TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 54, Issue I, 2011

CONTRIBUTION TO THE DETERMINING OF THE TRAJECTORIES DESCRIBED BY THE ENDS OF CONVEYORS ELASTIC ELEMENTS

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Abstract: *In this paper the authors have determined the trajectories of the points from the vibratory plate, considering the case of large oscillation. The numerical results were obtained with a C program. The paper also presents the graphical representation of the trajectories described by the end of the elastic element for two cases of constraints.*
Key words: *lamellar spring, linear vibratory conveyor, vibration, C program.*

1. INTRODUCTION

The transportation between two work places of different objects and materials (machine parts, food and pharmaceutical products) can be performed in many cases using vibratory conveyors.

In figure 1 and 2 are presented two different types of these machines:

Fig.1

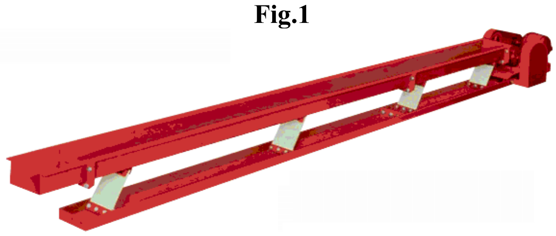

Fig.2

An exact study of the vibratory conveyor movements is based on the exact mathematical model, as close to reality as possible, followed by the exact numerical computational method to solve the simultaneous differential equations of first order.

In literature [7] [9] [14], was studied the problem of the relative movement of the parts transported on the vibratory conveyor plate, using the well known differential equation of the relative movement. In this differential equation appeared the term: inertial transport force F_{it}.

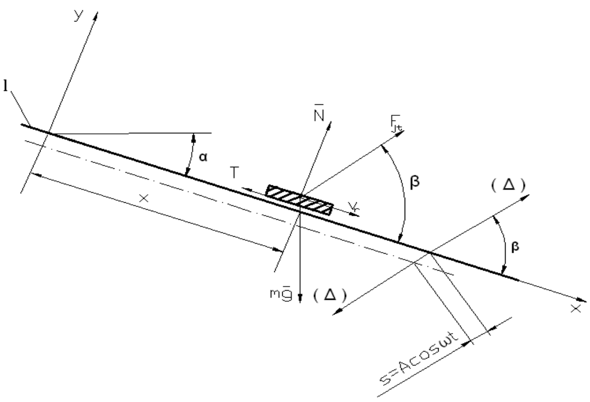

Fig.3

The vibratory conveyor plate makes a rectilinear motion of harmonic law and the inertial force has a fixed direction.

The presumption is correct if the amplitude is small and linear and has rectilinear direction.

In the case of vibrating plate displacement with large amplitudes, due to the crank mechanism or to the inertial generator the assumption about the fixed direction of the inertial transport force has to be abandoned.

The trajectories described by the vibratory plate points will be established exactly and as consequence the direction of inertial transported force can be calculated, in each point if the speed is known.

There are three possible modes of vibratory plate displacement with respect of cradle, depending of the elements that link the conveyor plate with the fixed element.

1. The conveyor plate is linked with the cradle through articulated bars and the plate is actuated by a crank mechanism. In this case all the trajectories are circular.

2. The second possible case is when the vibratory plate is linked with the cradle through elastic bars embedded in cradle and articulated in the plate. The plate is actuated by the inertial generator.

3. The third case consists of the vibratory plate linked with the cradle through the elastic bars embedded at both ends. The inertial generator is also used to actuate the plate.

In the following paragraph are determined the trajectories described by the points belonging to the vibratory plate considering the cases 2 and 3.

2. THEORETICAL ASPECTS

In figure 4 and 5 are represented the shapes of the elastic element embedded at one end and articulated at the other and the shape of the elastic element embedded at both ends when the forces F_1 respectively F_2 and the moment M_2 are acting on the ends of these elements.

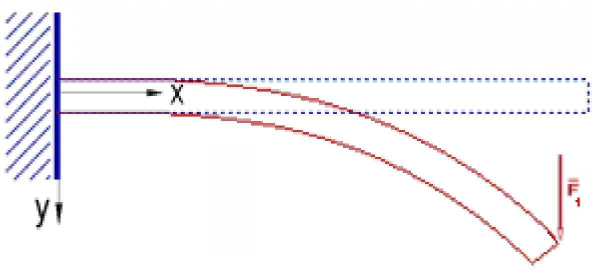

Fig.4

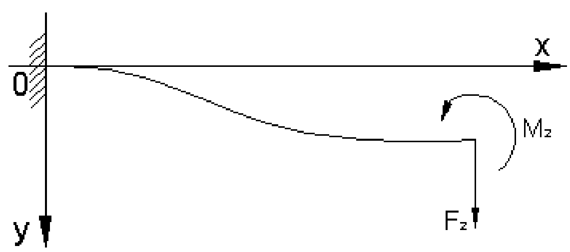

Fig.5

Because, in both cases the elastic bars in point O has a horizontal tangent result that they will be expressed considering the third degree polynomial in which is missing the first degree and the free term.

$$y = ax^3 + bx^2 \qquad (1)$$

The values for the coefficients a_1, b_1 and a_2, b_2 define the deformed shape of the elastic element and they depend of the load at the end of these elements.

Knowing the formula of the deformed shape and the angle of deflection for the first case

$$y = \frac{F_1 L^3}{3EI}, \ \frac{dy}{dx} = \frac{F_1 L^2}{2EI} \qquad (2)$$

on obtains the following system (x=L)

$$\begin{cases} a_1 L^3 + b_1 L^2 = \dfrac{F_1 L^3}{3EI} \\ 3a_1 L^2 + 2b_1 L = \dfrac{F_1 L^2}{2EI} \end{cases} \qquad (3)$$

and considering the system, we can determine the coefficients a_1 and b_1 obtaining

$$a_1 = -\frac{F_1 L^3}{6EI}, \ b_1 = \frac{F_1 L}{2EI}$$

The values for the shape deformation in every point of the elastic element can be determined with the following formula:

$$y = -\frac{F_1}{6EI} x^3 + \frac{F_1 L}{2EI} x^2 \qquad (4)$$

In the second case (the elastic element is embedded at both ends), the moment is: $M_2 = \frac{F_2 L}{2}$, if at the right end the tangent is horizontal.

The method used to solve the problem is similar as in the first case. The coefficients a_2 and b_2 from the system were determined and have the following expressions:

$$\begin{cases} a_2 L^3 + b_2 L^2 = \dfrac{F_2 L^3}{12EI} \\ 3a_2 L^2 + 2b_2 L = 0 \end{cases} \qquad (5)$$

After the solving of the system result the coefficients values a_2 and b_2 are:

$$a_2 = -\frac{F_2}{6EI}, \ b_2 = \frac{F_2 L}{4EI}$$

The equation of the medium fiber is:

$$y = -\frac{F_2}{6EI}x^3 + \frac{F_2 L}{4EI}x^2 \quad (6)$$

Depending on the elastic element material that is used and knowing the allowed strength σ_{adm}, were determined, for both cases of constraints, the maximal value of the forces F_1 and F_2. Considering the intervals $[0, F_{1max}]$, $[0, F_{2max}]$, for each value of the two forces, result different values for the coefficients a_1, b_1, a_2 and b_2. The right end of the elastic element may be situated in various positions on the trajectory that can be described by it. Obviously, the trajectories are not rectilinear or circular, but their shapes will be determined.

The length of elastic element is noted with L and with x_{max} is noted the abscissa of the elastic element end.

For each value of the force were determined the values for the abscissa x_{max} using the following equation:

$$f(x_{max}) = L - \int_0^{x_{max}} \sqrt{1 + (3ax^2 + 2bx)^2}\,dx = 0, \quad (7)$$

were L, a and b are known values. This equation has one root in the interval (0, L) and was numerically solved [14].

The values of the definite integral, from equation (7) were calculated with a numerical method of generalized rectangles.

For each value of the abscissa x_{max} and knowing the equations (4) and (6) have been determined the values for the coordinates y of the elastic element.

The values gradually determined for the abscissa x_{max} and the corresponding ordinate y in the two cases presented in this paper, will be saved in files. The files can be accessed during the numerical solving of the differential equation system of the particle relative motion. The inertial force direction varies depending of the trajectory shape.

3. NUMERICAL RESULTS

The dimensions for the elastic element are:
- -L=1 [m] – the elastic element length;
- -d=0.050 [m] - the elastic element width;
- -h=0.04 [m]- the elastic element thickness.

The material of the elastic element is steel 51Si17A, STAS 795-87 and the allowed strength is $\sigma_{adm} = 600 \cdot 10^6$ [N/m²].

In the first case the force F_1 belongs to the interval [0, 80] N and in the second case the force F_2 belongs to the interval [0, 160] N.

In figures 6 and 7 are presented the trajectories described by the ends of the bars corresponding to the studied case.

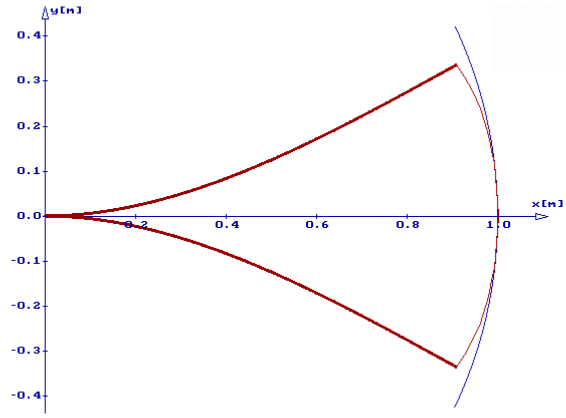

Fig.6 L=1 [m], d= 0.050 [m], h=0.004[m], F=80 [N]

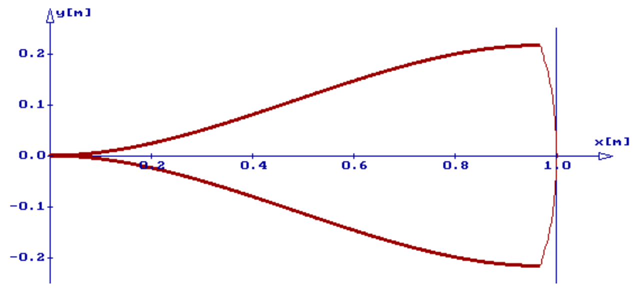

Fig.7 L=1 [m], d=0.050[m], h=0.004 [m], F=160 [N]

Observing the figures 6 and 7, results noticeable differences between the real trajectory described by the elastic beam ends considering large deflection compared to trajectories (rectilinear or circular) considered by different authors.

4. CONCLUSIONS

Based on the presented procedures and programmed in a C program, authors have solved the problem of the exact transport trajectory determination of the points belonging to the vibratory conveyor plate considering the case of the elastic element large oscillations.

The differential equation of relative movement of the parts will be solved based on the obtained numerical results.

In literature the case of large relative amplitude of the transport motion wasn't studied yet.

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CONTRIBUȚIA LA STABILIREA TRAIECTORIILOR DESCRISE DE CAPETELE ELEMENTELOR ELASTICE DIN COMPONENȚA VIBROTRANSPORTOARELOR

Rezumat: *În cadrul acestui articol s-a prezentat modul în care s-a determinat traiectoria de transport a punctelor plăcii vibrotransportoare în cazul unor oscilații mari. Rezolvarea acestei probleme a fost posibilă datorită unui program creat în limbajul de programare C de către autori. Lucrarea prezintă de asemenea şi reprezentările grafice ale traiectoriilor descrise de vârful elementului elastic pentru două cazuri de încastrare.*

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