, 5, 6, 8, 10, 11, 12, 14]. The power transfer from the engine to the wheels is done by the systems that compose the transmission (clutch, gearbox, reducers, distribution boxes, main transmission, differential, final transmissions) and which consume a part of the effective power to overcome component friction.

As a consequence, obtaining a tangential traction force in the driving wheel needed to counteract the forward resistance (running resistance forces, resistance forces due to longitudinal slopes in the road, air resistance forces, start resistance forces, resistance forces due to towing trailers and semi-trailers) depends directly on the engine torque that reaches the driving wheel, respectively the power that reaches them [10].

The movement of the vehicle is possible when its driving wheel transmit an active torque big enough to counteract the forward resistance and to ensure its movement [1, 4, 5, 6, 8, 10, 11, 12, 14]. The abovementioned forces (both the ones from the driving wheel as well as the forward ones) influence the start ability of vehicles. In the case of overtaking, it is important for the vehicles to have good acceleration so that they can overtake in the shortest time and space.

Thus, engine performance, that are also in connection to the vehicles dynamic performances, can strongly contribute to the increase in traffic safety [10]. We should also mention that the possibility of getting higher vehicle speed can also influence the drivers to reach them and they also represent a risk factor in car accident occurrence [10, 13].

Shifting from a lower gear to a higher one is done after decoupling the clutch and after the engine acceleration has stopped by removing the gearing that was coupled in the kinematic circuit of the gearbox and connecting to the desired gear. Shifting from one gear to another takes between 1 and 3 seconds [8, 11, 12], and during this time, since the engine is idle, the speed of the vehicle does not remain the same and decreases under the forward resistance. As a consequence, the vehicle speed when shifting to the next gear is smaller than the vehicle speed that the vehicle has reached at the end of the start of the lower gear ($v_{max_1} > v_{min_2}$; $v_{max_2} > v_{min_3}$; ...; $v_{max_{(k-1)}} > v_{min_k}$).

Starting the engine is regarded as done during the engine speed span of $0 \div n_M$. In the case of a vehicle with a gearbox with k gearbox, the speed span covered with gear I is between v_I, corresponding to speed n_M and v_{Imax} corresponding to speed n_p. The second gear covers the span between v_{min_2} and v_{max_2}, the gear $(k\text{-}1)$ the span $v_{min_{(k-1)}} \dots v_{max_{(k-1)}}$, in the case of gear (k) the span is over v_{min_k}, corresponding to engine speeds n_M and n_p [8, 11, 12].

The optimal shift between gearbox is done [8, 11, 12] in the engine speed interval $[n_M, n_p]$, and the minimum/maximum vehicle speeds in km/h for each gear is determined using the relations:

$$v_{min_j} = \begin{cases} 0.377 \cdot \dfrac{n_{min} \cdot r_d}{i_{t_j}}, \ for \ j = 1 \\ 0.377 \cdot \dfrac{n_M \cdot r_d}{i_{t_j}}, \ for \ j \geq 2 \end{cases}, \tag{1}$$

$$v_{max_j} = \begin{cases} \neg \ if \ \ d.g. \rightarrow gear \ k : \\ 0.377 \cdot \dfrac{n_p \cdot r_d}{i_{t_j}}, \ for \ j \leq (k-1) \\ 0.377 \cdot \dfrac{n_{max} \cdot r_d}{i_{t_j}}, \ for \ j = k \\ \neg \ if \ \ d.g. \rightarrow gear \ (k-1) : \\ 0.377 \cdot \dfrac{n_p \cdot r_d}{i_{t_j}}, \ for \ j \leq (k-2) \\ 0.377 \cdot \dfrac{n_{max} \cdot r_d}{i_{t_j}}, \ for \ j = (k-1) \\ 0.377 \cdot \dfrac{n_{lim} \cdot r_d}{i_{t_j}}, \ for \ j = k \end{cases}, \tag{2}$$

in wich:

$$n_{lim} = \frac{v_{max_c} \cdot i_{t_k}}{0.377 \cdot r_d}, \ in \ rpm, \tag{3}$$

$$i_{t_j} = i_o \cdot i_{cv_j} \cdot i_{tf}. \tag{4}$$

If the engine studied has a speed limitative regulator, then in relation (2), instead of speed n_p we will use speed n_r. The engine speed n_{lim}, used in relation (2), has a value of about n_{ec}.

Based on the minimal and maximal vehicle speeds for each gear, determined by the relations (1) and (2), one can evaluate the ground covered by each gear.

4. THE NUMERICAL EVALUATION METHOD REGARDING THE DESIGN OF THE SAW DIAGRAM

To exemplify, in the numerical calculus we take into account a vehicle with the following characteristics: the engine is a spark combustion engine; the weight is 1536 kg; it has front traction; v_{max} = 174 km/h; tyre marking 185/65 R 15; tyre pressure 0.22 MPa; P_{max} = 64 kW at n_p = 5500 rpm; M_{max} = 128 Nm at n_M = 3000 rpm; k = 5, if (d.g.) is in gear k, respectively k = 6, if (d.g.) is in gearbox (k-1); wheelbase, 2.589 m; front wheel gauge, 1.480 m; rear wheel gauge, 1.470 m; height, 1.534 m; total length, 4.020 m.

The gear spans are presented in the saw diagram of the vehicle $v = f(n)$ (Fig. 1, Fig. 2), which can be constructed [8] by defining the y, respectively z, according to the minimal and maximal speeds of each gear and the engine speeds n_{min}, n_M, n_p, n_{max}, thus:

a) For the situation in which the last gear k from the gearbox is regarded as a direct grip (d.g.), (Fig. 1),

- $y_{j,x_1} = v_{max_j} \cdot \dfrac{x_1}{n_p}$,

  where: $j = 1...(k\text{-}1)$; $x_1 = 0...n_p$;

- $y_{k,x_2} = v_{max_k} \cdot \dfrac{x_2}{n_{max}}$,

  where: $x_2 = 0...n_{max}$;

- $z_{j,x_3} = a_j \cdot x_3 + b_j$,

where: $j = 1...(k\text{-}1);\ x_3 = n_M...n_p$;

$a_j = \dfrac{A1_j}{n_p - n_M}$; $A1_j = v_{max_j} - v_{min_{(j+1)}}$;

$b_j = v_{min_{(j+1)}} - n_M \cdot a_j$;

$\Rightarrow z_{j,x_3} = (x_3 - n_M) \cdot \dfrac{A1_j}{n_p - n_M} + v_{min_{(j+1)}}$;

- $y_f = 0 \dots v_{min_1}$; $y_m = 0 \dots v_{min_k}$;

$y_p = 0 \dots v_{max_{(k-1)}}$; $y_{fm} = 0 \dots v_{max_k}$;

b) For the situation in which the last gear k from the gearbox is regarded as over grop and the direct grip (d.g.) is considered the gear before the last one (k-l), (Fig. 2),

- $y_{j,x_1} = v_{max_j} \cdot \dfrac{x_1}{n_p}$,

where: $j = 1...(k\text{-}2);\ x_1 = 0...n_p$;

- $y_{(k-1),x_2} = v_{max_{(k-1)}} \cdot \dfrac{x_2}{n_{max}}$,

where: $x_2 = 0...n_{max}$;

- $y_{k,x_{lim}} = v_{max_k} \cdot \dfrac{x_{lim}}{n_{lim}}$,

where: $x_{lim} = 0...n_{lim}$;

- $z_{j,x_3} = a_j \cdot x_3 + b_j$,

where: $j = 1...(k\text{-}2);\ x_3 = n_M...n_p$;

$a_j = \dfrac{A1_j}{n_p - n_M}$; $A1_j = v_{max_j} - v_{min_{(j+1)}}$;

$b_j = v_{min_{(j+1)}} - n_M \cdot a_j$;

$\Rightarrow z_{j,x_3} = (x_3 - n_M) \cdot \dfrac{A1_j}{n_p - n_M} + v_{min_{(j+1)}}$;

- $z1_{j1,x_3} = a1_{j1} \cdot x_3 + b1_{j1}$,

where: $j1 = (k\text{-}1);\ x_3 = n_M...n_p$;

$a1_{j1} = \dfrac{A11_{j1}}{n_p - n_M}$; $A11_{j1} = v_{max_{(j1-1)}} - v_{min_{(j1+1)}}$;

$b1_{j1} = v_{min_{(j1+1)}} - n_M \cdot a1_{j1}$;

$\Rightarrow z1_{j1,x_3} = (x_3 - n_M) \cdot \dfrac{A11_{j1}}{n_p - n_M} + v_{min_{(j1+1)}}$;

- $z2_{j2,x_4} = a2_{j2} \cdot x_4 + b2_{j2}$,

where: $j2 = k\ ;\ x_4 = n_M...n_{max}$;

$a2_{j2} = \dfrac{A12_{j2}}{n_{max} - n_M}$; $A12_{j2} = v_{max_{(j2-1)}} - v_{min_{j2}}$;

$b2_{j2} = v_{min_{j2}} - n_M \cdot a2_{j2}$;

$\Rightarrow z2_{j2,x_4} = (x_4 - n_M) \cdot \dfrac{A12_{j2}}{n_{max} - n_M} + v_{min_{j2}}$;

- $y_f = 0 \dots v_{min_1}$; $y_m = 0 \dots v_{min_k}$;

$y_{pv} = 0 \dots v_{max_{(k-2)}}$; $y_{lim} = y_p = 0 \dots v_{max_{(k-1)}}$;

$y_{fm} = 0 \dots v_{max_k}$; $x_5 = 0 \dots n_{max}$.

5. CONCLUSION

Applying the created method allows the design of the saw diagram for any type of vehicle on wheels as long as we know certain technical specifications (maximum engine power and its engine speed; the maximum torque of the engine and its engine speed; the minimum and maximum engine speed, the maximum velocity, total weight, the number of gearbox, the way the gearbox is organised – whether the last gear *k* in the gearbox is a direct grip or an overdrive gear, the size parameters of the vehicle, the type of tyres on the driving wheel, etc.).

Based on the diagram *v = f(n)* one can choose the shift moments from one gear to another during the vehicle running. Based on the results one can determine and compare the areas covered by the gearbox for various vehicles on wheels.

The results can be a good background for the study of vehicle performance in relation to the traction balance or power, dynamic characteristic etc.

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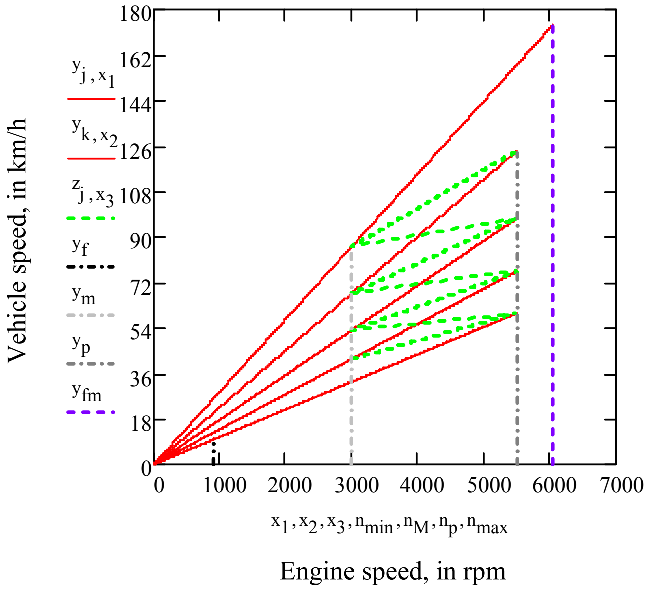

Fig. 1. Diagram $v = f(n)$ (saw) of the vehicle,
for the situation in which the last gear k from the gearbox is regarded as a direct grip (d.g.).

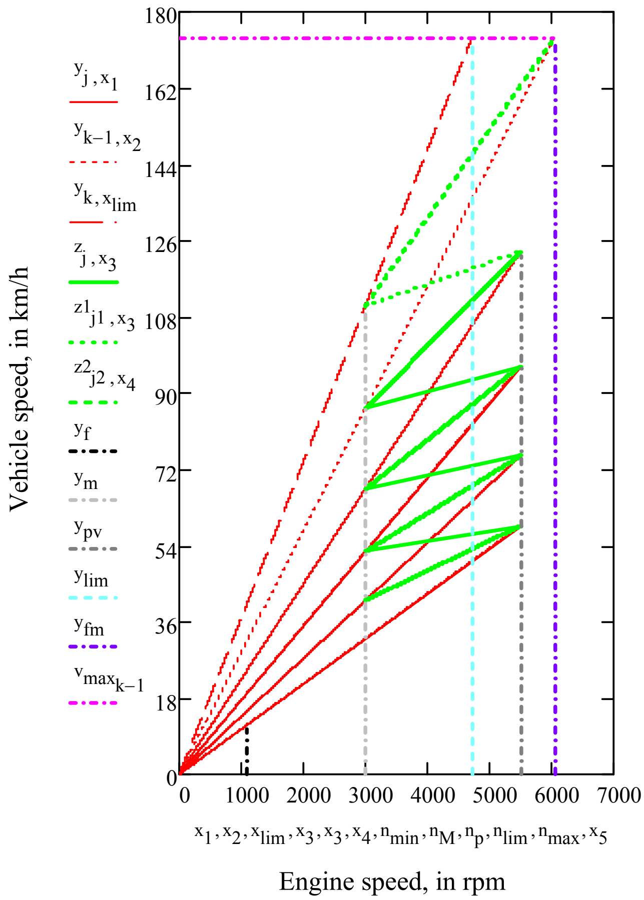

Fig. 2. Diagram $v = f(n)$ (saw) of the vehicle, for the situation in which the last gear k is regarded as the overdrive gear and the direct grip (d.g.) is the gear before the last (k-1).

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ALGORITM DE CALCUL CU PRIVIRE LA TRASAREA DIAGRAMEI FIERĂSTRĂU A AUTOVEHICULELOR

Rezumat: Lucrarea prezintă un algoritm de calcul, cu ajutorul căruia se poate trasa diagrama de variaţie a vitezelor corespunzătoare treptelor de viteze, în funcţie de turaţia motorului, la diferite regimuri de funcţionare ale acestuia. Diagrama astfel obţinută, permite identificarea zonei optime de schimbare a treptelor de viteze, respectiv a acoperirilor la schimbarea treptelor de viteze. Algoritmul de calcul este dezvoltat atât pentru cazul în care ultima treaptă a cutiei de viteze este considerată priză directă cât și supraprizǎ. Rezultatele obţinute pot sta la baza studiului performanţelor autovehiculelor, referitoare la bilanţul de tracţiune sau putere, caracteristica de tracţiune sau putere, caracteristica dinamică etc. Metoda de lucru are ca scop de a fi utilizată pentru orice tip de autovehicul.

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