



## INVERSE AND DIRECT GEOMETRIC MODEL OF A NEW PARALLEL ROBOT WITH SIX DEGREES OF FREEDOM AND THREE GUIDING KINEMATIC CHAINS OF THE PLATFORM

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*ABSTRACT: The paper presents a new parallel robot with six degrees of freedom and three guiding kinematic chains of the platform. Starting from an initial solution with actuators on levels one and three, a new configuration is proposed in order to position all the actuators on the fixed platform. The inverse and direct geometric models are presented. While the inverse geometric model is obtained in closed form, the direct geometric model cannot be solved in closed form, the direct geometric solutions being computed using the Newton-Raphson numerical method. The robot can be used in assembly tasks and as a module for minimally invasive surgical systems.*

*Key Words: parallel robot, six freedom degrees, kinematics chain*

### 1. INTRODUCTION

A parallel manipulator typically consist of a moving platform and a fixed base that are connected together by several legs. Because of the closed-loop architecture not all joints can be independently actuated. In general, the number of actuated joints is equal to the number of degrees of the robot.

Though the serial robots possess a large workspace, the positioning capability is rather poor. So, where high load carrying capacity and precise positioning is the prime concern, an alternative is provided by parallel actuated and closed-loop robots. Parallel robots provide high structural rigidity, load carrying capacity, and good positioning capabilities.

Various six DOF manipulators have been considered in literature [6,9]. In [7] is presented a 6 DOF manipulator consisting of three inextensible links, one end of each link being connected to a planar actuator using a spherical joint and at the other end by a revolute joint. In [8] the authors propose a novel six degrees of freedom system – a tripod structure with inextensible limbs actuated at the

base by two-dimensional linear stepper motors. Similar 6 DOF manipulators having different joint geometries have been considered in [3, 18, 19]. A tripod like manipulator with extensible limbs and rotary motors at the base was considered in [4]. A parallel manipulator with three active revolute joints and three active prismatic joints was proposed in [20].

In [1] and [2] are presented new architectures of a six degrees of freedom parallel robot suitable for micro-electromechanical systems (MEMS) fabrication. The robots consists of revolute joints for the passive joints, and linear actuators located at the base for the active ones, both being characterized by the fact that are easier to manufacture in MEMS technology.

In [12, 13, 15, 16] the authors present several parallel microrobots with six degrees of freedom and two guiding kinematic chains [15], three guiding kinematic chains [12], four and five guiding kinematic chains [13] and respectively with six guiding kinematic chains of the platform [16] with linear active joints (piezoelectric linear actuators) and elastic passive joint. In [14] is presented a structure with five degrees of freedom where three

degrees of freedom are for general positioning of the camera, the active couples being mounted on the first level, module that is similar to MOTBAT6 (structure presented in this paper) mounting active couples and connection between the rods of the each limb. The robot structure is based on a registered patent [17].

The determination of the inverse geometric model is in the case of parallel robots, relatively easy. Some general methods have been developed in [11]. Due to non-linearities in the equations, the direct geometric model is rather difficult to obtain. Some numeric methods, like Newton-Raphson, have been successfully used [10]. In parallel robot research area, the problem of Jacobian matrix determination is an open and interesting problem. Indeed, the inverse Jacobian matrix computation is known [5, 10] but its analytical expression remains relatively complex. Its expression is generally obtained by a numerical method using a classic algorithm of matrix inversion or by a method based on a iterative scheme.

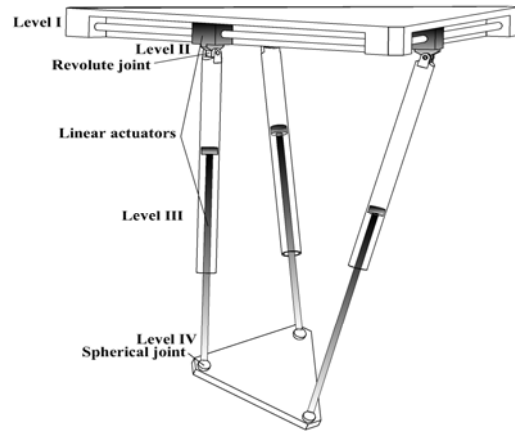
The paper is organized as follows: Section 2 is dedicated to the description and design of the parallel robot with six degrees of freedom and three guiding kinematic chains of the platform. The inverse and direct geometric problems are the subject of the Section 3. In the last section are presented the conclusions.

**2. DESIGN CONSIDERATIONS**

A definite advantage of parallel manipulators is the fact that in most cases actuators can be placed on or near the base, thus imposing a limited weight on the moving parts, which makes it possible for parallel robots to achieve higher speed and acceleration. In addition, by using three limbs instead of six other benefits such as lower possibility of mechanical interference between limbs can be realized. For the fact that all of the actuators are base-mounted, higher payload capacity, smaller actuator size, and lower power dissipation can be obtained. Very often for spatial manipulators, the joints that connect the limbs and the moving platform are spherical joints. More precisely, the inertial position of the mobile platform center and the orientation of

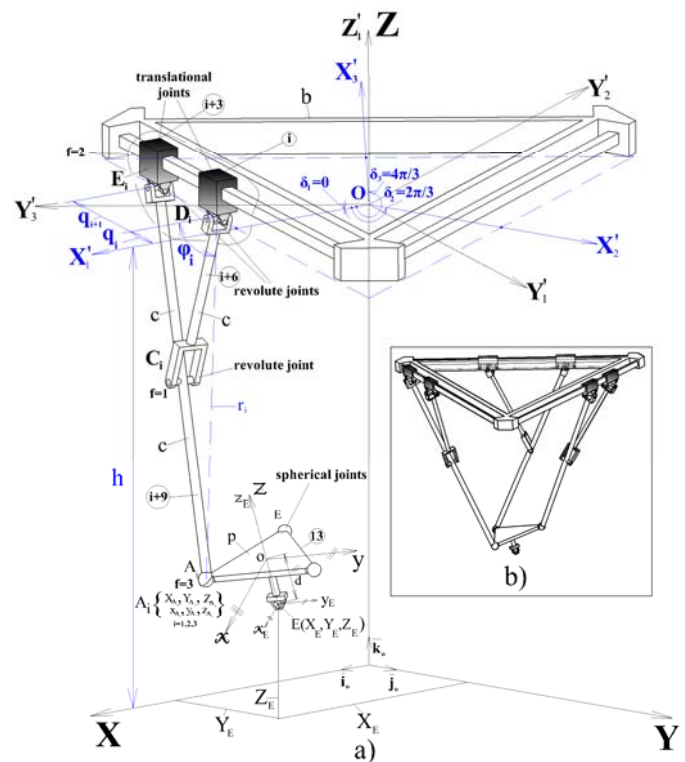
this platform is defined only by the coordinates of the three points  $A_1;A_2;A_3$ .

As a conclusion of the above, starting from an initial parallel structure (Figure 1) with 6 DOF and three guiding chains of the platform with the active linear joints placed on levels I and III



**Figure 1.** Parallel structure with 6 DOF and three guiding kinematic chains of the platform with active linear joints on levels I and III

in Figure 2b) is presented a structure of a new 6-DOF parallel robot with three guiding kinematic chains called MOTBAT6 (active couples/ motors mounted at the base).



**Figure 2. a)** Geometric scheme of MOTBAT6; **b)** Virtual prototype of MOTBAT6 parallel robot. The moving platform has an equilateral triangular form  $A_1A_2A_3$ . The two platforms are connected through three 2 DOF modules with the active joints positioned on the base. The connection of each limb to the base is achieved

by two prismatic joints at points  $D_i$  respectively  $E_i$  ( $i=1, 2, 3$ ), and to the mobile platform by a spherical joint at points  $A_i$  ( $i=1, 2, 3$ ). The robot has six linear actuated joints mounted at the base. There are also the following passive joints: six revolute joints between the active joints and each rod of each limb, and three revolute joints between rods (10, 11, 12) respectively rods (7, 8, 9). The geometric parameters of the robot are represented by  $h, b, \delta_1, \delta_2, \delta_3, c, p, d$  (Fig. 2a) and the coordinates of point E ( $X_E, Y_E, Z_E$ ).

### 3. THE GEOMETRIC MODEL

#### 3.1. The inverse geometric model

The inverse geometric model consist in the determination of the active coordinates (variables)  $q_i$  and  $q_{i+3}$  ( $i=1,2,3$ ) of the mechanism when the position and orientation of the manipulated object, respectively his coordinates are known:  $X_p, Y_p, Z_p, \psi, \theta, \varphi$ .

Starting from the relationship that defines the height of the equilateral triangle of the mobile platform:

$$A_3H = \frac{\sqrt{3}p}{2}$$

$$oH = \frac{1}{3}A_3H = \frac{p\sqrt{3}}{6}$$

Are obtained the  $A_i$  ( $i=1, 2, 3$ ) coordinate points in relation with the oxyz system attached to the mobile platform:

$$x_{A_1} = \frac{p\sqrt{3}}{6}, \quad y_{A_1} = -\frac{p}{2}, \quad z_{A_1} = 0$$

$$x_{A_2} = oH = \frac{p\sqrt{3}}{6}, \quad y_{A_2} = \frac{p}{2}, \quad z_{A_2} = 0$$

$$x_{A_3} = \frac{p\sqrt{3}}{3}, \quad y_{A_3} = 0, \quad z_{A_3} = 0,$$

And the point coordinates E to the same coordinate system  $OX_i, Y_i, Z_i$  (Fig. 2) are:

$$x_E = 0, \quad y_E = 0, \quad z_E = -d$$

With the notation for the equilateral triangle sides:

$$A_1A_2 = A_2A_3 = A_3A_1 = p$$

The variable radius on which the point  $A_i$  rotates will be:  $r_i = \sqrt{(2c)^2 - (q_{i+3} - q_i)^2}$ ,  $i=1, 2, 3$

The angles between the oxyz axis system attached to the fixed base and the OXYZ fixed

system of the robot are given in the table below:

**Table 1.**

	<b>ox</b>	<b>oy</b>	<b>oz</b>
<b>OX</b>	$\alpha'$	$\alpha''$	$\alpha'''$
<b>OY</b>	$\beta'$	$\beta''$	$\beta'''$
<b>OZ</b>	$\gamma'$	$\gamma''$	$\gamma'''$

Whose cosines are given by the table bellow:

**Table 2.**

$c\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$	$c\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi$	$c\alpha''' = s\psi s\theta$
$c\beta' = s\psi c\varphi + c\psi c\theta s\varphi$	$c\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$	$c\beta''' = -c\psi s\theta$
$c\gamma' = s\theta s\gamma$	$c\gamma'' = s\theta c\gamma$	$c\gamma''' = c\theta$

\* Where s stand for sine and c for cosine.

From the Table 1 results the following relations:

$$\begin{cases} \overline{EA}_i = (X_{A_i} - X_E)\bar{i}_0 + (Y_{A_i} - Y_E)\bar{j}_0 + (Z_{A_i} - Z_E)\bar{k}_0 \\ \overline{EA}_i = (x_{A_i} - x_E)\bar{i} + (y_{A_i} - y_E)\bar{j} + (z_{A_i} - z_E)\bar{k} \end{cases}$$

$$i=1, 2, 3 \tag{7}$$

Respectively, based on Table 2 are obtained the relationships:

$$(1) \begin{bmatrix} X_{A_i} - X_E \\ Y_{A_i} - Y_E \\ Z_{A_i} - Z_E \end{bmatrix} = \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}, i=1,2,3$$

$$(2)$$

From which are obtained the coordinates of the points  $A_i$ ,  $i=1, 2, 3$  according to the generalized coordinates of the end-effector:

$$(3) \begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}, i=1,2,3 \tag{9}$$

The angles between the axes attached to the fixed platform are:

$$\delta_1 = 0; \quad \delta_2 = \frac{2\pi}{3}; \quad \delta_3 = \frac{4\pi}{3} \tag{10}$$

(4) Using (10) is determined from (9):

$$(5) \begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \\ 1 \end{bmatrix} = \begin{bmatrix} c\delta_i & -s\delta_i & 0 & 0 \\ s\delta_i & c\delta_i & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X'_{A_i} \\ Y'_{A_i} \\ Z'_{A_i} \\ 1 \end{bmatrix} \tag{11}$$

(6)

$$\begin{cases} X_{A_i} = X'_{A_i} c\delta_i - Y'_{A_i} s\delta_i \\ Y_{A_i} = Y'_{A_i} s\delta_i + X'_{A_i} c\delta_i \\ Z_{A_i} = Z'_{A_i} + h \end{cases} \quad i=1, 2, 3 \quad (12)$$

$$\begin{cases} X'_{A_i} = r_i c\varphi_i = \sqrt{(2c)^2 - (q_{i+3} - q_i)^2} \cdot c\varphi_i \\ Y'_{A_i} = q_i \\ Z'_{A_i} = -r_i s\varphi_i = -\sqrt{(2c)^2 - (q_{i+3} - q_i)^2} \cdot s\varphi_i \end{cases} \quad i=1, 2, 3 \quad (13)$$

And the coordinates  $X'_{A_i}; Y'_{A_i}; Z'_{A_i}$ , ( $i=1, 2, 3$ ) of the point  $A_i$  depending on the coordinates  $X_{A_i}; Y_{A_i}; Z_{A_i}$ , ( $i=1, 2, 3$ ):

$$\begin{bmatrix} X'_{A_i} \\ Y'_{A_i} \\ Z'_{A_i} \\ 1 \end{bmatrix} = \begin{bmatrix} c\delta_i & s\delta_i & 0 & 0 \\ -s\delta_i & c\delta_i & 0 & 0 \\ 0 & 0 & 1 & -h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \\ 1 \end{bmatrix} \quad (14)$$

From (14) results the coordinates:

$$\begin{cases} X'_{A_i} = X_{A_i} c\delta_i + Y_{A_i} s\delta_i \\ Y'_{A_i} = -X_{A_i} s\delta_i + Y_{A_i} c\delta_i \\ Z'_{A_i} = Z_{A_i} - h_i \end{cases} \quad (15)$$

Where:

$$h_i = \frac{b\sqrt{3}}{6}, \quad i=1, 2, 3 \quad (16)$$

The equation of the mobile circle described by the variable radius is written below as a intersection between the variable radius  $r_i$  and the plane  $Y_{A_i}=-q_i$ :

$$\begin{cases} (X'_{A_i} - X'_{D_i})^2 + (Z'_{A_i} - Z'_{D_i})^2 = r_i^2 \\ Y'_{A_i} = -q_i \end{cases} \quad (17)$$

$$X'_{D_i} = O'_i H'_i = \frac{p\sqrt{3}}{6}, \quad Y'_{D_i} = -q_i \quad (18)$$

$$\left( X'_{A_i} - \frac{p\sqrt{3}}{6} \right)^2 + (Z'_{A_i})^2 = (2c)^2 - (q_{i+3} - q_i)^2, \quad i=1, 2, 3 \quad (19)$$

Using the relations (18) and (19) results the equations for the active coordinates:

$$q_i = -Y'_{A_i}, \quad i=1, 2, 3 \quad (20)$$

$$q_{i+3} = q_i \pm \sqrt{(2c)^2 - \left( X'_{A_i} - \frac{p\sqrt{3}}{6} \right)^2 - (Z'_{A_i})^2} \quad (21)$$

The coordinates of the  $A_i$ ,  $i=1, 2, 3$  point are determined in relation with coordinate system  $O'_i X'_i Y'_i Z'_i$ :

$$\begin{cases} X'_{A_i} = oH_i + r_i c\varphi_i = \frac{b\sqrt{3}}{6} + \sqrt{(2c)^2 - (q_{i+3} - q_i)^2} \\ Y'_{A_i} = -q_i \\ Z'_{A_i} = -r_i s\varphi_i = -\sqrt{(2c)^2 - (q_{i+3} - q_i)^2} \end{cases} \quad (22)$$

$$\begin{cases} u_i = s\varphi_i = -\frac{Z'_{A_i}}{r_i} = \frac{Z'_{A_i}}{\sqrt{(2c)^2 - (q_{i+3} - q_i)^2}} \\ w_i = c\varphi_i = \frac{X'_{A_i} - \frac{b\sqrt{3}}{6}}{r_i} = \frac{X'_{A_i} - \frac{b\sqrt{3}}{6}}{\sqrt{(2c)^2 - (q_{i+3} - q_i)^2}} \end{cases} \quad (23)$$

Using the relations (22) and (23) are determined the  $\varphi_i$  angles:

$$\varphi_i = a \tan 2(u_i, w_i), \quad i=1, 2, 3 \quad (24)$$

**Table 3.** The inverse geometric model for the 6 DOF and three guiding kinematic chains of the platform parallel robot. Algorithm

<b>Given:</b> $X_E, Y_E, Z_E, \psi, \theta, \varphi$										
<b><math>h, b, \delta_1, \delta_2, \delta_3</math> (<math>\delta_1 + \delta_2 + \delta_3 = 2\pi</math>), <math>c, p, d</math>;</b>										
<b>Unknowns:</b> $q_1, q_2, q_3, q_4, q_5, q_6, \quad (q_i = 1 \div 6)$										
<b>Variables:</b>	<b>Solving equations:</b>									
$x_{A_i}, y_{A_i}, z_{A_i}, \quad i=1,2,3$	$x_{A_1} = \frac{p\sqrt{3}}{6}, y_{A_1} = -\frac{p}{2}, z_{A_1} = 0$ $x_{A_2} = \frac{p\sqrt{3}}{6}, y_{A_2} = \frac{p}{2}, z_{A_2} = 0$ $x_{A_3} = -\frac{p\sqrt{3}}{3}, y_{A_3} = 0, z_{A_3} = 0$									
$\delta_1, \delta_2, \delta_3$	$\delta_1 = 0, \delta_2 = \frac{2\pi}{3}, \delta_3 = \frac{4\pi}{3}$									
$x_E, y_E, z_E$	$x_E = 0, y_E = 0, z_E = -d$									
$c\alpha', \dots, c\gamma''''$	<table border="1" style="width:100%; border-collapse: collapse;"> <tr> <td style="width:33%;"><math>c\alpha' = c\psi c\varphi - s\psi c\theta s\varphi</math></td> <td style="width:33%;"><math>c\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi</math></td> <td style="width:33%;"><math>c\alpha' = s\psi s\theta</math></td> </tr> <tr> <td><math>c\beta' = s\psi c\varphi + c\psi c\theta s\varphi</math></td> <td><math>c\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi</math></td> <td><math>c\beta''' = -c\psi s\theta</math></td> </tr> <tr> <td><math>c\gamma' = s\theta s\gamma</math></td> <td><math>c\gamma'' = s\theta c\gamma</math></td> <td><math>c\gamma'''' = c\theta</math></td> </tr> </table>	$c\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$	$c\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi$	$c\alpha' = s\psi s\theta$	$c\beta' = s\psi c\varphi + c\psi c\theta s\varphi$	$c\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$	$c\beta''' = -c\psi s\theta$	$c\gamma' = s\theta s\gamma$	$c\gamma'' = s\theta c\gamma$	$c\gamma'''' = c\theta$
$c\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$	$c\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi$	$c\alpha' = s\psi s\theta$								
$c\beta' = s\psi c\varphi + c\psi c\theta s\varphi$	$c\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$	$c\beta''' = -c\psi s\theta$								
$c\gamma' = s\theta s\gamma$	$c\gamma'' = s\theta c\gamma$	$c\gamma'''' = c\theta$								
$X_{A_i}, Y_{A_i}, Z_{A_i}, \quad i=1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha'''' \\ c\beta' & c\beta'' & c\beta'''' \\ c\gamma' & c\gamma'' & c\gamma'''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$									

$\begin{cases} X'_{A_i}, \\ Y'_{A_i}, Z'_{A_i} \\ i=1, 2, 3 \end{cases}$	$\begin{cases} X'_{A_i} = X_{A_i} c\delta_i + Y_{A_i} s\delta_i \\ Y'_{A_i} = -X_{A_i} s\delta_i + Y_{A_i} c\delta_i \\ Z'_{A_i} = Z_{A_i} - h_i \end{cases}$ <p>and <math>h_i = \frac{b\sqrt{3}}{6}</math></p>
$q_i, \\ i=1, 2, 3$	$q_i = -Y'_{A_i}, \quad i=1, 2, 3$
$q_{i+3}, \\ i=1, 2, 3$	$q_{i+3} = q_i \pm \sqrt{(2c)^2 - \left(-X_{A_i} - \frac{\sqrt{3}b}{6}\right)^2} - (Y'_{A_i} + q_i)^2$
$r_i \\ i=1, 2, 3$	$r_i = \sqrt{(2c)^2 - (q_{i+3} - q_i)^2}$
$u_i, w_i \\ i=1, 2, 3$	$\begin{cases} u_i = s\varphi_i = \frac{Z'_{A_i}}{\sqrt{(2c)^2 - (q_{i+3} - q_i)^2}} \\ w_i = c\varphi_i = \frac{X'_{A_i} - \frac{b\sqrt{3}}{6}}{r_i} \end{cases}$
$\varphi_i, \\ i=1, 2, 3$	$\varphi_i = a \tan 2(u_i, w_i)$

### 3.2. The direct geometric model

The direct geometric model of a mechanism (DGM) called also as the direct positional problem, consist in determining the generalized coordinates of the end-effector depending on the variable parameters.

Are known the active coordinates  $q_i$ , and  $q_{i+3}$  ( $i=1, 2, 3$ ) and the geometrical characteristics of the robot:  $h, b, \delta_1, \delta_2, \delta_3, c, p, d$ .

The unknowns are the coordinates that define de position of the mobile platform:  $X_E, Y_E, Z_E, \psi, \theta, \varphi$ .

From the six position equations:

$$F_i(q_{i+3}, q_i, X_E, Y_E, Z_E, \psi, \beta, \varphi) \equiv (q_{i+3} - q_i)^2 - (2c)^2 + (X'_{A_i} - h_i)^2 + (Z'_{A_i})^2 = 0$$

$$F_{i+3}(q_i, X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv Y'_{A_i} + q_i = 0, \quad i=1, 2, 3 \quad (25)$$

where:

$$\begin{cases} X'_{A_i} = X_{A_i} c\delta_i + Y_{A_i} s\delta_i \\ Y'_{A_i} = -X_{A_i} s\delta_i + Y_{A_i} c\delta_i \\ Z'_{A_i} = Z_{A_i} - h \end{cases} \quad (26)$$

Using the expressions from the Table 1 of the cosines  $\alpha', \beta', \gamma', \alpha'', \beta'', \gamma'', \alpha''', \beta''', \gamma'''$  we obtain the coordinates of the points  $A_i$ :

$$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\psi c\varphi - s\psi c\theta s\varphi & -c\psi s\varphi - s\psi c\theta c\varphi & s\psi s\theta \\ s\psi c\varphi + c\psi c\theta s\varphi & -s\psi s\varphi + c\psi c\theta c\varphi & -c\psi s\theta \\ s\theta s\varphi & s\theta c\varphi & c\theta \end{bmatrix} \cdot \begin{bmatrix} X_{A_i} - X_E \\ Y_{A_i} - Y_E \\ Z_{A_i} - Z_E \end{bmatrix}, \quad i=1, 2, 3 \quad (27)$$

Hence the following relations:

$$\begin{cases} X_{A_i} = X_E + (X_{A_i} - X_E)(c\psi c\varphi - s\psi c\theta s\varphi) + \\ (Y_{A_i} - Y_E)(-c\psi s\varphi - s\psi c\theta c\varphi) + (Z_{A_i} - Z_E)s\psi s\theta \\ Y_{A_i} = Y_E + (X_{A_i} - X_E)(s\psi c\varphi + c\psi c\theta s\varphi) + \\ (Y_{A_i} - Y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - (Z_{A_i} - Z_E)c\psi s\theta \\ Z_{A_i} = Z_E + (X_{A_i} - X_E)s\theta s\varphi + (Y_{A_i} - Y_E)s\theta c\varphi + \\ (Z_{A_i} - Z_E)c\theta \end{cases} \quad (28)$$

Are introduced the equations of the point  $A_i$  (28) into the positional equations (25).

Using the relations (26) are obtained from (25):

$$\begin{cases} (q_{i+3} - q_i)^2 - (2c)^2 + (X_{A_i} c\delta_i + Y_{A_i} s\delta_i - h_i)^2 + (Z_{A_i} - h_i)^2 = 0 \\ -X_{A_i} s\delta_i + Y_{A_i} c\delta_i + q_i = 0, \\ i=1, 2, 3 \end{cases} \quad (29)$$

From the relation (29) the expressions for the functions  $F_1, \dots, F_6$  can be written as:

$$\begin{aligned} F_1(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv (q_4 - q_1)^2 - (2c)^2 + \{X_E + (X_{A_1} - X_E) \cdot \\ &\cdot (c\psi c\varphi - s\psi c\theta s\varphi)(Y_{A_1} - Y_E)(-c\psi s\varphi - s\psi c\theta c\varphi) + (Z_{A_1} - Z_E)s\psi s\theta\} \cdot \\ &\cdot c\delta_1 + \{Y_E + (X_{A_1} - X_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (Y_{A_1} - Y_E) \cdot \\ &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (Z_{A_1} - Z_E)c\psi s\theta\} s\delta_1 - h_1\}^2 + \{Z_E + \\ &+ (X_{A_1} - X_E)s\theta s\varphi + (Y_{A_1} - Y_E)s\theta c\varphi + (Z_{A_1} - Z_E)c\theta - h_1\}^2 = 0 \\ F_2(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv (q_5 - q_2)^2 - (2c)^2 + \{X_E + (X_{A_2} - X_E) \cdot \\ &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (Y_{A_2} - Y_E)(-c\psi s\varphi - s\psi c\theta c\varphi) + (Z_{A_2} - Z_E) \cdot \\ &\cdot s\psi s\theta\} c\delta_2 + \{Y_E + (X_{A_2} - X_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (Y_{A_2} - Y_E) \cdot \\ &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (Z_{A_2} - Z_E)c\psi s\theta\} s\delta_2 - h_2\}^2 + \{Z_E + \\ &+ (X_{A_2} - X_E)s\theta s\varphi + (Y_{A_2} - Y_E)s\theta c\varphi + (Z_{A_2} - Z_E)c\theta - h_2\}^2 = 0 \\ F_3(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv (q_6 - q_3)^2 - (2c)^2 + \{X_E + (X_{A_3} - X_E) \cdot \\ &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (Y_{A_3} - Y_E)(-c\psi s\varphi - s\psi c\theta c\varphi) + (Z_{A_3} - Z_E) \cdot \\ &\cdot s\psi s\theta\} c\delta_3 + \{Y_E + (X_{A_3} - X_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (Y_{A_3} - Y_E) \cdot \\ &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (Z_{A_3} - Z_E)c\psi s\theta\} s\delta_3 - h_3\}^2 + \{Z_E + \\ &+ (X_{A_3} - X_E)s\theta s\varphi + (Y_{A_3} - Y_E)s\theta c\varphi + (Z_{A_3} - Z_E)c\theta - h_3\}^2 = 0 \\ F_4(X_E, Y_E, Z_E, \psi, \theta, \varphi) &\equiv -\{X_E + (X_{A_1} - X_E)(c\psi c\varphi - s\psi c\theta s\varphi) + \\ &+ (Y_{A_1} - Y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (Z_{A_1} - Z_E)s\psi s\theta\} s\delta_1 + \{Y_E + \\ &+ (X_{A_1} - X_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (Y_{A_1} - Y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - \\ &- (Z_{A_1} - Z_E)c\psi s\theta\} c\delta_1 + q_1 = 0 \end{aligned}$$

$$\begin{cases}
 F_5(X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv -[X_E + (x_{A_2} - x_E)(c\psi c\varphi - s\psi c\theta s\varphi) + \\
 + (y_{A_2} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_2} - z_E)s\psi s\theta]s\delta_2 + [Y_E + \\
 + (x_{A_2} - x_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_2} - y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 - (z_{A_2} - z_E)c\psi s\theta]c\delta_2 + q_2 = 0 \\
 F_6(X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv -[X_E + (x_{A_3} - x_E)(c\psi c\varphi - s\psi c\theta s\varphi) + \\
 + (y_{A_3} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_3} - z_E)s\psi s\theta]s\delta_3 + [Y_E + \\
 + (x_{A_3} - x_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_3} - y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 - (z_{A_3} - z_E)c\psi s\theta]c\delta_3 + q_3 = 0
 \end{cases} \quad (30)$$

From the six nonlinear equations (30) are obtained the six generalized coordinates of the end-effector  $X_E, Y_E, Z_E, \psi, \beta, \varphi$  by applying the Newton-Raphson method.

With notations:  $X = \begin{bmatrix} X_E \\ Y_E \\ Z_E \\ \psi \\ \theta \\ \varphi \end{bmatrix}, F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$

$$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\
 \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\
 \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\
 \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\
 \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\
 \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \quad (31)$$

are obtained:

$$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)}), \quad p=0, 1, 2, \dots \quad (32)$$

**Table 4.** The direct geometric model for the M=6 DOF and three guiding kinematic chains of the platform parallel robot. Algorithm

<b>Given:</b> $q_1, q_2, q_3, q_4, q_5, q_6, (q_i = 1 \div 6),$	
<b>h, b, <math>\delta_1, \delta_2, \delta_3</math> (<math>\delta_1 + \delta_2 + \delta_3 = 2\pi</math>), c, p, d</b>	
<b>Unknowns:</b> $X_E, Y_E, Z_E, \psi, \theta, \varphi$	
Variables:	Solving equations:
$\alpha', \dots, \gamma'''$	

$x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3$	$x_{A_1} = \frac{p\sqrt{3}}{6}, y_{A_1} = -\frac{p}{2}, z_{A_1} = 0$ $x_{A_2} = \frac{p\sqrt{3}}{6}, y_{A_2} = \frac{p}{2}, z_{A_2} = 0$ $x_{A_3} = -\frac{p\sqrt{3}}{3}, y_{A_3} = 0, z_{A_3} = 0$									
$h_i, i=1, 2, 3$	$h_1 = h_2 = h_3 = \frac{b\sqrt{3}}{6}$									
$x_E, y_E, z_E$	$x_E = 0, y_E = 0, z_E = -d$									
$\delta_1, \delta_2, \delta_3$	$\delta_1 = 0, \delta_2 = \frac{2\pi}{3}, \delta_3 = \frac{4\pi}{3}$									
$X_E^{(p+1)}, Y_E^{(p+1)}, Z_E^{(p+1)}, \psi^{(p+1)}, \theta^{(p+1)}, \varphi^{(p+1)}$	$  \begin{bmatrix} X_E^{(p+1)} \\ Y_E^{(p+1)} \\ Z_E^{(p+1)} \\ \psi^{(p+1)} \\ \theta^{(p+1)} \\ \varphi^{(p+1)} \end{bmatrix} = \begin{bmatrix} X^{(p)} \\ Y^{(p)} \\ Z^{(p)} \\ \psi^{(p)} \\ \theta^{(p)} \\ \varphi^{(p)} \end{bmatrix} - W(X) \begin{bmatrix} (F_1)^{(p)} \\ (F_2)^{(p)} \\ (F_3)^{(p)} \\ (F_4)^{(p)} \\ (F_5)^{(p)} \\ (F_6)^{(p)} \end{bmatrix}  $ <p>Where: <math>p=0, 1, 2, 3 \dots</math> And:</p> $  \begin{cases}  F_i(X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv (q_{i+3} - q_i)^2 - (2c)^2 + \\  \{ [X_E + (x_{A_i} - x_E)(c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_i} - y_E) \cdot \\  \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_i} - z_E)s\psi s\theta]c\delta_i + [Y_E + \\  + (x_{A_i} - x_E)(s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_i} - y_E)(-s\psi s\varphi + \\  + c\psi c\theta c\varphi) - (z_{A_i} - z_E)c\psi s\theta]s\delta_i - h_i \}^2 + [Z_E + (x_{A_i} - \\  - x_E)s\theta s\varphi + (y_{A_i} - y_E)s\theta c\varphi + (z_{A_i} - z_E)c\theta - h_i]^2 = 0 \\  F_{i+3}(X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv -[X_E + (x_{A_i} - x_E) \cdot \\  \cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_i} - y_E)(-c\psi s\varphi - s\psi c\theta c\varphi) + \\  + (z_{A_i} - z_E)s\psi s\theta]s\delta_i + [Y_E + (x_{A_i} - x_E)(s\psi c\varphi + \\  + c\psi c\theta s\varphi) + (y_{A_i} - y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - \\  - (z_{A_i} - z_E)c\psi s\theta]c\delta_i + q_i = 0  \end{cases}  $									
$\alpha', \dots, \gamma'''$	<table border="0"> <tr> <td><math>\alpha' = c\psi c\varphi - s\psi c\theta s\varphi</math></td> <td><math>\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi</math></td> <td><math>\alpha' = s\psi s\theta</math></td> </tr> <tr> <td><math>\beta' = s\psi c\varphi + c\psi c\theta s\varphi</math></td> <td><math>\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi</math></td> <td><math>\beta''' = -c\psi s\theta</math></td> </tr> <tr> <td><math>\gamma' = s\theta s\gamma</math></td> <td><math>\gamma'' = s\theta c\gamma</math></td> <td><math>\gamma''' = c\theta</math></td> </tr> </table>	$\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$	$\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi$	$\alpha' = s\psi s\theta$	$\beta' = s\psi c\varphi + c\psi c\theta s\varphi$	$\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$	$\beta''' = -c\psi s\theta$	$\gamma' = s\theta s\gamma$	$\gamma'' = s\theta c\gamma$	$\gamma''' = c\theta$
$\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$	$\alpha'' = -c\psi s\varphi - s\psi c\theta s\varphi$	$\alpha' = s\psi s\theta$								
$\beta' = s\psi c\varphi + c\psi c\theta s\varphi$	$\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$	$\beta''' = -c\psi s\theta$								
$\gamma' = s\theta s\gamma$	$\gamma'' = s\theta c\gamma$	$\gamma''' = c\theta$								

$X_{A_i},$ $Y_{A_i},$ $Z_{A_i},$ $i=1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} +$ $\begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} X_{A_i} - X_E \\ Y_{A_i} - Y_E \\ Z_{A_i} - Z_E \end{bmatrix},$	1
$X'_{A_i},$ $Y'_{A_i},$ $Z'_{A_i}$ $i=1,2,3$	$\begin{cases} X'_{A_i} = X_{A_i} c\delta_i + Y_{A_i} s\delta_i \\ Y'_{A_i} = -X_{A_i} s\delta_i + Y_{A_i} c\delta_i \\ Z'_{A_i} = Z_{A_i} - h \end{cases}$	2
$r_i,$ $i=1,2,3$	$r_i = \sqrt{(2c)^2 - (q_{i+3} - q_i)^2}$	2
$u_i, w_i$ $i=1,2,3$	$\begin{cases} u_i = s\varphi_i = \frac{Z'_{A_i}}{\sqrt{(2c)^2 - (q_{i+3} - q_i)^2}} \\ w_i = c\varphi_i = \frac{X'_{A_i} - \frac{b\sqrt{3}}{6}}{r_i} \end{cases}$	3
$\varphi_i,$ $i=1,2,3$	$\varphi_i = a \tan 2(u_i, w_i)$	4

#### 4. CONCLUSIONS

This paper presents the development of a new parallel robot which can be used for assembly operations or as a module for minimally invasive surgical systems. A definite advantages of this new parallel structure is that the actuators are placed on the base, thus imposing a limited weight for the moving parts, smaller actuator size, lower power dissipation, and for the fact that structure has three limbs instead of six a reduction of the mechanical interference between the links. The inverse and direct geometric models were solved. The inverse geometric model was achieved using an analytical method and the direct geometric model by using the Newton-Raphson numerical method.

The parallel structures are in course of patenting.

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MODELUL GEOMETRIC INVERS ȘI DIRECT AL UNUI NOU ROBOT PARALEL CU ȘASE GRADE DE MOBILITATE ȘI TREI LANȚURI CINEMATICE DE GHIDARE A PLATFORMEI

Rezumat: Lucrarea prezintă un nou robot paralel cu șase grade de mobilitate și trei lanțuri cinematice de ghidare a platformei. Pornind de la o soluție inițială cu actuatorii poziționali pe nivelele I și III, se propune o nouă configurație ce prezintă actuatorii montați pe batiu. Este prezentat modelul geometric invers și modelul geometric direct. În timp ce modelul geometric invers este obținut în formă închisă, modelul geometric direct nu se poate rezolva în formă închisă, fiind rezolvat utilizând metoda numerică Newton-Raphson. Robotul se poate utiliza în operațiile de asamblare sau ca și modul pentru sistemele chirurgicale minim invazive.

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