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STRUCTURES WITH SYMMETRIES USED IN CIVIL ENGINEERING

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Abstract: *In many engineering application the designed device or product posses parts with some kind of symmetries. It is the case of automotive engineering or tools machines. We can find these type of symmetries too in the civil engineering. The existence of the symmetries leads to some properties that can simplify the vibrations calculus. In the paper are presented the main properties of a structure with transversal symmetry.*
Key words: *symmetry, eigenvalues, eigenmodes, transversal symmetry*

1. INTRODUCTION

The structures used in civil engineering have, many times, parts that present some kind of symmetries. These symmetries are determined both from esthetic reasons and engineering considerations (to simplify the construction and the calculus of such structures). In this paper we want to prove that the symmetries can be used to simplify the vibrations and strength calculus.

We consider a civil engineering structure like in fig.1, (can be the strength structure of a block of flats). This structure can be considered composed by two identical parts, the nodes from symmetry plane being common. (fig.2)

2. SYMMETRICAL SYSTEM USED IN CIVIL ENGINEERING

We propose to solve the eigenvalues and eigenvectors problem for the structure represented in fig.1. We denote this structure with (S). This structure can be considered composed by two identical substructure, denoted with (S_1) (fig.2). We denote by Δ_a the common nodes of the two structures (S_1), with Δ_l the nodes of the left structure (S_1) different from Δ_a and with Δ_r the nodes of the right structure (S_1) different from Δ_a. The motion equations of the undamped free vibrations for the entire structure (S), the left substructure (S_1) and the right substructure (S_1) are, respectively [1],[2]:

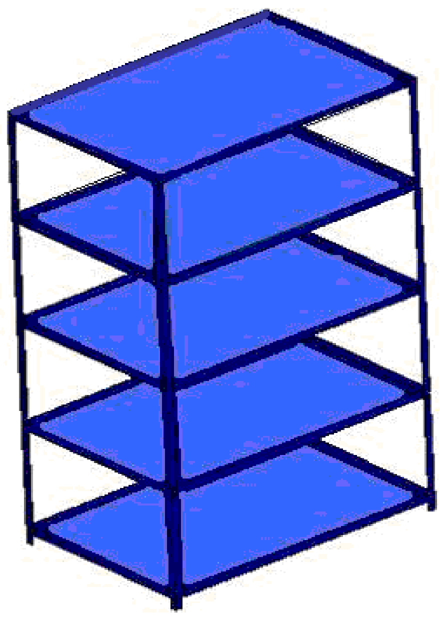

Figure 1

System (S):

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 2m_a \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}_l \\ \ddot{\Delta}_r \\ \ddot{\Delta}_a \end{Bmatrix} +$$

$$+\begin{bmatrix} k & 0 & -k_a \\ 0 & k & -k_a \\ -k_a^T & -k_a^T & 2k_a \end{bmatrix} \begin{Bmatrix} \Delta_l \\ \Delta_r \\ \Delta_a \end{Bmatrix} = 0 \qquad (1)$$

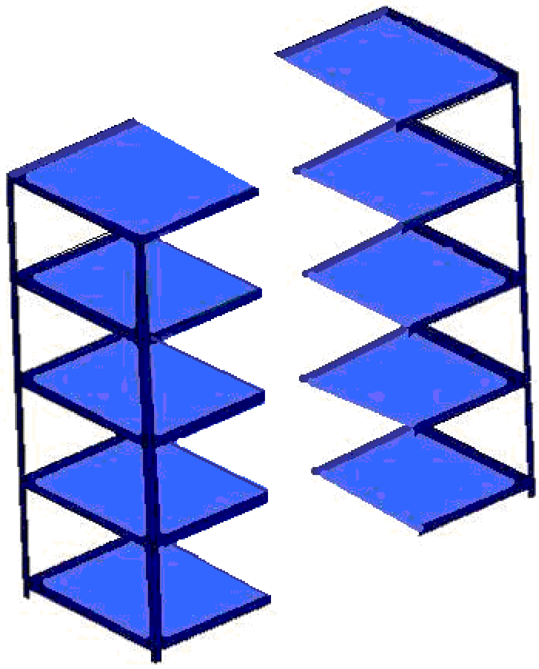

Figure 2

Left subsystem (S_1):

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}_l \\ \ddot{\Delta}_a \end{Bmatrix} + \begin{bmatrix} k & -k_a \\ -k_a^T & k_a \end{bmatrix} \begin{Bmatrix} \Delta_l \\ \Delta_a \end{Bmatrix} = 0 \qquad (2)$$

Right subsystem (S_1):

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{\Delta}_r \\ \ddot{\Delta}_a \end{Bmatrix} + \begin{bmatrix} k & -k_a \\ -k_a^T & k_a \end{bmatrix} \begin{Bmatrix} \Delta_r \\ \Delta_a \end{Bmatrix} = 0 \qquad (2')$$

In the paper we prove the following properties[3]:

P1 – The eigenvalues for the subsystem (S_1) (for the differential equations (1) or (1')) are eigenvalues for the system (S).

P2 – The eigenvectors for the subsystem (S_1) have the same form like the corresponding components of the eigenvectors of the system (S), for the common eigenvalues (according to **P1**).

The first property expresses the fact that the eigenvalues corresponding to the subsystem (S_1):

$$\begin{vmatrix} k-\omega^2 m & -k_a \\ -k_a^T & k_a-\omega^2 m_a \end{vmatrix} = 0$$

verify the eigenvalues problem for the entire system (S):

$$\begin{vmatrix} k-\omega^2 m & 0 & -k_a \\ 0 & k-\omega^2 m & -k_a \\ -k_a^T & -k_a^T & 2k_a-2\omega^2 m_a \end{vmatrix} = 0$$

3. EXAMPLE

For the sample shown in figure 1 are computed the eigenvalues and are presented in Table 1. In the same table are presented the eigenvalues computed for the substructure (S_1)(Fig.2). All the eigenvalues of the substructure (S_1) can be found between the eigenvalues of (S) (in table are presented only 10 eigenvalues).

Table 1. Eigenmodes

Mode	Structure (S) Eigenmodes (Hz)	Substructure (S_1) Eigenmodes (Hz)
1	9,92	11,04
2	11,04	17,75
3	16,01	17,78
4	17,75	17,94
5	17,88	17,98
6	17,94	18,05
7	17,98	36,30
8	18,05	45,50
9	32,95	46,22
10	34,22	46,59

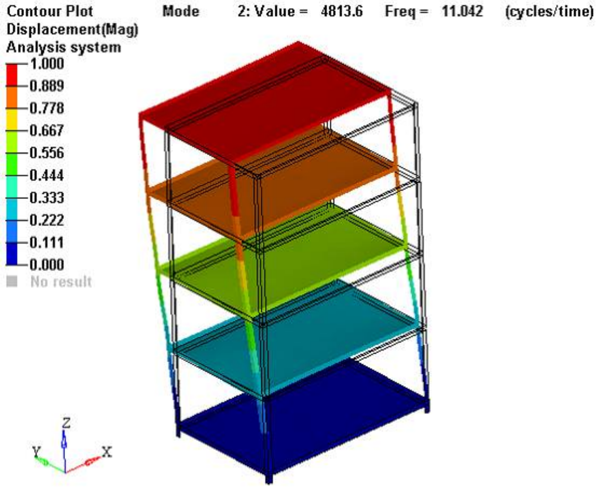

Figure 3. System (S). Second eigenmode

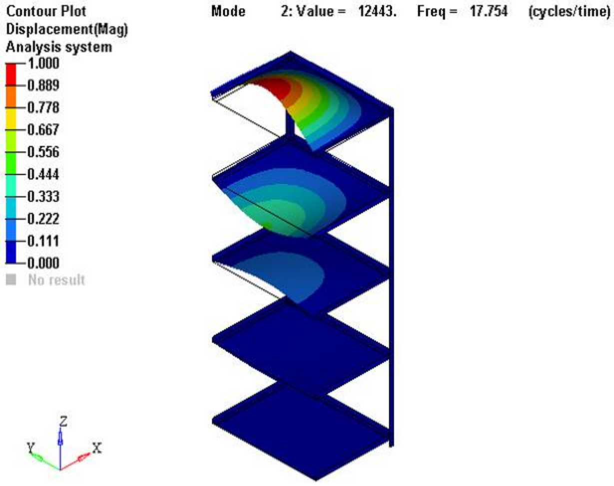

Figure 6. Subsystem (S1). Second eigenmode

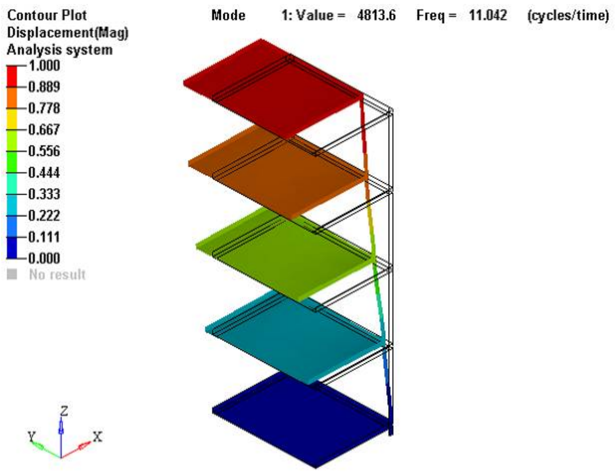

Figure 4. Subsystem (S1). First eigenmode

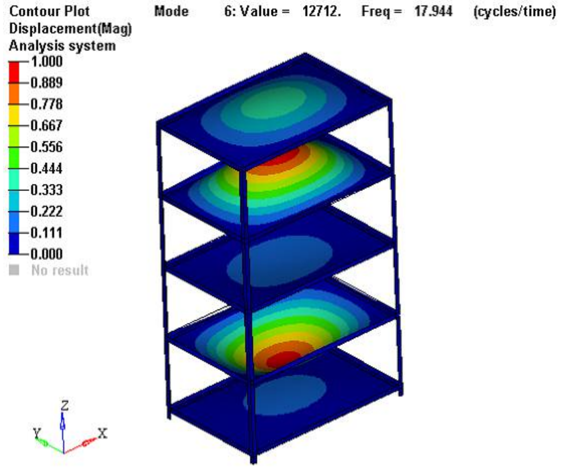

Figure 7. System (S). 6th eigenmode

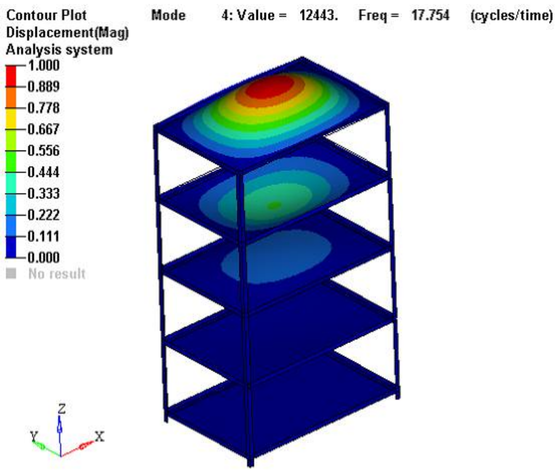

Figure 5. System (S). 4th eigenmode

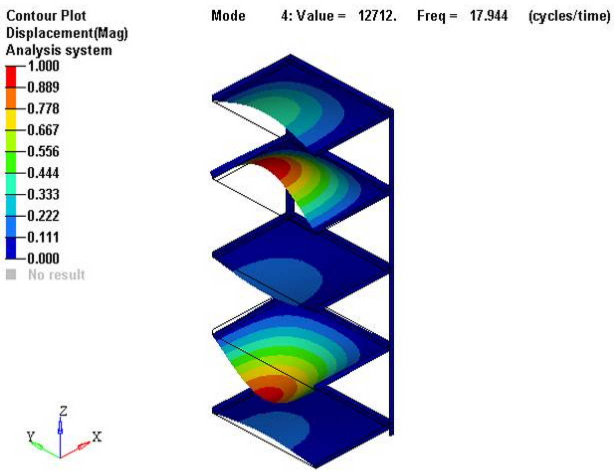

Figure 8. Subsystem (S1). $th eigenmode

In figures 3-8 are represented the eigenvectors for the first 2 eigenvalues of (S_1). We can observe that these eigenmodes are identical with the eigenmodes of the structure (S) for the same eigenvalues. The properties presented in the paper can be used to simplify the calculus to vibration of structures with symmetries.

4. CONCLUSIONS

There are many technical applications where the studied system can be considered composed by two or many identical parts or systems that present some symmetries. These symmetries can be used to simplify the analysis of such systems in order to reduce the dimension of the equations that describe the motion. In the paper are identified, for the systems with three identical parts, some properties to vibrations. The use of the above mentioned properties permit to simplify the steady state analysis of the model. In the paper the damping was neglected. If we consider a damping respecting the Caughey conditions, the presented properties remain.

5. REFERENCES

1. Vlase, S., *A Method of Eliminating Lagrangean Multipliers from the Equation of Motion of Interconnected Mechanical System*. ASME Transaction. Journal of Applied Mechanics, Vol. 54, 1987, p.235-236.
2. Vlase, S., *Elimination of Lagrangean Multipliers. Mech. Research Communications*, Vol. 14, 1987, p.17-20.
3. Mangeron, D., Goia, I., Vlase, S., *Symmetrical Branched Systems Vibration*. Memoriile sectiilor stiintifice ale Academiei, Buc., seria IV, Tomul XII, Nr.1, 1991.

Structuri cu simetrii utilizate in construcții

Rezumat: *In multe aplicatii ingineresti, dispozitivele sau produsele in faza de concept poseda componente cu anumite simetrii. Este cazul domeniului de automobile sau masini unelte. De asemenea, putem intalni aceste tipuri de simetrii si in domeniul constructiilor civile. Existenta simetriilor poate conduce la simplificarea unui calculului de vibratii. In acest articol sunt prezentate principalele proprietati ale unei structuri cu simetrie transversala.*

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