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AN ANALYSIS OF A FLUID FLOW IN A CHANNEL WITH UNIFORM HEATED WALLS

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Abstract: *A problem of a mixed convection flow in a channel filled with a porous medium is presented in the paper. The walls of the channel are heated by a uniform heat flux. Mathematical models are presented for the case of vertical parallel plates and in the case of an inclined channel. The velocity at the channel entrance is given for the upward (assisting) flow. Analytical and numerical solutions of the problem for different parameters involved are presented and discussed.The problem of replacing the fluid which saturates the porous matrix by a nanofluid is also considered by a mathematical model.*
Key words: *Mathematical Model, Mixed Convection, Porous Medium, Differential Equations, Nanofluids.*

1. INTRODUCTION

Many engineering applications are concerned to combined convective heat transfer in fluid saturated porous media. The interest in this subject has been stimulated, to a large extent, by the fact that thermally driven flows in porous media have considerable applications in mechanical, chemical and civil engineering. Applications include fibrous insulation, food processing and storage, thermal insulation of buildings, geophysical systems, electro chemistry, metallurgy, the design of pebble bed nuclear reactors, underground disposal of nuclear or non-nuclear waste, solar power collectors, geothermal applications, nuclear reactors, etc.

One of the fundamental problems concerning heat transfer in porous media is the mixed convection flow through a channel or past a given plate embedded in a fluid-saturated porous medium. A wide application of porous media in many practical applications can be found in the well known books by Bejan *et al.* [4], Ingham and Pop [11], Nield and Bejan [14] and Pop and Ingham [18].

The conventional heat transfer fluids, including oil, water and ethylene glycol mixture, are poor heat transfer fluids, since the thermal conductivity of these fluids play an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. An innovative technique for improving heat transfer by using ultra fine solid particles in the fluids has been used extensively during the last several years. The term nanofluid refers to these kinds of fluids by suspending nanoscale particles in the base fluid. Details about this subject were introducedin 1995 by Choi [8]. Nanofluids have attracted enormous interest from researchers due to their potential for high rate of heat exchange incurring either little or no penalty in the pressure drop. The convective heat transfer characteristics of nanofluids depends on the thermo-physical properties of the base fluid and the ultra fine particles, the flow pattern and flow structure, the volume fraction of the suspended particles, the dimensions and the shape of these particles.

The utility of a particular nanofluid for a heat transfer application can be established by suitably modeling the convective transport in the nanofluid (Kumar et al. [13]). The particles are different from conventional particles (millimeter or micro-scale) in that they stay in suspension in the fluid and no sedimentation occurs. Some numerical and experimental studies on nanofluids include thermal conductivity (Kang et al. [12]), separated flow, Abu-Nada [1] and convective heat transfer (see Tiwari and Das [19] , Abu-Nada and Oztop [2] and Oztop and Abu-Nada [17]). Studies on natural convection using nanofluids are limited and they are related with differentially heated enclosures.

Eastman et al. [10] have used pure copper nanoparticles of less than 10 nm in size and achieved a 40 percents increase in thermal conductivity for only 0.3 percent volume fraction of the solid dispersed in ethylene glycol. They showed the particle size effect and the potential of nanofluids with smaller particles. Comprehensive references on nanofluid can be found in the recent book by Das et al. [9] and in the review paper by Buongiorno [5]. A recent study of a mixed convection over a vertical plate embedded in a nanofluid was given by Ahmand and Pop [3].

Therefore, the aim of the present paper is to study the mixed convection flow through a channel with uniform heated walls and filled with a porous medium. Mathematical models of both, a clear Newtonian fluid which saturates the porous solid matrix and the case when the base fluid is replaced by a nanofluid, for a given assisting flow, are presented.

2. MATHEMATICAL MODELS OF A MIXED CONVECTION FLOW IN A CHANNEL

2.1 Mixed convection flow in a channel filled with a fluid-saturated porous medium

Consider the mixed convection flow in an inclined infinitely long two-dimensional channel bounded by parallel plane walls and filed with a fluid-saturated porous medium. The x axis is considered up lengthways and the y axis is oriented into the channel. The viscous fluid and porous media properties are constant except for the variation of density in the buoyancy term of the Darcy equation. The flow is fully developed and steady. The fluid has an uniform upward streamwise velocity at the channel entrance and the walls are at uniform heat flux, q, see Figure 1.

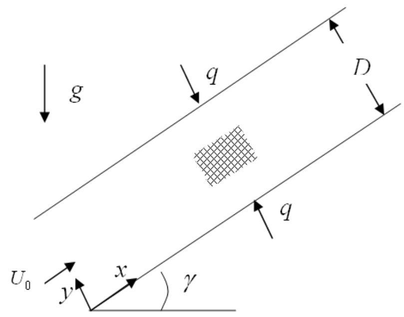

Fig. 1. The channel configuration

Under these assumptions, and with the use of the Darcy's law and the Boussinesq approximation, the governing equations are written as follows (see Cimpean [6]):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=\frac{g\beta K}{\nu}\left(\frac{\partial T}{\partial y}\sin\gamma-\frac{\partial T}{\partial x}\cos\gamma\right) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (3)$$

Where u and v are the cartesian velocity components, T is the fluid temperature. The coefficients are β the fluid thermal expansion, K the specific permeability of the medium, ν the kinematic viscosity and α_m the effective fluid thermal diffusivity. Also, the tilt angle, measured counterclockwise from the horizontal is denoted by γ in the considered equations.
The equations (1) - (3) are to be solved subject to the boundary conditions:

$$v=0,\ \frac{\partial T}{\partial y}=-\frac{q}{k}\ \text{ on } y=0 \text{ and}$$
$$v=0,\ \frac{\partial T}{\partial y}=\frac{q}{k}\ \text{ on } y=D \quad (4)$$

where q is the heat flux to the wall, D is the channel width and k is the thermal conductivity of the fluid.

2.2 Conservation equations for a porous medium saturated by a nanofluid

We consider, now, a nanofluid instead the classical Newtonian fluid. It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix.

A two-dimensional problem of a steady flow is considered. We select a coordinate frame in which the x-axis is aligned vertically upwards and take a vertical plate for y = 0. At this boundary the temperature T and the nanoparticle fraction Φ take constant values T_w and Φ_w, respectively. The ambient values, attained as y tends to infinity, are denoted by T_∞ and Φ_∞, respectively. The Oberbeck-Boussinesq approximation is employed, see Cimpean [6]. Homogeneity and local therma l equilibrium in the porous medium is assdumed, as for the previous case. We consider a porous medium whose porosity is denoted by ϵ and permeability by K. The Darcy velocity is denoted by **v**= (u, v). The following four field equations embody the conservation of total mass, momentum, thermal energy, and nanoparticles, respectively. The field variables are the Darcy velocity **v**, the temperature T and the nanoparticle volume fraction Φ (see Nield and Kuznetsov [16]):

$$\nabla \cdot \mathrm{v} = 0 \tag{5}$$

$$\frac{\mu}{K}\mathrm{v} = \{\Phi\rho_p + (1-\Phi)\rho_f[1-\beta(T-T_0)]\}\mathrm{g} - \nabla p \tag{6}$$

$$(\rho c)_f \mathrm{v}\cdot\nabla T = \varepsilon(\rho c)_p[D_B\nabla\Phi\cdot\nabla T + k_m\nabla^2 T + +(D_T/T_\infty)\nabla T\cdot\nabla T] \tag{7}$$

$$\frac{1}{\varepsilon}\mathrm{v}\cdot\nabla\Phi = D_B\nabla^2\Phi + (D_T/T_\infty)\nabla^2 T \tag{8}$$

Here ρ_f, μ and β are the density, viscosity, and volumetric volume expansion coefficient of the fluid while ρ_p is the density of the particles. The gravitational acceleration is denoted by **g**. We have introduced the effective heat capacity $(\rho c)_m$, and the effective thermal conductivity k_m of the porous medium. The coefficients that appear in Eqs. (7) and (8) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T. Details of the derivation of these equations are given in the papers by Buongiorno [5] and Nield and Kuznetsov [15], [16].

We consider **v**= (u, v) and then the boundary conditions are given by

$$v = 0, T = T_w, \Phi = \Phi_w \ at\ y = 0$$
$$u = v = 0, T \to T_\infty, \Phi \to \Phi_w \ as\ y \to \infty \tag{9}$$

2.3 Mixed convection flow of a nanofluid in a channel filled by porous medium

We consider now, the fully developed mixed convection flow of a nanofluid in an inclined channel. The channel configuration is considered as in Figure 1. By following the paragraph 2.2 we write the governing equations of the problem, as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$\frac{\partial p}{\partial x} + \frac{\mu}{K}u = -\{\Phi\rho_p + (1-\Phi)\rho_f[1 - -\beta(T-T_0)]\}g_x \sin\gamma \tag{11}$$

$$\frac{\partial p}{\partial y} = -\{\Phi\rho_p + (1-\Phi)\rho_f[1 - -\beta(T - T_0)]\}g_y \cos\gamma \tag{12}$$

$$u\frac{\partial T}{\partial x} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \varepsilon\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\nabla\Phi\cdot\nabla T + (D_T/T_0)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{13}$$

$$\frac{1}{\varepsilon}u\frac{\partial\Phi}{\partial x} = D_B\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right) + (D_T/T_0)\frac{\partial^2 T}{\partial y^2} \tag{14}$$

The equations (10) - (14) are to be solved subject to the boundary conditions:

$$v = 0, \frac{\partial T}{\partial y} = -\frac{q}{k_m}, \Phi = \Phi_1 \text{ on } y = 0 \text{ and}$$
$$v = 0, \frac{\partial T}{\partial y} = \frac{q}{k_m}, \Phi = \Phi_2 \text{ on } y = D \tag{15}$$

Here q is the heat flux to the walls of the channel, D is the channel width, Φ_1 and Φ_2 are the nanoparticles volume fractions to the lower and upper wall, respectively and k_m is the effective thermal conductivity of the porous medium.

We present in the table bellow, the thermo-physical properties for different phases. We consider cooper nanoparticles for improving the fluid properties.

Table 1.

Thermo-physical properties of different phases

Property	Fluid phase (water)	Solid phase (copper)

c_p (J/kgK)	4179	383
ρ (kg/m3)	997.1	8954
k (W/mK)	0.6	400
β(K^{-1})	2.1×10^{-4}	1.67×10^{-5}

3. ANALYSIS OF THE MODEL

To analyze the model from the fluid mechanics side, in the equations (1) – (3), we introduce, first, the following non-dimensional variables:

$$X = \frac{x}{D},\ Y = \frac{y}{D},\ U = \frac{u}{U_0},\ \theta = \frac{T - T_0}{qD/k} \qquad (16)$$

Further, by taking into account the given fully developed flow, in the model presented in paragraph 2.1, we consider the non-dimensional velocity $U = U(Y)$ and the non-dimensional temperature $\theta(X,Y) = C_1 X + F(Y)$. Then, by following the paper by Cimpean *et al.* [7], from the given conditions, we have $C_1 = 2/Pe$ and a third order ordinary differential equation is obtained:

$$\frac{d^3F}{dY^3} - (2\lambda \sin\gamma)\frac{dF}{dY} + \frac{4\lambda\cos\gamma}{Pe} = 0 \qquad (17)$$

which has to be solved, subject to the boundary conditions:

$$\frac{dF}{dY} = -1 \ at\ Y = 0,\ \frac{dF}{dY} = 1\ at\ Y = 1 \qquad (18)$$

In the Eq. (17) the parameters are the mixed convection parameter $\lambda = \frac{g\beta Hq D}{U_0 \nu k}$ and the *Péclet* number $Pe = \frac{U_0 D}{\alpha_m}$, respectively. We assume that $Pe > 0$ throughout.

We, also, consider channels inclined in an upward direction then we can limit γ to the range $0 \leq \gamma \leq \pi/2$. Hence, in Eq. (17) we have the terms $\sin\gamma \geq 0$ and $\cos\gamma \geq 0$. Also, we notice that for γ=0 we have a horizontal channel.

4. AN ANALYTICAL SOLUTION

4.1 General case γ > 0

Further, we will take into account the general case of the inclination of the channel (γ > 0) and the analytical solutions for this case are given by:

$$\frac{dF}{dY} = \frac{2\cot\gamma}{Pe} + \left(1 - \frac{2\cot\gamma}{Pe}\right)\frac{\sinh\alpha Y}{\sinh\alpha} - \left(1 + \frac{2\cot\gamma}{Pe}\right)\frac{\sinh\alpha(1-Y)}{\sinh\alpha} \qquad (19)$$

where $\alpha = \sqrt{2\lambda\sin\gamma} > 0$.

The velocity profile has the expression:

$$U(Y) = \frac{\alpha}{2\sinh\alpha}\left(1 - \frac{2\cot\gamma}{Pe}\right)\cosh\alpha Y + \left(1 + \frac{2\cot\gamma}{Pe}\right)\cosh\alpha(1-Y) \qquad (20)$$

On integrating Expression (19) and applying the forced flow condition $\int_0^1 F(Y)U(Y)dY = 0$ (see Cimpean *et al.* [7]), we obtain the temperature expression:

$$F(Y) = \frac{1}{\alpha\sinh\alpha}\left(1 - \frac{2\cot\gamma}{Pe}\right)\cosh\alpha Y + \frac{1}{\alpha\sinh\alpha}\left(1 + \frac{2\cot\gamma}{Pe}\right)\cosh\alpha(1-Y) + \frac{2\cot\gamma}{Pe}Y + A_0 \qquad (21)$$

where the constant A_0 is obtained from the given conditions and has the form:

$$A_0 = -\frac{(\cosh\alpha + 1)(\sinh\alpha + \alpha)}{2\alpha\sinh^2\alpha} - \frac{\cot\gamma}{Pe} - \frac{2\cot^2\gamma}{Pe^2} - \frac{2\cot^2\gamma(\cosh\alpha - 1)(3\sinh\alpha - \alpha)}{Pe^2\,\alpha\sinh^2\alpha} \qquad (22)$$

4.2 Vertical channel γ = π/2

For the vertical channel, the solutions from expressions (19) and (20) become:

$$\frac{dF}{dY} = \frac{\sinh\alpha Y}{\sinh\alpha} - \frac{\sinh\alpha(1-Y)}{\sinh\alpha}$$

$$U(Y) = \frac{\alpha}{2\sinh\alpha}[\cosh\alpha Y + \cosh\alpha(1-Y)] \qquad (23)$$

4.3 Results

The analytical solution given by (19) - (21) were, also, obtained numerically and are graphically presented by using the Matlab programming. The Figures 2 and 3 show the velocity and temperature profiles, respectively. We consider the numerical results for the inclination of the channel $\gamma = \pi/4$ for Péclet number Pe = 10.0. The velocity profiles increase to the walls and decrease to the center of the channel, for all the parameters considered. Also, we notice that, for a vertical channel, the profiles are symmetric about the center line of the channel.

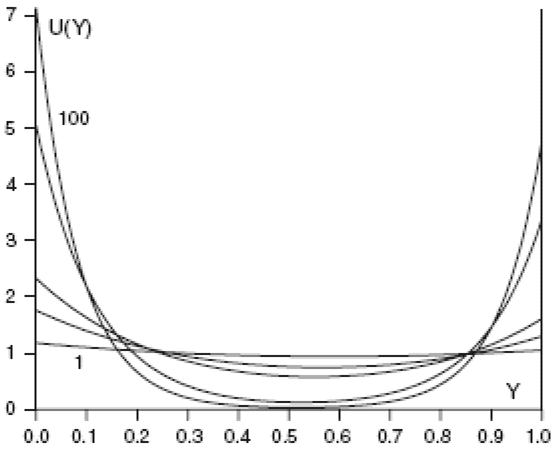

Fig. 2. The velocity profiles for a channel inclined at an angle $\gamma = \pi/4$, for λ = 1, 5, 10, 50, 100 and for Péclet number Pe = 10.0.

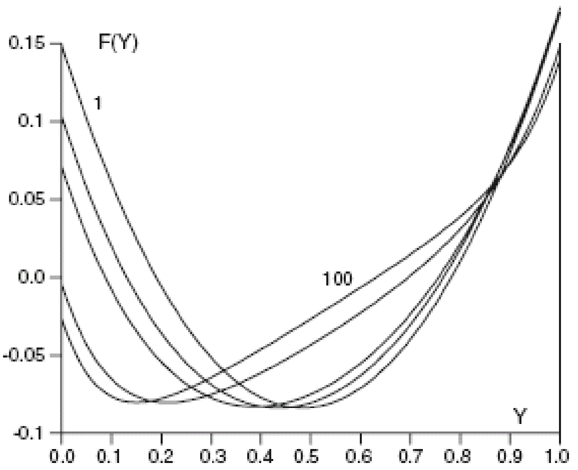

Fig. 3. The temperature profiles for a channel inclined at an angle $\gamma = \pi/4$, for λ = 1, 5, 10, 50, 100 and for Péclet number Pe = 10.0.

5. CONCLUSIONS

The paper presents a problem of the steady, fully developed mixed convection flow which arises between parallel plates filled with a porous medium, when both plates are subject to the same constant heat flux. Mathematical models for both, the classical fluid and a nanofluid, respectively, are given with the corresponding boundary conditions. Some analytical results are presented in terms of a mixed convection parameter λ and the Péclet number Pe, as well as the inclination angle γ to the horizontal. We found that, for vertical channels ($\gamma = \pi/2$), the solution is independent of the Péclet number and is symmetric about the centre line. Also, we conclude that, there was always a temperature increase across the channel, see Figure 3.

The basic assumption behind our models is that the flow develops from some (uniform) inflow at the entrance to the channel, with our solution then being valid at large distances along the channel from this entrance region.

For a future work, by considering the presented models, we intend to show and discuss that, the using of a small concentration of nanoparticles, in a base fluid, enhances considerably the thermal properties of the fluid.

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O analiza a unei miscari fluide intr-un canal cu pereti incalziti uniform

In lucrare se prezinta o problema de convective mixta a unei miscari fluide intr-un canal inclinat umplut cu mediu poros. Peretii canalului sunt incalziti de un flux uniform de caldura. Se prezinta modele matematice corespunzatoare pentru canal cu pereti verticali si pentru canal inclinat. Se considera cazul miscarii asistate cunoscandu-se viteza initiala la intrarea fluidului in canal. Se discuta solutii numerice si analitice ale problemei pentru diferiti parametri implicati in problema. Se considera de asemenea, un model matematic corespunzator inlocuirii cu un nanofluid a fluidului clasic ce satureaza matricea solida poroasa.

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