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KINEMATIC MODEL OF A NEW SURGICAL PARALLEL ROBOT WITH FIVE DEGREES OF FREDOM

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Abstract: *The paper presents the kinematics of an innovative parallel structure designed for the manipulation of surgical instruments in minimally invasive surgery. Some advantages of using parallel robots in medicine could be emphasized: precision of the movements, absence of the laparoscope operator's natural tremor, precise, stable view of the internal surgical field for the surgeon, reduces eye fatigue, eliminates the need for a second surgeon to be present for the entire procedure.*

Key words: *parallel robot, robotic assisted surgery, laparoscopic surgery*

1. INTRODUCTION

The paper presents the kinematical model of a new surgical robot designed for manipulation of surgical instruments in laparoscopic surgery. In recent years, more effort and attention has been given to the development of parallel structure robots. The basic reference for parallel mechanisms is the research by Stewart in 1965, who used the parallel mechanism as an aircraft simulator motion base, hence the so-called "Stewart platform" [1].

In the past decade, a significant amount of research has been done on developing the parallel robot for different medical applications.

A minimally invasive procedures (MIS) refers to the introduction of surgical instruments inside the human body through small incision trying to minimize the damage of healthy tissue [5].

Tanikawa et al. in 1999 developed a parallel mechanism on a dexterous micro-manipulation system for use in assembling micro-machines, manipulating biological cells, and performing micro-surgery [2].

Merlet developed a micro robot called MIPS with a parallel mechanical architecture having three degrees of freedom that allows fine positioning of a surgical tool. The purpose of MIPS is to act as an active wrist at the tip of an endoscope, providing to the surgeon an accurate tool that may further offers a partial force-feedback [3].

The DA VINCI robotic system is a robotic platform designed to enable complex surgery using minimally invasive approach. The da Vinci system consist of an ergonomic surgeon's console, a patient-side cart with four interactive robotic arms, a high-performance three dimension high definition vision system [4]. This robotic system has been approved to be used on human patients and although it brings many benefits, some drawback are reported [5] which encourage further researches in this field.

Another robot designed for minim invasive surgery is PARAMIS, which has been developed at Technical University of Cluj-Napoca, Romania and it used for laparoscope camera positioning [6]. The control input allows the user to give commands in for the positioning of the laparoscope using different interfaces: joystick, microphone, keyboard, mouse and haptic device [7].

Parallel robots offer higher stiffness and smaller mobile mass than serial robots, thus they allow faster and more precise manipulations. In the field of robotics assisted surgery, the drawbacks of serial robots motivate the search of task oriented robot architectures that best fit a specific group of medical applications [9].

2. DESCRIPTION OF ROBOTIC SYSTEM

The geometry of this parallel surgical robot, illustrated in figure 1, is composed [10] of a fixed base and two guiding kinematics chains of the platform. The fixed base coordinate system is placed on the base with Z-axis perpendicular to the base plane. The laparoscope coordinate system is located at the end of kinematics chains. All kinematics chains are connected by two splined shafts from the fixed base. The movements of parallel robot are archived by using a rotational motor and four linear motors, as seen in the figure 1. The rotational motor is positioned on the fixed base and moves the splined shaft with the first active coordinate q_1. The next active coordinates q_2, q_3, q_4, q_5 are actuated by linear motors, which slide on two ball screws [8].

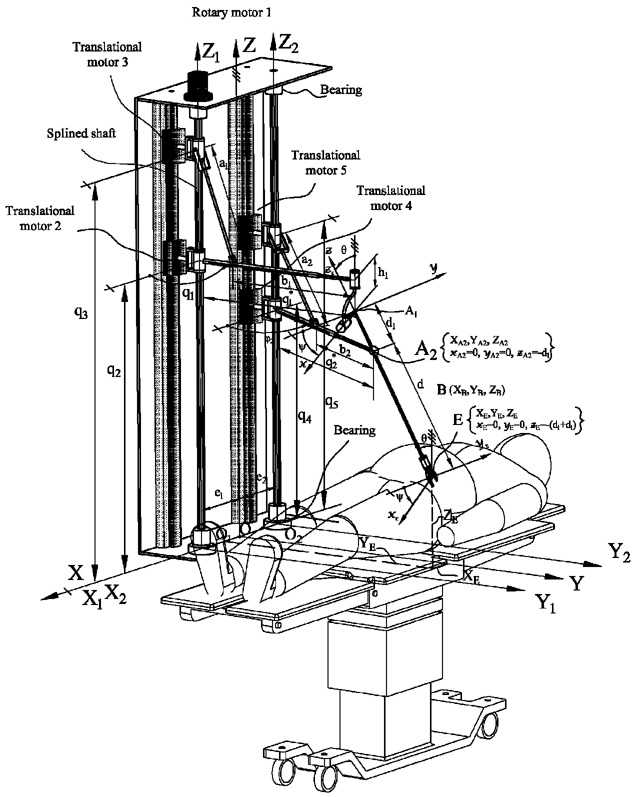

Fig. 1. Kinematic scheme

3. KINEMATICS

To determine the relationships for speed and acceleration it is used the equations of parallel robot's position, where each equation have just one generalized coordinate of the robot as:

$$F_i(q_{i,}X_E, Y_E, Z_E, \psi, \theta) = 0,\ i = 1,2,3,4,5 \qquad (1)$$

First relation of the system of implicit functions results from the next system:

$$\begin{cases} q_1^* \cdot Cq_1 = X_E - z_E \cdot C\alpha''' - e_1 \\ q_1^* \cdot Sq_1 = Y_E - z_E \cdot C\beta''' \\ q_2^* - h_1 = Z_E - z_E \cdot C\gamma''' \end{cases} \qquad (2)$$

$$F_1(q_{i,}X_E, Y_E, \psi, \theta) \equiv q_1^* \cdot Sq_1 - Y_E + z_E \cdot C\beta''' = 0 \qquad (3)$$

where:

$$q_1^* = \sqrt{(X_E - z_E \cdot C\alpha''' - e_1)^2 + (Y_E + z_E \cdot C\beta''')^2} \qquad (4)$$

Replacing the values of $C\alpha'''$ and $C\beta'''$ result:

$$q_1^* = \sqrt{(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2} \qquad (5)$$

Using relation (5) first implicit function is:

$$F_1(q_{i,}X_E, Y_E, \psi, \theta) \equiv \left[\sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + z_E \cdot C\psi \cdot S\theta)^2\end{matrix}}\right] \cdot Sq_1 -$$
$$-Y_E - z_E \cdot C\psi \cdot S\theta = 0 \qquad (6)$$

The second implicit function of the system is:

$$F_2(q_{2,}Z_E, \theta) \equiv q_2 - h_1 - Z_E + z_E \cdot C\gamma''' = 0 \qquad (7)$$

Replacing the value of $C\gamma'''$ it result:

$$F_2(q_{2,}Z_E, \theta) \equiv q_2 - h_1 - Z_E + z_E \cdot C\theta = 0 \qquad (8)$$

Expression of q_1^* can be written as follows:

$$\begin{aligned} &[a_1^2 - (q_3 - q_2)^2] = (q_1^* - b_1)^2 \\ &(q_3 - q_2)^2 = a_1^2 - (q_1^* - b_1)^2 \\ &q_3 - q_2 = \sqrt{a_1^2 - (q_1^* - b_1)^2} \\ &q_3 - q_2 = \sqrt{a_1^2 - (q_1^* - b_1)^2} = 0 \end{aligned} \qquad (9)$$

Using relation (9) result third implicit function of the system of implicit equations:

$$q_3 - Z_E + z_E \cdot C\theta - h_1 - \sqrt{a_1^2 - \left[\sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + Z_E \cdot C\psi \cdot S\theta)^2\end{matrix}} - b_1 \right]^2} = 0 \quad (10)$$

$$F_3(q_{3,} X_E, Y_E, Z_E, \psi, \theta) \equiv q_3 - Z_E + z_E \cdot C\theta - h_1 - \sqrt{a_1^2 - \left[\sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + Z_E \cdot C\psi \cdot S\theta)^2\end{matrix}} - b_1 \right]^2} = 0 \quad (11)$$

From the next relation:

$$q_4 = Z_E + d \cdot C\theta \quad (12)$$

Result the fourth implicit function:

$$F_4(q_{4,} Z_E, \theta) \equiv q_4 - Z_E - d \cdot C\theta = 0 \quad (13)$$

From the next relation:

$$q_5 = q_4 - \sqrt{a_2^2 - \left[-b_2 + \sqrt{(X_{A_2} - e_2)^2 + Y_{A_2}} \right]^2} \quad (14)$$

Insert in relation (13) the expresion of q_4, X_{A_2} and Y_{A_2} result fifth implicit function:

$$q_5 - Z_E - d \cdot C\theta - \sqrt{a_2^2 - \left[-b_2 + \sqrt{\begin{matrix}(X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot \theta)\end{matrix}} \right]^2} = 0 \quad (15)$$

$$F_5(q_{5,} X_E, Y_E, Z_E, \psi, \theta) \equiv q_5 - Z_E - d \cdot C\theta - \sqrt{a_2^2 - \left[-b_2 + \sqrt{\begin{matrix}(X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)\end{matrix}} \right]^2} = 0 \quad (16)$$

Using relations (6), (8), (11), (13) and (16) result five implicit equations:

$$F_1(q_{i,} X_E, Y_E, \psi, \theta) \equiv \left[\sqrt{(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E - z_E \cdot C\psi \cdot S\theta)^2} \right] \cdot Sq_1 - Y_E - z_E \cdot C\psi \cdot S\theta = 0 \ ,$$

$$F_2(q_{2,} Z_E, \theta) \equiv q_2 - h_1 - Z_E - z_E \cdot C\theta = 0 \ ,$$

$$F_3(q_{3,} X_E, Y_E, Z_E, \psi, \theta) \equiv q_3 - Z_E + z_E \cdot C\theta - h_1 - \sqrt{a_1^2 - \left[\sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + z_E \cdot C\psi \cdot S\theta)^2\end{matrix}} - b_1 \right]^2} = 0 \ ,$$

$$F_4(q_{4,} Z_E, \theta) \equiv q_4 - Z_E - d \cdot C\theta = 0 \ ,$$

$$F_5(q_{5,} X_E, Y_E, Z_E, \psi, \theta) \equiv q_5 - Z_E - d \cdot C\theta - \sqrt{a_2^2 - \left[-b_2 + \sqrt{\begin{matrix}(X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)\end{matrix}} \right]^2} = 0 \ , \quad (17)$$

The first equation of system (17) can be written:

$$\left[\sqrt{(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2} \right] \cdot Sq_1 = Y_E + z_E \cdot C\psi \cdot S\theta \quad (18)$$

By squaring the relation (16) result:

$$\left[(X_E - z_E \cdot s\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \right] \cdot S^2 q_1 = (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \quad (19)$$

$$\left[(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \right] \cdot S^2 q_1 - (Y_E + z_E \cdot C\psi \cdot S\theta)^2 = 0 \quad (20)$$

The third equation of system (17) can be written as follows:

$$q_3 - Z_E + z_E \cdot C\theta - h_1 = \sqrt{a_1^2 - \left[-b_1 + \sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + z_E \cdot C\psi \cdot S\theta)^2\end{matrix}} \right]^2} \quad (21)$$

By squaring the relation (21) result:

$$(q_3 - Z_E + z_E \cdot C\theta - h_1)^2 = a_1^2 - \left[-b_1 + \sqrt{\begin{matrix}(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + \\ +(Y_E + z_E \cdot C\psi \cdot S\theta)^2\end{matrix}} \right]^2 \quad (22)$$

$$(q_3 - Z_E + z_E \cdot C\theta - h_1)^2 = a_1^2 - b_1^2 + 2b_1 \cdot$$
$$\cdot\sqrt{(X_E - z_E \cdot S\psi \cdot C\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 +}$$
$$+(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \quad (23)$$

$$(q_3 - Z_E - z_E \cdot C\theta - h_1)^2 - a_1^2 + b_1^2 + (X_E -$$
$$-z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 = 2b_1 \cdot$$
$$\cdot\sqrt{(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2} \quad (24)$$

By squaring the relation (24) result:

$$\begin{bmatrix} (q_3 - Z_E - z_E \cdot C\theta - h_1)^2 - a_1^2 + b_1^2 + \\ +(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 - (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \end{bmatrix}^2$$
$$= 4b_1^2 \cdot \left[(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \right] \quad (25)$$

The fifth equation of the system (17) can be written:

$$q_5 - Z_E - d \cdot C\theta =$$
$$= \sqrt{a_2^2 - \left[-b_2 + \sqrt{\begin{matrix} (X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)^2 \end{matrix}} \right]^2} \quad (26)$$

By squaring relation (26) result:

$$(q_5 - Z_E - d \cdot C\theta)^2 = a_2^2 -$$
$$-\left[-b_2 + \sqrt{\begin{matrix} (X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)^2 \end{matrix}} \right]^2 \quad (27)$$

The relation (27) can be written as follows:

$$(q_5 - Z_E - d \cdot C\theta)^2 = a_2^2 - b_2^2 - (X_E + d \cdot$$
$$\cdot S\psi \cdot S\theta - e_2)^2 - (Y_E - d \cdot C\psi \cdot S\theta)^2 + 2b_2 \cdot$$
$$\cdot\sqrt{\begin{matrix} (X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)^2 \end{matrix}} \quad (28)$$

$$(q_5 - Z_E - d \cdot C\theta)^2 - a_2^2 + b_2^2 + (X_E + d \cdot$$
$$\cdot S\psi \cdot S\theta - e_2)^2 + (Y_E - d \cdot C\psi \cdot S\theta)^2 =$$
$$= 2b_2 \sqrt{\begin{matrix} (X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)^2 \end{matrix}} \quad (29)$$

By squaring relation (29) result:

$$\begin{bmatrix} (q_5 - Z_E - d \cdot C\theta)^2 - a_2^2 + b_2^2 + (X_E + d \cdot S\psi \cdot \\ \cdot S\theta - e_2)^2 + (Y_E - d \cdot C\psi \cdot S\theta)^2 \end{bmatrix}^2 -$$
$$-4b_2^2 \begin{bmatrix} (X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + \\ +(Y_E - d \cdot C\psi \cdot S\theta)^2 \end{bmatrix} = 0 \quad (30)$$

Introducing relations (20), (25) and (30) in the system of the implicit equations (15) result a new form of the equations as follow:

$$F_1(q_i, X_E, Y_E, \psi, \theta) \equiv$$
$$\left[(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \right]$$
$$\cdot S^2 q_1 - (Y_E + z_E \cdot C\psi \cdot S\theta)^2 = 0$$
$$F_2(q_2, Z_E, \theta) \equiv q_2 - h_1 - Z_E + z_E \cdot C\theta = 0$$
$$F_3(q_3, X_E, Y_E, Z_E, \psi, \theta) \equiv$$
$$\begin{bmatrix} (q_3 - Z_E + z_E \cdot C\theta - h_1)^2 - a_1^2 + b_1^2 + (X_E - \\ -z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \end{bmatrix}^2 - 4b_1^2 \cdot$$
$$\cdot\left[(X_E - z_E \cdot S\psi \cdot S\theta - e_1)^2 + (Y_E + z_E \cdot C\psi \cdot S\theta)^2 \right] = 0$$
$$F_4(q_4, Z_E, \theta) \equiv q_4 - Z_E - d \cdot C\theta = 0$$
$$F_5(q_5, X_E, Y_E, Z_E, \psi, \theta) \equiv$$
$$\begin{bmatrix} (q_5 - Z_E - d \cdot C\theta)^2 - a_2^2 + b_2^2 + (X_E + d \cdot \\ \cdot S\psi \cdot S\theta - e_2)^2 + (Y_E - d \cdot C\psi \cdot S\theta)^2 \end{bmatrix}^2 - 4b_2^2 \cdot$$
$$\cdot\left[(X_E + d \cdot S\psi \cdot S\theta - e_2)^2 + (Y_E - d \cdot C\psi \cdot S\theta)^2 \right] = 0$$
$$(31)$$

Deriving the equations of the systems (17) or (31) in relation with time result:

$$A\dot{X} + B\dot{q} = 0 \quad (32)$$

where:

$$\dot{X} = \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix},$$

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \quad (33)$$

Expression (32) can be written as follow:

$$\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} +$$

$$+ \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = 0$$

(34)

By multiplying with A^{-1} and B^{-1}, from relation (32) obtain:

$$\dot{X} = -A^{-1} \cdot B\dot{q} \quad (35)$$

$$\dot{q} = -B^{-1} A\dot{X} \quad (36)$$

Detailed forms of the expressions (35) and (36) are given by the next relation:

$$\begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix}^{-1} \cdot$$

$$\cdot \left\{ \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \right\} \quad (37)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix}\right\} \quad (38)$$

Deriving the expression (32) in relation with time result:

$$\dot{A}\dot{X} + A\ddot{X} + \dot{B}\dot{q} + B\ddot{q} = 0 \quad (39)$$

By multiplying with A^{-1} respectively B^{-1} result:

$$\ddot{X} = -A^{-1}(B\ddot{q} + \dot{A}\dot{X} + \dot{B}\dot{q}) \quad (40)$$

$$\ddot{q} = -B^{-1}(A\ddot{X} + \dot{A}\dot{X} + \dot{B}\dot{q}) \quad (41)$$

where:

$$\ddot{X} = \begin{bmatrix} \ddot{X}_P \\ \ddot{Y}_P \\ \ddot{Z}_P \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix}, \; \ddot{q} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix},$$

$$\dot{\boldsymbol{A}} = \begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix},$$

$$\dot{\boldsymbol{B}} = \begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix}, \quad (42)$$

Expressions (40), (41) can be written as follow:

$$\begin{bmatrix} \ddot{X}_P \\ \ddot{Y}_P \\ \ddot{Z}_P \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} + \right.$$

$$+ \begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} +$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \Bigg\} \tag{43}$$

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_P \\ \ddot{Y}_P \\ \ddot{Z}_P \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} + \right.$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} +$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_5}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_1}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_2}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_3}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_4}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \Bigg\} \tag{44}$$

A few of partial derivatives written in system of implicit equation (29) are null. Matrix A, B and $\dot{A}$, $\dot{B}$ given the relations (31) and (40) become:

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ 0 & 0 & \frac{\partial F_2}{\partial Z_E} & 0 & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ 0 & 0 & \frac{\partial F_4}{\partial Z_E} & 0 & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \tag{45}$$

$$\dot{A}=\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix}$$

$$\dot{B}=\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & 0 & 0 & 0 & 0 \\ 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & 0 & 0 & 0 \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & 0 & 0 \\ 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & 0 \\ 0 & 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix} \tag{46}$$

Relations (37), (38), (43) and (44) become:

$$\begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ 0 & 0 & \frac{\partial F_2}{\partial Z_E} & 0 & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ 0 & 0 & \frac{\partial F_4}{\partial Z_E} & 0 & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix}\right\} \tag{47}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ 0 & 0 & \frac{\partial F_2}{\partial Z_E} & 0 & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ 0 & 0 & \frac{\partial F_4}{\partial Z_E} & 0 & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix}\right\} \tag{48}$$

$$\begin{bmatrix} \ddot{X}_P \\ \ddot{Y}_P \\ \ddot{Z}_P \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ 0 & 0 & \frac{\partial F_2}{\partial Z_E} & 0 & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ 0 & 0 & \frac{\partial F_4}{\partial Z_E} & 0 & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix}^{-1} \cdot$$

$$\cdot\left\{\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} + \right.$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} +$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & 0 & 0 & 0 & 0 \\ 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & 0 & 0 & 0 \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & 0 & 0 \\ 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & 0 \\ 0 & 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \Bigg\} \tag{49}$$

$$\ddot{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} \end{bmatrix}^{-1} \cdot$$

$$\cdot \left\{ \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ 0 & 0 & \frac{\partial F_2}{\partial Z_E} & 0 & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ 0 & 0 & \frac{\partial F_4}{\partial Z_E} & 0 & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix} \cdot \begin{bmatrix} \ddot{X}_P \\ \ddot{Y}_P \\ \ddot{Z}_P \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} + \right.$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial Y_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_1}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_3}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_3}{\partial \theta}\right) \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial Z_E}\right) & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial \theta}\right) \\ \frac{d}{dt}\left(\frac{\partial F_5}{\partial X_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Y_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial Z_E}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \psi}\right) & \frac{d}{dt}\left(\frac{\partial F_5}{\partial \theta}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix}$$

$$+\begin{bmatrix} \frac{d}{dt}\left(\frac{\partial F_1}{\partial q_1}\right) & 0 & 0 & 0 & 0 \\ 0 & \frac{d}{dt}\left(\frac{\partial F_2}{\partial q_2}\right) & 0 & 0 & 0 \\ 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_3}{\partial q_3}\right) & 0 & 0 \\ 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_4}{\partial q_4}\right) & 0 \\ 0 & 0 & 0 & 0 & \frac{d}{dt}\left(\frac{\partial F_5}{\partial q_5}\right) \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \Bigg\} \tag{50}$$

4. CONCLUSION

The paper presents the concept of an advanced parallel robot designed for medical application. The relations presented give the equations for the kinematic model of this parallel mechanism. Unlike other robots designed for minimally invasive surgery, this robot can position the laparoscope without support on the abdominal wall. The design of this robot allows a large working space. Also, this robot can have attached an orientation system designed for active surgical instruments. Then, two parallel robots would act as the right hand and left hand of the surgeon and the third one can be used for laparoscopic camera positioning in surgery field. This robotic structure is the objective of a patent presented in [10].

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MODELAREA CINEMATICĂ A UNUI NOU ROBOT PARALEL CU CINCI GRADE DE LIBERTATE DESTINAT CHIRURGIEI MINIM INVAZIVE

Abstract: *Lucrarea prezintă cinematica unei structuri paralele inovative destinată pentru manipularea instrumentelor chirurgicale în chirurgia minim invazivă. Printre avantajele utilizarii robotilor paraleli in chirurgie ar putea fi subliniate: precizia miscărilor, absența tremourului natural, precizie, o bună vizualizare a câmpului operator de către chirurg, reduce oboseala ochilor, nu mai este nescesară prezența celui de al doilea chirurg in timpul procedurii chirurgicale.*

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