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CONTRIBUTIONS TO DYNAMIC FUNCTIONS FOR A MOBILE ROBOT

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Abstract: *In the paper, on the basis of the dynamics equations developed using new concepts in advanced mechanics, will be determined the dynamic control functions, for a mobile robot, and called PatrolBot. To express the dynamic control functions, the mathematical model had been developed in keeping the fact that the approach of the mobile platform is different besides the other robots types, due to nonholonomic constraints.*
Key words: *dynamic control functions, mobile robot, acceleration energy, working process.*

1. INTRODUCTION

In the last years, much effort has been spent in developing of mobile robot structures used for different tasks as transportation of materials or tools over longer distances, working in dangerous environments, presently they are being implemented in the most of human activities due to their benefits. The main task of mobile robots is to describe certain motion trajectories, based on control functions, which is to displace from a point, to a programmed position. As a result, the dynamic control functions for the structure of the mobile robots are fundamental for accomplishing these types of tasks.

2. MODELING OF ROBOT PATROL BOT

This part of the paper presents the physical properties and mathematical coordinate for a mobile robot, called PatrolBot, (see Fig.1) that have two independently driven wheels on one axle, known as "differentially-driven" robot.

2.1. PatrolBot`s Structure

The common characteristic of PatrolBot is that cannot autonomously produce a velocity which is transversal to the axle of it's wheels, this constrain being called nonholonomic constraint. The nonholonomic constraints reduce the mobile robot's instantaneous velocity degrees of freedom, and hence most robots have only two actuated joints, two driven wheels in the case of a differentially-driven robot. The mechanical structure of the robot, has two driving wheels, which are rolling around O_1 and O_2 ,and two driven wheels which according to the Fig.1, are rotating around O_3 and O_4, respectively around vertical axis $(\overline{z})$ axis.

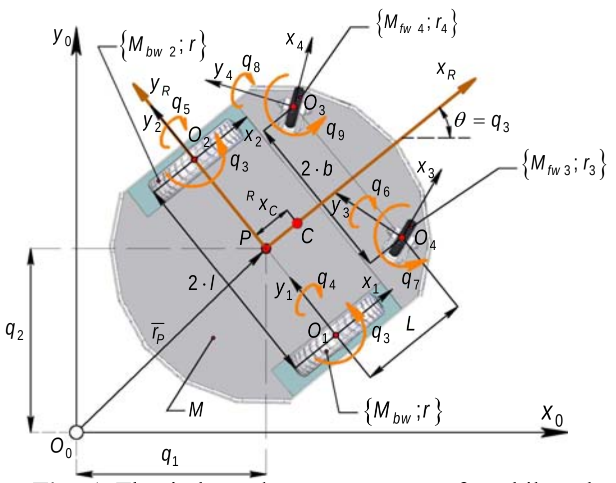

Fig. 1 The independent parameters of mobile robot

According to above statements, results:

$$\overline{X}(t) = \left[q_i(t);\ i = 1 \rightarrow 9 \right]^T \tag{1}$$

which characterizes the independent parameters of mobile structure PatrolBot in finite displacements.

2.2 Dynamics equations of PatrolBot

To determine the dynamic equations of mobile structure PatrolBot, there are used fundamental notions of advanced mechanics like acceleration energy,[2], which is integrated in the following expression specific to nonholonomic systems:

$$\frac{\partial E_A^{tot}}{\partial \ddot{q}_j} + Q_f^j + Q_g^j = Q_m^j + \sum_{i=1}^{7} \lambda_i \cdot a_{ij} \tag{2}$$

where, Q_f^j, Q_g^j and Q_m^j are the inertial, gravitational and driving forces; λ_j are the Lagrange`s parameters, and a_{ij} are the displacement coefficients. In keeping with the kinematical restrictions according to Fig. 1 and [1], there have been determined the dynamics moving equations as:

$$\begin{aligned} &M \cdot \ddot{q}_1 - \left(\ddot{q}_3 \cdot sq_3 + \dot{q}_3^2 \cdot cq_3\right)\cdot\left(M_A \cdot {}^R x_C + 2\cdot M_f \cdot L\right) = \\ &= -\lambda_1 \cdot sq_3 + cq_3 \cdot (\lambda_2 + \lambda_3) - \lambda_4 \cdot s(q_3 + q_7) - \\ &-\lambda_5 \cdot s(q_3 + q_9) + \lambda_6 \cdot c(q_3 + q_7) + \lambda_7 \cdot c(q_3 + q_9) \end{aligned} \tag{3}$$

$$\begin{aligned} &M \cdot \ddot{q}_2 + \left(\ddot{q}_3 \cdot cq_3 - \dot{q}_3^2 \cdot sq_3\right)\cdot\left(M_A \cdot {}^R x_C + 2\cdot M_f \cdot L\right) = \\ &= \lambda_1 \cdot cq_3 + sq_3 \cdot (\lambda_2 + \lambda_3) + \lambda_4 \cdot c(q_3 + q_7) + \\ &+\lambda_5 \cdot c(q_3 + q_9) + \lambda_6 \cdot s(q_3 + q_7) + \lambda_7 \cdot s(q_3 + q_9) \end{aligned} \tag{4}$$

$$\begin{aligned} &\ddot{q}_3\left[I_{\Delta_P} + 2M_f\left(b^2 + L^2\right)\right] - \\ &-\left(M_A{}^R x_C + 2M_f L\right)\left(\ddot{q}_1 sq_3 + \dot{q}_1\dot{q}_3 cq_3 - \ddot{q}_2 cq_3 + \dot{q}_2\dot{q}_3 sq_3\right) + \\ &+\dot{q}_3\left(M_A{}^R x_C + 2M_f L\right)\left(\dot{q}_1 cq_3 + \dot{q}_2 sq_3\right) = \\ &= l(\lambda_2 - \lambda_3) + \lambda_4(Lcq_7 + bsq_7) + \lambda_5(Lcq_9 + bsq_9) + \\ &+\lambda_6(Lsq_7 - bcq_7) + \lambda_7(Lsq_9 - bcq_9) \end{aligned} \tag{5}$$

$$\left\{\begin{aligned} &\frac{M_b \cdot r_b^2}{2}\cdot\ddot{q}_4 + Q_f^{bw}\cdot \operatorname{sgn}(\dot{q}_4) = Q_m^4 - \lambda_2 \cdot r_s; \\ &\frac{M_s \cdot r_s^2}{2}\cdot\ddot{q}_5 + Q_f^{bw}\cdot \operatorname{sgn}(\dot{q}_5) = Q_m^5 - \lambda_3 \cdot r_s \\ &\frac{M_f \cdot r_f^2}{2}\cdot\ddot{q}_6 + Q_f^{fw}\cdot \operatorname{sgn}(\dot{q}_6) = -\lambda_6 \cdot r_f; \\ &\frac{M_f \cdot r_f^2}{4}\cdot\ddot{q}_7 + Q_f^{7}\cdot \operatorname{sgn}(\dot{q}_7) = 0 \\ &\frac{M_f \cdot r_f^2}{2}\cdot\ddot{q}_8 + Q_f^{fw}\cdot \operatorname{sgn}(\dot{q}_8) = -\lambda_7 \cdot r_f; \\ &\frac{M_f \cdot r_f^2}{4}\cdot\ddot{q}_9 + Q_f^{9}\cdot \operatorname{sgn}(\dot{q}_9) = 0 \end{aligned}\right\} \tag{6}$$

In previous relations, the terms are expressing as follows: $M = M_A + 2\cdot M_{bw} + 2\cdot M_{fw}$ the total mass of the mobile system, M_A the mass of the platform (without wheels), M_b and M_f is the mass of the back respectively of the front wheels, r_b and r_f the ray of previous wheels, I_{Δ_P} is the inertia moment with respect to O_z axis, ${}^R x_C$ the position of the mass center, $\lambda_j, j = 1 \to 7$ Lagrange parameters, $Q_f^{1,2} = Q_f^{bw}$ back wheels friction moment; $Q_m^{4,5}$ wheels driving forces. [3]

2.3 Determining of wheels driving moments

The dynamics equations presented above are highlighting the complexity of the dynamic control. According to the relations, in order to express the dynamic control functions of the mobile structure, on an established trajectory, the actuating motors must overlap the generalized forces , as inertial forces, or gravitational forces (of the handled object). The moving of the structure has to be analyzed in keeping with the fact that the robot PatrolBot, due to its structure in order to achieve the goal point can`t realize simultaneous a translation and a orientation, hence the two displacements will be analyzed independently. Hence, to realize a straight line motion, according to its design, an essential condition is that the driving moments of the wheels must be equal $\left\{Q_m^4 = Q_m^5\right\}$, so in keeping with this it can be deduced that [2]:

$$\left\{(\ddot{q}_4 = \ddot{q}_5),\ q_3 = cst., (\dot{q}_3, \ddot{q}_3) = 0, (\ddot{q}_6 = \ddot{q}_8), (q_7, q_9) = 0\right\} \tag{7}$$

According to (6) it can be seen that, $(\lambda_2 = \lambda_3)$ respectively $(\lambda_6 = \lambda_7)$ .It is introduced the notation:

$$\varepsilon_P = \ddot{q}_{4,5} = \left\{\frac{1}{\cos q_3}\cdot\frac{\ddot{q}_1}{r_{rf}}, \frac{1}{\sin q_3}\cdot\frac{\ddot{q}_2}{r_{rf}}\right\} \tag{8}$$

with the observation that $\{\varepsilon_P\}$ takes the first form from (8) in the case of translation along q_1, respectively the value of the second term in the case of displacement along q_2. In this case, the expressions of the driving moment of the robot PatrolBot in order to realize a translation are:

$$\begin{aligned} Q_m^4 = Q_m^5 = \frac{r_{rs}}{2}\cdot\big[&M\cdot(\ddot{q}_1 \cdot \cos q_3 + \ddot{q}_2 \cdot \sin q_3) + \\ &+\varepsilon_P \cdot (M_{rf}\cdot r_{rf} + M_{rs}\cdot r_{rs}) + \mu \cdot M \cdot g\big] \end{aligned} \tag{9}$$

To realize the orientation, the following condition has to be accomplished $\left\{Q_m^4 = -Q_m^5\right\}$. The conditions for orientation of the mobile robot PatrolBot are:

$$\left\{(q_1, q_2) = cst., (\dot{q}_1, \dot{q}_2) = 0, (\ddot{q}_1, \ddot{q}_2) = 0\right\} \tag{10}$$

This in keeping with the restriction conducts to:

$$\begin{aligned} &\{(\ddot{q}_5 = -\ddot{q}_4),\ (\ddot{q}_6 = \ddot{q}_8)\} \\ &\{(-\lambda_2 = \lambda_3),\ (\lambda_6 = \lambda_7)\} \end{aligned} \tag{11}$$

After a few transformations, there are obtained the expressions of the driving moments for the two wheels in the case of orientation, as being:

$$\begin{aligned} Q_m^4 = -Q_m^5 = &\left[\frac{I_{\Delta_P} + 2\cdot M_{rf}\cdot\left(b^2 + L^2\right)}{2\cdot l} + M_{rf}\cdot\frac{L^2 + b^2}{2\cdot l} + \right. \\ &\left. + M_{rs}\cdot\frac{l}{2}\right]\cdot r_{rs}\cdot\ddot{q}_3 + \frac{\mu\cdot M\cdot g\cdot r_{rs}}{2\cdot\left[\mu\cdot(r_{rf} - r_{rs}) + L\right]}\cdot Q_m^s \end{aligned} \tag{12}$$

where

$$Q_m^s = \left[\left({}^R x_C - \mu\cdot r_{rs}\right)\cdot\frac{\sqrt{L^2 + b^2}}{l} + \left(L - {}^R x_C + \mu\cdot r_{rf}\right)\right].$$

Relations (9) and (12) are the expressions of the driving moments of the mobile robot wheels. Analyzing the remaindered above expressions, it can be seen that the final form of the driving moments contents a static and a dynamic component.

3 THE WORKING PROCESS OF MOBILE ROBOT

Further is considered a working process, presented in Fig.2, where is integrated the mobile platform PatrolBot. The process contains $(j=1\rightarrow 9)$ working sequences, and consisting in five translations and four orientations.

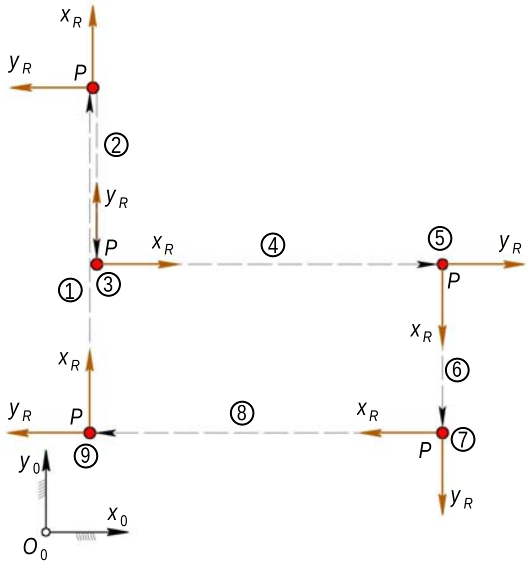

Fig. 2 The working process imposed to PatrolBot

Further, from the working process of the mobile robot described by Fig. 2, will be presented the sequences $(j=1,3,4)$, which are a translation (q_2), orientation (q_3) with $\varphi=-\pi/2$, another translation (q_1), and another orientation (q_3) with the same $\varphi=-\pi/2$.

3.1 Functions based on trapezoidal variation of velocities

To study the displacement of the robot PatrolBot between two intermediary points, it is imposed a trapezoidal variation law in velocities. Applying the conditions for continuity in velocities, it results:

$$\dot{q}_i=\frac{2}{t_i}\cdot(q_i-q_{i-1})+\sigma_i\cdot\dot{q}_{i-1} \tag{13}$$

where $\sigma_i=\{(+1;\ddot{q}_i\geq 0);(-1;\ddot{q}_i<0)\}$

According to aspects presented in [2], there are resulting the following functions to determine the velocities, accelerations and coordinates:

$$\begin{aligned}\dot{q}_i(\tau)&=\frac{\tau_i-\tau}{t_i}\cdot\dot{q}_{i-1}+\frac{\tau-\tau_{i-1}}{t_i}\cdot\dot{q}_i\\ \ddot{q}_i&=-\frac{1}{t_i}\cdot\dot{q}_{i-1}+\frac{1}{t_i}\cdot\dot{q}_i=\frac{1}{t_i}\cdot(\dot{q}_i-\dot{q}_{i-1})\\ q_i(\tau)&=-\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\dot{q}_{i-1}+\frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\dot{q}_i+\frac{1}{2}\cdot t_i\cdot\dot{q}_{i-1}+q_{i-1}\end{aligned} \tag{14}$$

The input parameters are presented in the Table 1.

Table 1

Values for coordinates and velocities.

Secq. j=1→9	Config. k=0→27	Time $\tau_{jk}\langle s\rangle$	Duration $t_i\langle s\rangle$	Input values for coordinates and velocities	
				$q_{jk}\langle m,rad\rangle$	$\dot{q}_{jk}\langle m/s,rad/s\rangle$
1	0	0	0	0	0
	1	2	2	0,5	0,5
	2	4	2	1,5	0,5
	3	6	2	2	0
3	6*	18	0	1,571	0
	7	18,5	0,5	1,0472	-2,0944
	8	18,75	0,25	0,5236	-2,0944
	9	19,25	0,5	0	0
4	9*	19,25	0	0	0
	10	21,25	2	0,5	0,5
	11	23,25	2	1,5	0,5
	12	25,25	2	2	0

On the basis of the previous considerations, the expressions for generalized coordinates, velocities and accelerations as follow in Table 2:

Table 2

Generalized expressions for generalized coordinates, velocities and accelerations

Secq. j=1→9	Interval i=1→3	Coord.	Expressions for generalized coordinates, velocities and accelerations		
			q_{jk} $\langle m,rad\rangle$	$\dot{q}_{jk}$ $\langle m,rad/s\rangle$	$\ddot{q}_{jk}$ $\langle m,rad/s^2\rangle$
1	1	q_2	$0,125\cdot\tau^2$	$0,25\cdot\tau$	0,25
	2		$0,5\cdot(\tau-1)$	0,5	0
	3		$2-0,2\cdot(\tau-6)^2$	$1,5-0,25\cdot\tau$	$-0,25$
3	1	q_3	$1,5-2,1\cdot(\tau-18)^2$	$-2,1\cdot(2\cdot\tau-36)$	$-4,1887$
	2		$-2,1\cdot\tau+39,8$	$-2,1$	0
	3		$2,1\cdot(\tau-19,25)^2$	$2,1\cdot(2\cdot\tau-38,5)$	4,1887
4	1	q_1	$0,125\cdot(\tau-19,25)^2$	$0,25\cdot\tau-4,9$	0,25
	2		$0,5\cdot\tau-10,125$	0,5	0
	3		$2-0,1\cdot(\tau-25,25)^2$	$6,4-0,25\cdot\tau$	$-0,25$

On the basis of information contained in Table 2, further are represented the time variation of generalized coordinates, velocities and accelerations.

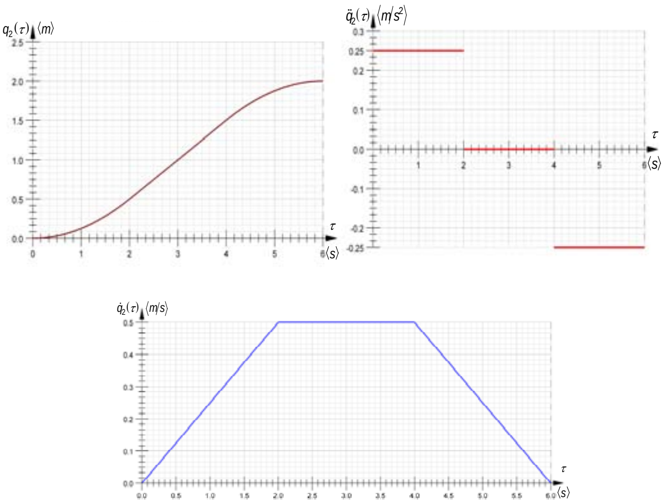

Fig. 3. Kinematical parameters on sequence $j=1$

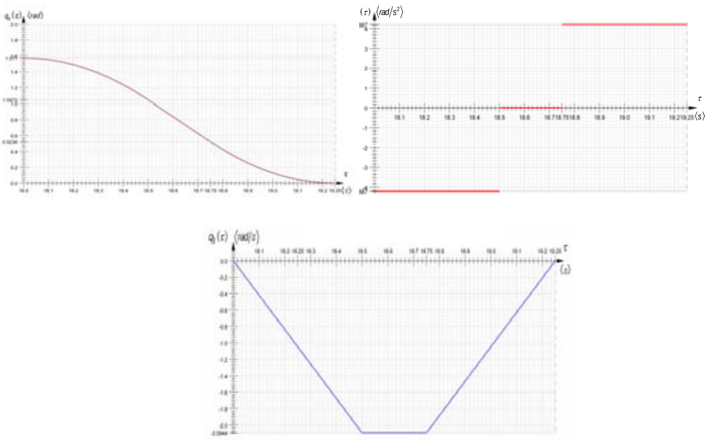

Fig. 4. Kinematical parameters on sequence $j=3$

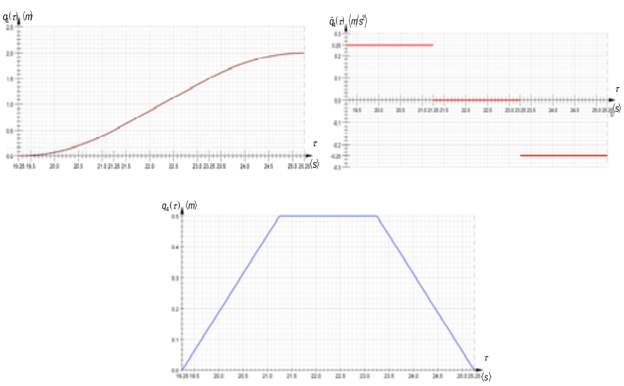

Fig. 5. Kinematical parameters on sequence $j=4$

On the basis of expression (9) and (12), in keeping with Table 1 and Table 2, there are represented graphically the variation for the driving moments, on the sequences $(j=1,3,4)$.

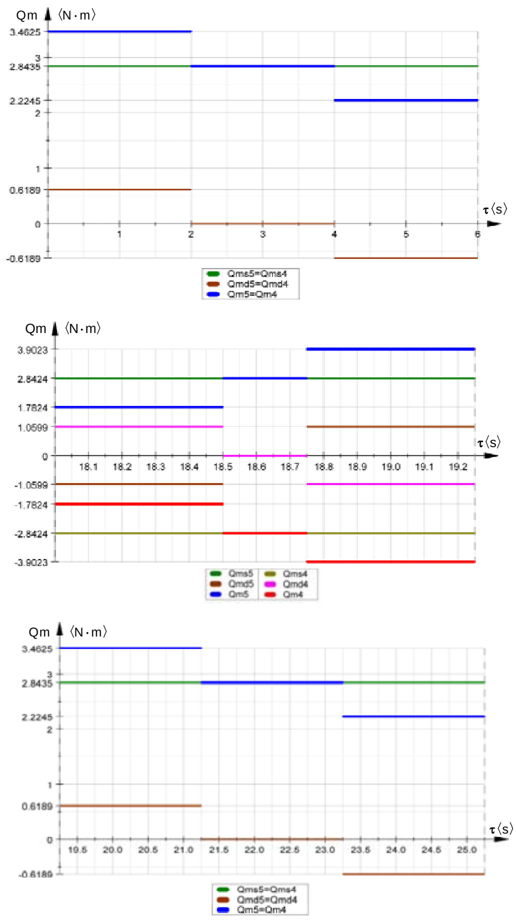

Fig.6. Variation of driving moments, on sequences $j=1,3,4$

3.2 $(3n)$ with restrictions polynomial functions

The interpolating functions are consisting in generation of linear functions with respect to time for generalized accelerations from every driving joint belonging to the robot. According to [2], it is generated a linear function with respect to time as:

$$\ddot{q}_{ji}(\tau) = \frac{\tau_i - \tau}{t_i} \cdot \ddot{q}_{ji}(\tau_{i-1}) + \frac{\tau - \tau_{i-1}}{t_i} \cdot \ddot{q}_{ji}(\tau_i) \qquad (15)$$

where $t_i = \tau_i - \tau_{i-1}$ represents the duration of each $(i=1 \rightarrow 3)$ segment of the trajectory.

The unknowns for the generalized accelerations at τ_{i-1} and τ_i are defined as:

$$\ddot{q}_{ji}(\tau_{i-1}) = \ddot{q}_{ji-1}\,; \quad \ddot{q}_{ji}(\tau_i) = \ddot{q}_{ji}\,. \qquad (16)$$

After a few transformations, are obtained the functions for generalized velocities and coordinations as shown in the following:

$$\dot{q}_{ji}(\tau) = -\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddot{q}_{ji-1} + \frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddot{q}_{ji} + \left(\frac{q_{ji}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji}\right) - \left(\frac{q_{ji-1}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji-1}\right) \quad (1$$

$$q_{ji}(\tau) = \frac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddot{q}_{ji-1} + \frac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddot{q}_{ji} + \left(\frac{q_{ji}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji}\right)\cdot(\tau-\tau_{i-1}) + \left(\frac{q_{ji-1}}{t_i}-\frac{t_i}{6}\cdot\ddot{q}_{ji-1}\right)\cdot(\tau_i-\tau) \quad (1$$

The input parameters for study are presented as follows in Table 3.

Table 3

Input parameters

Seq. j=1→9	Config. k=0→27	Time $\tau_{jk}\langle s\rangle$	Duration $t_i\langle s\rangle$	Coordinates values $q_{jk}\langle m,rad\rangle$
1	0	0	0	0
	1	2	2	
	2	4	2	
	3	6	2	2
3	6*	18	0	1,571
	7	18,5	0,5	
	8	18,75	0,25	
	9	19,25	0,5	0
4	9*	19,25	0	0
	10	21,25	2	
	11	23,25	2	
	12	25,25	2	2

On the basis of the parameters presented in Table 3, in keeping with (16)-(18), there are determined the expressions for generalized coordinates, velocities and accelerations as in Table 4.

Table 4

Expressions for generalized coordinates, velocities and accelerations

Secq. i=1→9	Interval=1→3	Coord.*	Expressions for generalized coordinates, velocities and accelerations		
			$q_{jk}\langle m,rad\rangle$	$\dot{q}_{jk}\langle m/s, rad/s\rangle$	$\ddot{q}_{jk}\langle m/s^2, rad/s^2\rangle$
1	1	q_2	$0,0417\cdot\tau^3$	$0,125\cdot\tau^2$	$0,25\cdot\tau$
	2		$-0,1\cdot\tau^3+0,75\cdot\tau^2-1,5\cdot\tau+1$	$-0,25\cdot\tau^2+1,5\cdot(\tau-1$	$1,5 - 0,5\cdot\tau$
	3		$0,0417\cdot(\tau-6)^3+2$	$0,125\cdot(\tau-6)^2$	$0,25\cdot\tau - 1,5$
3	1	q_3	$1,571-3,351\cdot(\tau-18)^3$	$-10,053\cdot(\tau-18)^2$	$-10,1\cdot(2\tau - 36)$
	2		$13,4\cdot\tau^3-748,9\cdot\tau^2+13945,7\cdot\tau-86540,1$	$40,2\cdot\tau^2-1497,8\cdot\tau+13945,7$	$80,4\cdot\tau - 1497,8$
	3		$-3,351\cdot(\tau-19,25)^3$	$-10,053\cdot(\tau-19,25)^2$	$-10,1\cdot(2\tau - 38,5)$
4	1	q_1	$0,04168\cdot(\tau-19,25)^3$	$0,125\cdot(\tau-19,25)^2$	$0,25\cdot\tau - 4,8125$
	2		$-0,1\cdot\tau^3+5,5\cdot\tau^2-123,1\cdot\tau+902,2$	$-0,25\cdot\tau^2+11,125\cdot\tau-123,0157$	$11,125 - 0,5\cdot\tau$
	3		$0,1\cdot(\tau-25,2)^3+2$	$0,125\cdot(\tau-25,25)^2$	$0,25\cdot\tau - 6,32$

On the basis of information contained in Table 4, there are represented the time variation of generalized coordinates, velocities and accelerations.

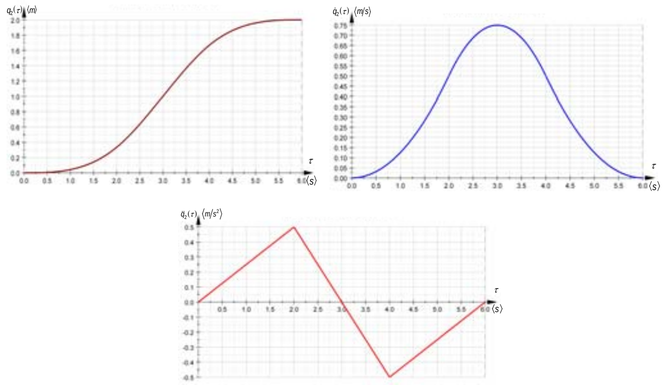

Fig. 7. Kinematical parameters on sequence $j=1$

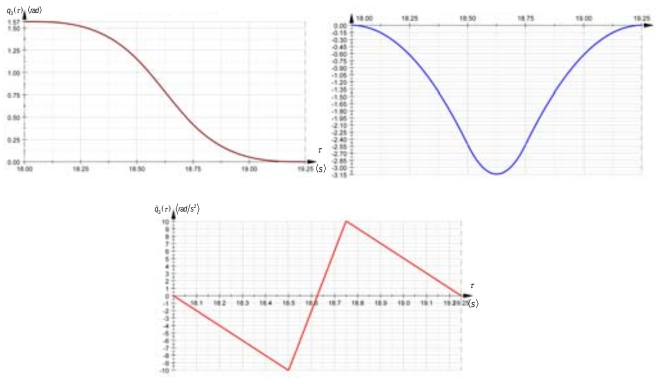

Fig.8. Kinematical parameters on sequence $j=3$

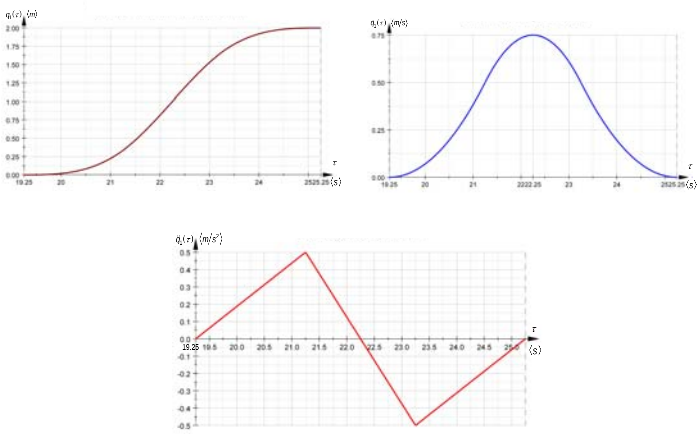

Fig. 9. Kinematical parameters on sequence $j=4$

On the basis of expression (9) and (12), in keeping with Table 4, there are represented graphically the variation for the driving moments, on the sequences $(j=1,3,4)$.

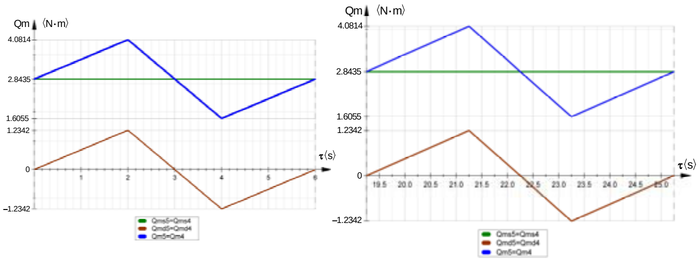

Fig. 10. Variation of driving moments, on sequence $j=1,4$

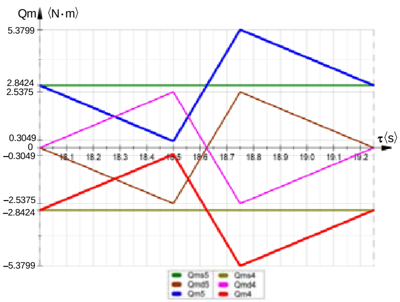

Fig.11. Variation of driving moments, on sequence $j=3$

4 CONCLUSIONS

In the paper there have been developed the dynamic control functions for the mobile robot PatrolBot. Using as starting point the dynamics equations, in keeping with the conditions which must be taken into account for straight line displacement or orientation, there have been deduced the driving moments of the wheels. Having the driving moments of the wheels, the structure was integrated in a technological process. The process has been modeled using in the first instance functions based on trapezoidal variation of velocities. The graphical representation of the variation of accelerations and driving moments on some sequences of the process shows that even if mathematically is correctly, graphically in the same moment the mentioned parameters have at least two different values. Hence, the same sequences have been modeled by $(3n)$ type with restrictions polynomial functions, where the representation is in accordance with the real working process of the mobile robot PatrolBot.

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Contribuţii privind funcţiile de control dinamic pentru un robot mobil

Rezumat: În lucrare, pe baza ecuaţiilor dinamicii, dezvoltate folosind concepte noi în mecanica avansata a sistemelor, au fost stabilite funcţiile de control dinamic, pentru un robot mobil, PatrolBot. Pentru a exprima funcţiile de control dinamic, modelul matematic a fost dezvoltat, ţinând seama de faptul că, abordarea platformei mobile este diferită de celelalte tipuri de roboţi, datorită constrângerilor neolonome.

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