TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue III , 2012

PARTICULARISATION OF THE GENERAL MATHEMATICAL MODEL 3T3R – 3T3R APPLIED TO A STRUCTURE TYPE 1T – 2T

Ioan VUSCAN, Iuliu NEGREAN, Kalman KACSO, Ancuta MIRCEA

Abstract*: Starting from the equations for a most complex structure type 3T3R-3T3R by using of certain restrictions on the general model can be accomplish any types of kinematic structures. Particularization is relatively easy to make because the required restrictions are applied to the geometrical-constructive parameters and of linear and angular coordinates, these being depend by the type of kinematic structure take on study. In this paper are presented the equations for the simplest and most widely used machine tool structure, and namely 1T-2T.*

Key words: *kinematic structures, applied mechanics, machine tool, 1T-2T structure.*

1. INTRODUCTION

The most complex kinematic structure *can be assigned as being two robots a commune platform which is in cooperation.* One of these structure represent the TOOL, and the other one represent the PIECE, both structures being in situation to have the all six degrees of freedom: 3T3R–3T3R (fig. 1).

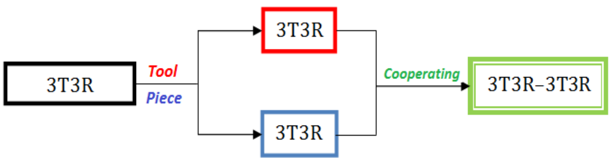

Fig. 1. The cooperation of the positioning and orientating structure of the TOOL and of the PIECE for a structure type 3T3R – 3T3R

Starting from this complex structure by apply of certain restriction on mathematical model on the geometrical-constructive parameters and of linear and angular coordinates can be obtain any type of machine tool structure.

In this case the particularization is made on kinematic structure type 1T – 2T (fig. 2).

As we can see from symbolization, the distributions of the motions are: one translation for the TOOL and two translations for the PIECE. It was choice this solution to be analyze, because is the most widely used on industry and the kinematic structure of this type is not present a very high grade of difficulty.

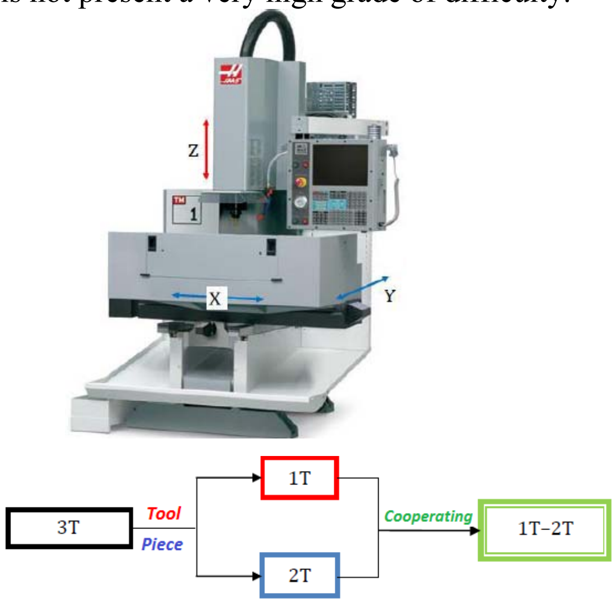

Fig.2. The cooperation of the positioning and orientating structure of the TOOL and of the PIECE for a structure type 1T – 2T

The present structure is represent one of 728 possible variants, which can be obtain from the most complex structure type 3T3R – 3T3R.

To realize a certain machining process, the TOOL and the PIECE must be brought together on a commune platform (can be the workspace of the machine tool) with positioning and orientations very well defined. To define these motions can be used the equations of direct and inverse kinematics.

2. GENERAL ASPECTS REGARDING THE PARTICULARIZATION OF GENERAL MODEL 3T3R–3T3R APPLIED TO A STRUCTURE TYPE 1T – 2T

To achieve the equations of the cooperating general model of structure type 3T3R–3T3R are facilitated by the fact, for both structures, of the TOOL and the PIECE was determinate independently the direct and inverse geometry equations. Having for both structures the equations, in a first step, it can be determinate the direct geometry equations for the structure type 3T3R–3T3R (see first paper). Is important to mention that, the general model 3T3R–3T3R is in two variants: *without errors* and *with errors*. Inserting the initial errors into model is important because in that way we have the generalized coordinates to allow to machine correctly and precisely the parts with corrections.

The second step is even the particularization itself of the structure type 1T–2T (fig. 3). This is having three coordinates linear (q_2, q_7 and q_8).

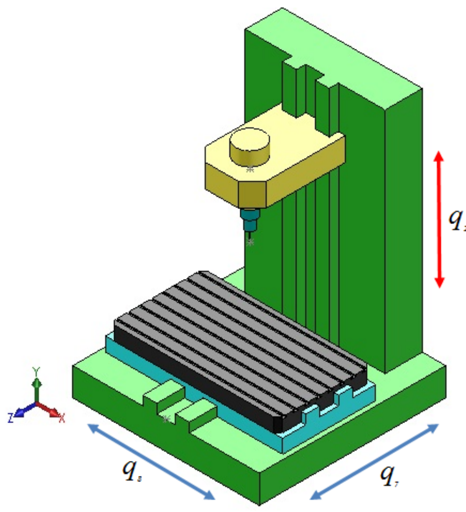

Fig. 3. The kinematic structure type 1T – 2T and distribution of the motions

By inserting the restrictions on mathematical model 3T3R–3T3R, as we can see on figure 3 the motions which remain are only the translations on X, Y and Z axis. This configurations of motions representing also the most widely and used structure of machine tool in manufacturing. Having now the new structure defined, we can go further to apply the mathematical model to the present structure type 1T–2T.

The mainly stages are:

- to process a piece correctly without inserting initial errors on the model;
- to process the same piece with inserting initial errors on the model.

In both stages, the output dates are the coordinates generalized.

In the case of machining *without errors* the input dates are the positioning and orientation vectors of the workpieces. In the other case, that of machining *with errors* beside the positioning and orientation vectors of the workpieces appears the errors. These can be imposed or measured.

The phenomenon of errors compensations is given by the fact, that knowing the initial errors, for the next parts which are machined, it will be realize correctly and without errors. The coordinates generalized are given the linear and orientation displacement along the X, Y and Z axis, necessary to machine correctly.

Also, in the both cases, the reporting of the systems is different.

In the case of the calculating of coordinates generalized without errors the reporting is made at the nominal system of the piece, noted with S_P. In the other case, with initial errors, the reporting is made on the real system of the piece, noted with S_{Preal}. Are use different type of systems because, the last one, S_{Preal} is affected by errors. These systems can be overlap or not, depending by the machining conditions.

2.1 The case of machining workpieces without initial errors

To realize that, in a first step are worked certain points from the piece without introducing of initial errors.

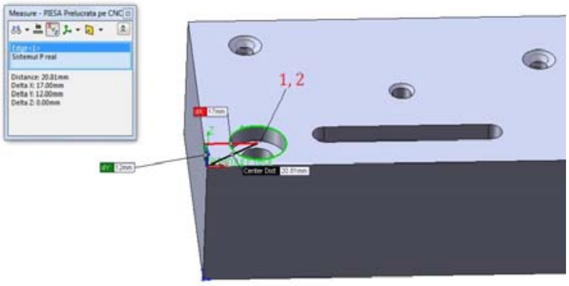

Fig. 4. Positioning and orientation coordinates for the machining points 1 and 2

Table 1

$^{(0)}X_{SP} = \left({}^{P}\overline{p}_{SP} \quad {}^{P}\overline{\psi}_{SP} \right)$					
${}^{P}\overline{p}_{SP}$ ⟨mm⟩			${}^{P}\overline{\psi}_{SP}$ ⟨°⟩		
$p_{x_{SP}}$	$p_{y_{SP}}$	$p_{z_{SP}}$	$\alpha_{z_{SP}}$	$\beta_{x_{SP}}$	$\gamma_{y_{SP}}$
17	12	0	0	0	0

The structure analyzed type 1T–2T being a simple structure, characterized only with translations, the orientation coordinates ($\alpha_{z_{SP}}, \beta_{x_{SP}}, \gamma_{y_{SP}}$) are zero (tab. 1).

The significations of the notations are as follow:

$p_{x_{SP}}, p_{y_{SP}}, p_{z_{SP}}$ – the positioning coordinate of the tool-piece cooperation along the X, Y and Z axis;

$\alpha_{z_{SP}}, \beta_{x_{SP}}, \gamma_{y_{SP}}$ – the orientation coordinate of the tool-piece cooperation along the X, Y and Z axis;

As a remark, the positioning and orientation vectors ($p_{x_{SP}}, p_{y_{SP}}, p_{z_{SP}}$) are equivalents with dX, dY and dZ the afferent notations of the SolidWorks software.

The coordinates generalized or the displacements on motor couples from the mathematical model 3T3R–3T3R customization for the present structure are:

Table 2

The generalized coordinates (displacements on the motor couples)				
⟨mm⟩			⟨°⟩	
q_2	q_7	q_8	q_{10}	q_{12}
-106,5	8	-17	0	0

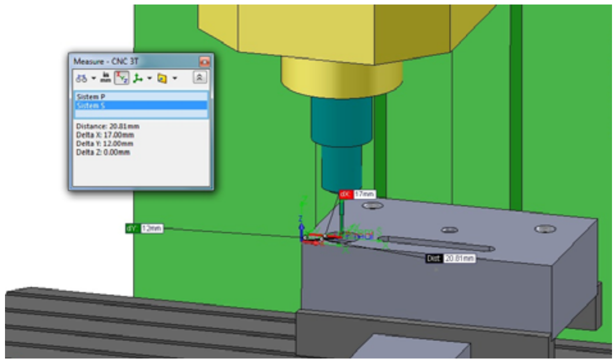

Fig. 5. The validation of the generalized coordinates for the machining points 1 and 2

Machining of another point from the surface of the piece is illustrated bellow. To work this point is necessary that the TOOL to make a translation on Y axis.

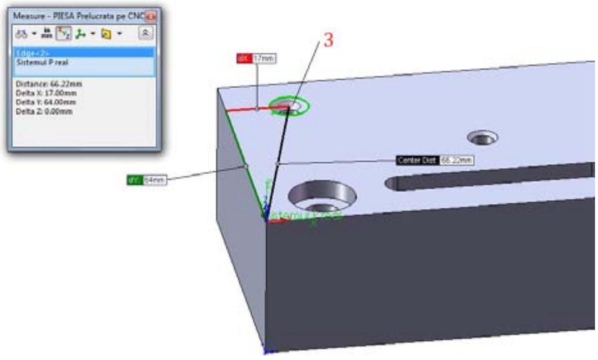

Fig. 6. Positioning and orientation coordinates for the machining point 3

The input parameters are presented in the table 3.

Table 3

$^{(0)}X_{SP} = \left({}^{P}\overline{p}_{SP} \quad {}^{P}\overline{\psi}_{SP} \right)$					
${}^{P}\overline{p}_{SP}$ ⟨mm⟩			${}^{P}\overline{\psi}_{SP}$ ⟨°⟩		
$p_{x_{SP}}$	$p_{y_{SP}}$	$p_{z_{SP}}$	$\alpha_{z_{SP}}$	$\beta_{x_{SP}}$	$\gamma_{y_{SP}}$
17	64	0	0	0	0

In the table 4 are listed the output parameters and are representing the new generalized coordinates achieve from the mathematical model.

Table 4

The generalized coordinates				
⟨mm⟩			⟨°⟩	
q_2	q_7	q_8	q_{10}	q_{12}
-106,5	44	-17	0	0

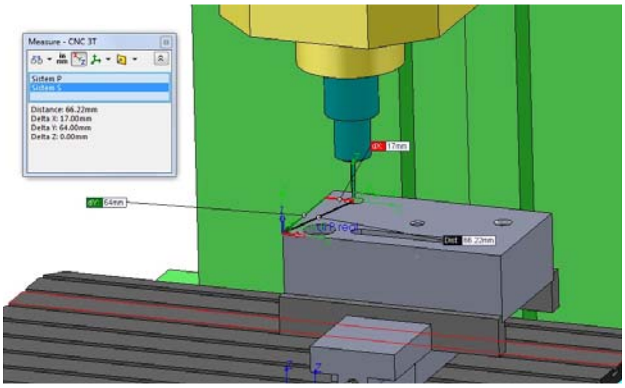

Fig. 7. The validation of the generalized coordinates for the machining point 3

The input parameters are presented in the table 5.

Table 5

$^{(0)}X_{SP} = \begin{pmatrix} ^{P}\overline{p}_{SP} & ^{P}\overline{\psi}_{SP} \end{pmatrix}$					
$^{P}\overline{p}_{SP}$ $\langle mm \rangle$			$^{P}\overline{\psi}_{SP}$ $\langle ^{o} \rangle$		
$p_{x_{SP}}$	$p_{y_{SP}}$	$p_{z_{SP}}$	$\alpha_{z_{SP}}$	$\beta_{x_{SP}}$	$\gamma_{y_{SP}}$
64,5	38,5	0	0	0	0

The new values of generalized coordinates are listed in table 6.

Table 6

The generalized coordinates (displacements on the motor couples)				
$\langle mm \rangle$			$\langle ^{o} \rangle$	
q_2	q_7	q_8	q_{10}	q_{12}
-106,5	-18,5	-64,5	0	0

From figures 4 and 5, respectively 6 and 7 it is clearly that, the positioning and orientation vectors are the same after the results obtained from the general mathematical model. The generalized coordinates calculated with the general model 3T3R–3T3R are introduced into the SolidWorks where are given the same values to allow to the tool to process the machining points desired.

Having identically results, it meaning that the general mathematical model 3T3R–3T3R is validate in the case of machining *without* introducing *initial errors* (fig. 5, 7).

2.2 The case of machining workpieces with initial errors

Just as the previously paragraph, the required input parameters are remain the same, only difference being that, in the mathematical model it will be inserted the initial errors. These are noted with d_x, d_y, d_z the *linear errors* and with $\delta_x, \delta_y, \delta_z$ the *angular errors*. These errors can be imposed or measured; a fact important is that to know the size of the errors to determinate the generalized coordinates.

Having a structure type 1T–2T it is obvious that the angular errors will be zero, and will not appear on the calculus of mathematical model.

The points which it will be machined are remaining unchanged.

The parameters which must be inserted into the model are the positioning and orientation vectors and the linear and angular errors. These are represented in the figure 10 and table 7.

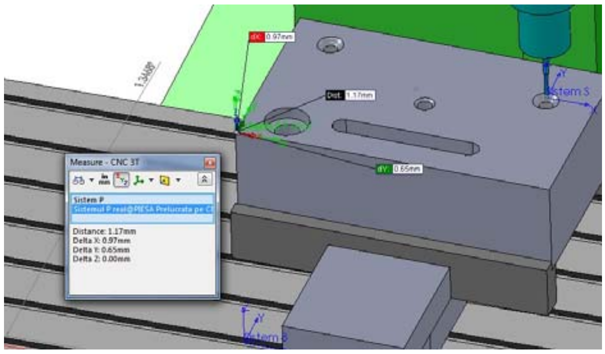

Fig. 8. The values of the initial errors

We must mention that, the new system of the part is the real system, noted with $S_{P_{real}}$ (fig. 8). In this case, the system is overlap on nominal system of the workpiece, noted with S_P.

The steps are remain the same like in the case without errors, which mean that we make the trajectory of the TOOL from points 1,2, and 3 to be machined, with the difference that it will be initial errors. The values of the imposed errors are listed in the table 7.

Table 7

Linear errors $\langle mm \rangle$			*Angular errors* $\langle ^{o} \rangle$		
d_x	d_y	d_z	δ_x	δ_y	δ_z
0,97	0,65	0,75	0	0	1,3468

The new coordinates for machining of point 1 and 2 are presented in figure 9.

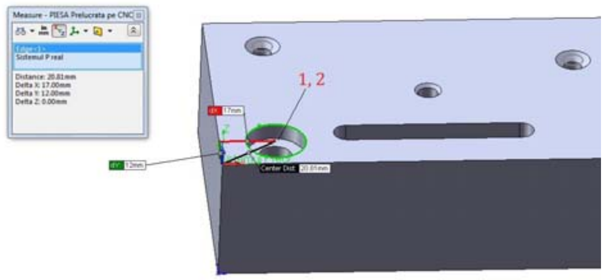

Fig. 9. Positioning and orientation coordinates with initial errors for points 1 and 2

These errors (tab. 7) and the positioning and orientating vectors (tab. 6) are introduced in the cooperating general model 3T3R–3T3R resulting the compensating generalized coordinates, which allow to accomplish a machining process correctly and precisely. The coordinates generalized obtained from the mathematical model are presented in the table 8.

The value of q_2, q_7 and q_8 represent the displacements necessary on X, Y and Z axis in such manner that, the workpieces to being correctly processed even these are wrong positioned into fixing devices are represent the compensating generalized coordinates presented in table 9.

Table 8

The generalized coordinates ⟨*mm*⟩		
q_2	q_7	q_8
-106,5	***8***	***-17***

Table 9

The compensating generalized coordinates ⟨*mm*⟩		
q_2	q_7	q_8
-106,5	***7,7183***	***-16,3305***

For machining of the point 3 (fig. 11) the positioning and orientation vectors remain the same (tab. 10). Additionally into the general mathematical model 3T3R–3T3R are introduced the initial errors (tab. 7).

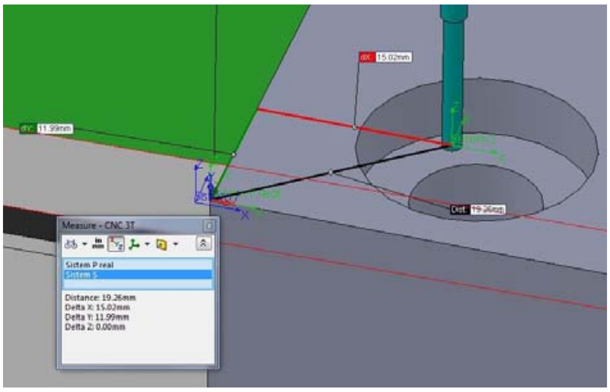

Fig. 10. The validation of the generalized coordinates for the machining points 1 and 2, the case with initial errors

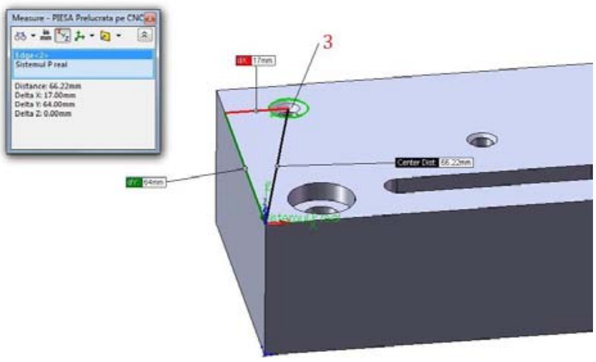

Fig. 11. Positioning and orientation coordinates with initial errors for point 3

After the calculating the new generalized coordinates, these are inserted on SolidWorks which give the new values of the displacements along the X, Y and z axis in such manner that, the piece it will be correctly and precisely machined (fig. 12 and tab. 11).

Table 10

The generalized coordinates ⟨*mm*⟩		
q_2	q_7	q_8
-106,5	***-44***	***-17***

Table 11

The compensating generalized coordinates ⟨*mm*⟩		
q_2	q_7	q_8
-106,5	***-45,1401***	***-17,5746***

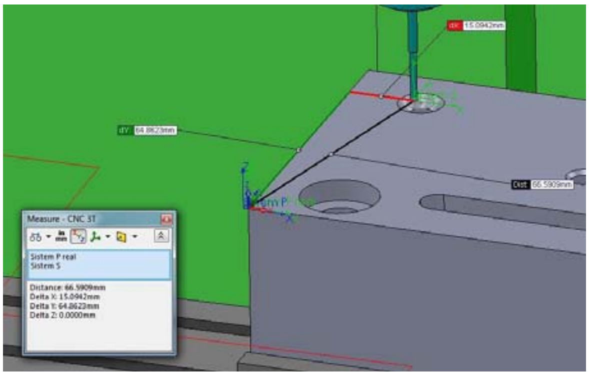

Fig. 12. The validation of the generalized coordinates for the machining point 3, the case with initial errors

In both variants *with errors* and *without errors* the particularization of the mathematical model 3T3R – 3T3R applied on a structure type 1T – 2T by results obtained was validate (fig. 5 and 7, fig. 10 and 12).

3. CONCLUSIONS

Mathematical modeling of the structure type 3T3R–3T3R represent an important realization, because starting from this it can be determinate the generalized coordinates for any type of kinematic structure.

An example in this way is particularization of the general model 3T3R – 3T3R applied on a structure 1T–2T, presented here.

The numerical analysis, by the results obtained confirm that the general model type 3T3R–3T3R, first is work properly and the second it can be applied for any type of kinematic structure.

A significant fact is also knowing the size of initial errors, even them, imposed or measured can be determinate the auxiliary displacement for the motor couples. Knowing this, the next parts, which must be process, will be correctly manufactured.

Also, is allowing to realize parts correctly, precisely even these are incorrectly, and imprecise installed on fixing devices.

4. SELECTIVE REFERENCES

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Particularizarea modelului matematic general 3T3R – 3T3R aplicat unei structuri de tipul 1T – 2T

Rezumat: Pornind de la ecuațiile celei mai complexe structuri de tip 3T3R – 3T3R prin utilizarea anumitor restricții în modelul generalizat se pot obține orice tip de structură cinematică. Particularizarea se face relativ simplu prin aplicarea unor restricții asupra parametrilor geometrici-constructivi şi a coordonatelor liniare şi unghiulare, acestea depinzând de structura cinematică luată în studiu. În această lucrare sunt prezentate ecuațiile pentru cea mai simplă şi des utilizată structură a unei maşini unelte, şi anume 1T – 2T.

Ioan Vuscan, Prof. Dr. Eng., Technical University of Cluj-Napoca, Manufacturing Engineering Department, Gheorghe.Vuscan@tcm.utcluj.ro, str. Aviator Badescu, no. 24A, Cluj-Napoca.

Iuliu Negrean, Prof. Dr. Eng., Head of Department of of Mechanical Systems Engineering, iuliu.negrean@mep.utcluj.ro, Office Phone 0264/401616.

Kalman Kacso, Assistant, Technical University of Cluj-Napoca, Department of Mechanical Systems Engineering Department, kacsokalman@gmail.com, Office Phone 0264/401616.

Ancuta Mircea, Dr. Eng., SC Alphametals, aecrimancuta@yahoo.com, str. Brates no. 5, Cluj-Napoca, 0748/362266.