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OPTIMAL DESIGN WITH EVOLUTIONARY ALGORITHMS OF A GEAR COUPLING

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Abstract: *In this paper an optimal design of a gear coupling is presented. The novelty of this paper consists in the optimal approach of the gear coupling. The chosen objective function was the mass of the gear coupling. For this optimization problem, eleven genes were taken into consideration and a set of twelve constraints were formulated. In solving the optimization problem we used a two-phase evolutionary algorithm (inspired from the evolutionary concept of "punctuated equilibrium") in a formulation that can be extended to include other important objectives.* **Key words:** *Evolutionary optimization, gear coupling design.*

1. INTRODUCTION

The main goal of this paper lies in emphasizing once again the advantages of the optimal design of all sorts of products as compared to the classical design. In this particular case we deal with the optimal design of a gear coupling (Figure 1). The complexity of a gear coupling lies in the strong and often intractable connections between the design variables defining its sub-systems. In other words, an optimal gear coupling is generally not an assembly of components optimized in isolation, a fact overlooked by many conventional design heuristics.

A series of researchers have reported solutions to this problem. Alfares et al. [1] present a study upon the clearance distribution between meshing teeth of the coupling and the effects of various geometrical parameters (angle of misalignment, tooth crowning, module, and pressure angle) on its performance. Drăghici and Olteanu in [2] presented a method for calculus and optimization of a gear coupling. Renzo et al. [6] investigated the contact characteristics of meshing teeth, tooth load distribution, coupling load capacity, optimal backlash, and tooth bearing.

2. A TWO-PHASE EVOLUTIONARY ALGORITHM

For solving the optimization problem we used a two-phase evolutionary algorithm [7] inspired from the evolutionary concept of "punctuated equilibrium" [4] (a theory about how new species evolve [3]). The main idea behind this algorithm is its operation in two phases. In each phase, the individual's fitness is determined by another factor. In *Phase* 1, the individual's fitness depends only on the way in which an individual is more suitable (or not) in terms of constraints. This phase is a kind of "feasible individual generator". The algorithm moves into the second phase when the number of feasible individuals of the population exceeds a preset threshold. *Phase* 2 is a common evolutionary algorithm. In the following we present, in short, how to determine an individual's fitness in both phases of the algorithm. The optimization problem consists of an objective function *f* accompanied by certain number of constraints. The search space is considered the space of the *n* – dimensional decision vectors:

$$\bar{x} = \left(x^{(1)}, x^{(2)}, \ldots, x^{(n)}\right) \qquad (1)$$

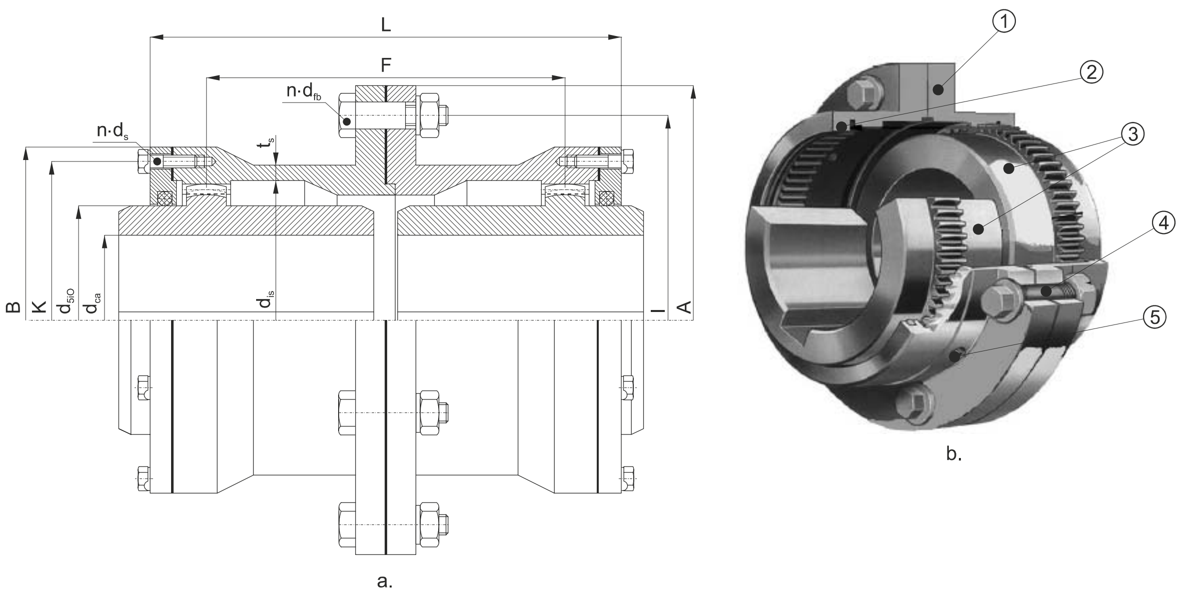

Fig. 1 The sketch of the gear coupling
1 – sleeve, **2** – coupling seal (O – ring), **3** – hub, **4** – fitted bolts, **5** – lubrication fitting

Table 1

The 11 design variable describing the gear coupling

Symbol	Range	Description
D_p	∈ {50 … 561}	Preliminary pitch diameter of the gear coupling. Integer values.
iO	∈ {0 … 63}	Catalogue index of standardized coupling seal (O – rings). Integer values
cK	[1.05 … 1.3]	Screw arrangement diameter (*K*) to pitch diameter ratio. Real values.
cB	[1.2 … 1.4]	Hub exterior diameter (*B*) to pitch diameter ratio. Real values.
n_s	∈ {4, 6, 8, 10, 12, 14, 16, 18}	Number of fitted bolts. Integer values.
id_{fb}	∈ {10, 12, 14, 16}	Diameter of the fitted bolt (d_{fb}). Integer values.
cI	[1.35 … 1.45]	Fitted bolt arrangement diameter (*I*) to pitch diameter ratio. Real values.
cA	[1.6 … 1.7]	Flange diameter (*A*) to pitch diameter ratio. Real values.
ct	[0.04 … 0.095]	Coupling sleeve thickness (*t*) to pitch diameter ratio. Real values.
cF	[0.8 … 1.1]	Tooth center distance (*F*) to pitch diameter ratio. Real values.
cL	[2.255 … 2.64]	Coupling length (*L*) to pitch diameter ratio. Real values.

The constraints of the problem are: n_u – *inequality type constraints*: $g_i(\bar{x}) \le 0,\ i = \overline{1, n_u}$; n_s, – *strict inequality type constraints*: $g_i(\bar{x}) < 0,\ i = \overline{n_u + 1, n_u + n_s}$ and n_e – *equality type constraints*: $g_i(\bar{x}) = 0,\ i = \overline{n_u + n_s + 1, n_u + n_s + n_e}$.

In order to use the use this constraints in our algorithm we needed to aggregate them in the following form:

$$G_i(\bar{x}) = \begin{cases} \left.\begin{matrix} 0, & g_i(\bar{x}) \le 0 \\ g_i(\bar{x}), & g_i(\bar{x}) > 0 \end{matrix}\right\} i = \overline{1, n_u} \\ \left.\begin{matrix} 0, & g_i(\bar{x}) < 0 \\ g_i(\bar{x}) + \varepsilon, & g_i(\bar{x}) \ge 0 \end{matrix}\right\} i = \overline{n_u + 1, n_u + n_s} \\ \left.\begin{matrix} 0, & g_i(\bar{x}) = 0 \\ \left|g_i(\bar{x})\right|, & g_i(\bar{x}) \ne 0 \end{matrix}\right\} i = \overline{n_u + n_s + 1, n_u + n_s + n_e} \end{cases} \quad (2)$$

In each phase, for each individual a so called score is computed. The partial score of an individual (from those *N* individuals of the population) $\bar{x}_j,\ j = \overline{1, N}$, regarding to the constraint *i*, $i = \overline{1, n_u + n_s + n_e}$ is calculated as follows: $PS_i(\bar{x}_j) = G_i(\bar{x}_j) \Big/ \sum_{k=1}^{N} G_i(\bar{x}_k)$. Eventually, the (individual) score of each individual $\bar{x}_j,\ j = \overline{1, N}$ of the population is: $S(\bar{x}_j) = \sum_{i=1}^{n_u + n_s + n_e} PS_i(\bar{x}_j)$. Obviously, any feasible individual has null score. During *Phase* 1 the population is sorted by the score, and during *Phase* 2, the population is sorted by score and objective value. In both phases the fitness of an individual is set according to its rank.

3. A GEAR COUPLING OPTIMIZATION PROBLEM

Let us now consider the following design problem. A gear coupling (Figure 1) is to be designed for minimum weight, given an input speed of 4500 RPM and a power of 4.5 kW. The shaft dimensions are Ø50 × 110 mm. The gear coupling (hubs and sleeves, Figure 1) is made of quenched and tempered 34CrMo4.

In the following we will consider the “genotype”, or the set of design variables, that describe the coupling design problem.

3.1 The “genotype” of the gear coupling

The set of 11 design variable that describe the gear coupling is shown in Table 2. The resulting design space is vast, around 3×10^{17} possible gear coupling designs (it is notoriously hard to gain an intuitive “feel” for such numbers). Two conclusions can be drawn from here. Firstly, it is clear that, although the computational cost of evaluating the objective function (the mass of the gear coupling) and the constraints for a given design is quite low, an exhaustive (full factorial) search of the design space is not feasible. Secondly, of all the search heuristics, a population-based evolutionary algorithm appears to be the most suitable.

3.2 The constraints of the optimization problem

There are a total of 12 constraints which should be viewed with reference to the sketch presented in Figure. 1. In the interest of conciseness we shall not dwell on the details of their calculation (the interest reader may find all the details about the sleeves and hubs toothing geometry in [5]). As it will become apparent, they are all of the inequality type, mostly involving geometrical or structural considerations. The solutions of the optimization problem have to satisfy those 12, constraints which are listed bellow (note that all values of these constraints have to be negative or at least zero).

C1. *The capable torque of the gear coupling should be greater then the nominal torque.*

C2. *The shearing stress of the gear coupling flanges must not exceed a specified value.*

C3,4. *The shearing and bearing stresses must not exceed a specified value on the fitted bolts (used for the assembly of the gear coupling) and mounting holes of the gear coupling flanges.*

C5,6. *The shearing and crushing stresses must not exceed a specified value on the keys* (*used for the assembly of the gear coupling*) *and keyways of the shafts.*

C7. *Geometrical constraint relating to the space required between two hubs of the gear coupling.*

C8. *The diameter* d_{5iO} *should be lower then the root diameter of the hub.*

C9. *The diameter* d_{is} *must be lower then the screws arrangement diameter K of the side cover.*

C10. *The screws arrangement diameter K of the side cover should be lower then the exterior diameter of the hub B.*

C11. *The diameter B must be lower then fitted bolts arrangement diameter I.*

C12. *The fitted bolts arrangement diameter I must be lower then the exterior diameter A of the coupling flange.*

4. RESULTS

The values of all considered genes, after optimization are shown in Table 2.

Table 2

Gene values obtained after optimization

No.	Gene	Value	No.	Gene	Value
1	D_p	93	7	*cI*	1.447
2	*iO*	21	8	*cA*	1.603
3	*cK*	1.178	9	*ct*	0.0684
4	*cB*	1.284	10	*cF*	0.926
5	n_s	4	11	*cL*	2.64
6	id_{fb}	0			

In Table 3 the geometrical dimensions of the optimal design solution are shown (these should be viewed with reference to the sketch presented in Figure. 1).

Table 3

Geometrical dimensions of the optimal solution of the gear coupling

d_{ca}	D	d_{5iO}	d_{is}	K	B
50	95	70	102	112	122

d_{fb}	I	A	t	F	L
10	137	152	6	88	132

The optimal solution of the gear coupling weighted 8.51 kg. This solution was very near to four additional constraint boundaries. These are highlighted in Figure 2. In this figure the *x* symbols indicate the values of the constraints of the problem at the optimum design. With black points are highlighted the four constraints whose boundaries are closest to that optimum (note that the value g_i of constraint number *i* is defined as $g_i = a_i / b_i - 1$, where the constraint is of the form $a_i < b_i$).

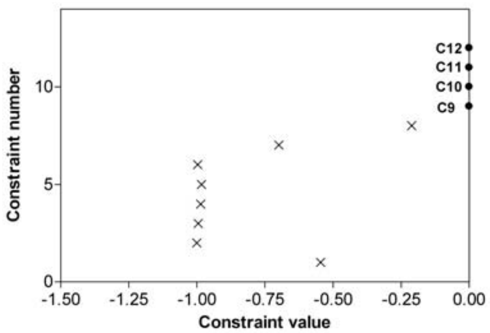

Fig. 2 The values of the constraints for the optimal design solution

5. CONCLUSIONS

In this paper we have shown how an evolutionary algorithm, can make solving such complex structural design problems. There is room for further development in terms of the fidelity of the analysis, which drives the optimization process described here. Also other objective functions could also be considered – coupling load capacity or manufacturing cost are a simply two potential examples.

6. ACKNOWLEDGMENT

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7. REFERENCES

[1] Alfares, M.A., Falah, A.H., Elkholy, A.H., *Clearance distribution of misaligned gear coupling teeth considering crowning and geometry variations*, Mechanism and Machine Theory, Elsevier, Vol. 41, 2005.11.004, p. 1258–1272, 2006.

[2] Drăghici, I, Olteanu, F., *Metoda generală pentru calculul cuplajelor dințate cu optimizarea soluțiilor constructive*, ICTCM, Bucureşti, 1981.

[3] Eldredge, N., Gould, S. – *Punctuated equilibria: An alternative to phyletic gradualism*, in Schopf, T. (editors) Models in Paleontology. San Francisco: Freeman Cooper, 1972.

[4] McGrath, K.A., Blachford, S. (editors) – *The Gale Encyclopedia of Science*, Gale Group, 2003.

[5] Mony, L., *Cuplaje dințate în construcțiile moderne de maşini*, ICDT, Bucureşti, 1967.

[6] Renzo, P.C., Kaufman, S., De Rocker, D.E., *Gear couplings*, Journal of Engineering for Industry, Transactions of ASME, 1967.

[7] Tudose, L., Buiga O., Ştefanache C., Sobester A., *Automated optimal design of a two-stage helical gear reducer*, Structural and Multidisciplinary Optimization, Springer, Vol. 42, ISSN1615-147X, p. 429-435, 2010.

PROIECTAREA OPTIMALĂ CU AJUTORUL ALGORITMILOR EVOLUTIVI A UNUI CUPLAJ DINȚAT

În această lucrare este prezentată proiectarea optimală cu ajutorul algoritmilor evolutivi a unui cuplaj dințat destinat compensării nealinierilor unghiulare din liniile tehnologice. Noutatea acestei lucrări constă în abordarea optimală a problemei. Funcția obiectiv este masa cuplajului. Pentru această problemă de optimizare s-au luat în considerare 11 gene şi 12 restricții. Pentru rezolvarea problemei de optimizare s-a utilizat un algoritm evolutiv în două faze (inspirat după conceptul „punctuated equilibrium”) într-o formulare care permite luarea în considerare şi a altor obiective.

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