TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue III, 2012

CONTRIBUTIONS TO THE GEOMETRIC MODELING OF INDUSTRIAL MODULAR, ARTICULATED, SERIAL ROBOTS WITH FIVE DEGREES OF FREEDOM, USING SYMBOLIC COMPUTATION

Florin BUGNAR, Ovidiu-Aurelian DETEŞAN, Viorel ISPAS

Abstract: *The aim of this paper is to present the way the symbolic computation in MATLAB can be used for determining the geometric model equations of the modular, articulated, serial industrial robots, with five degrees of freedom. A MATLAB script was created in this purpose, using as input data the position-orientation matrices describing the geometry of the considered robot, the output data being the geometric model equations, written in the algebraic form.*
Key words: *serial robot, geometric modelling, symbolic computation.*

1. INTRODUCTION

The geometric, kinematic and dynamic modelling of robots is a complex process that involves the use of iterative matrix computation. The complexity of this process is increased as the number of the rotation joints from the mechanical structure of the robot is bigger.

In the case if articulated robots, all their joints are rotation joints. An example of an articulated robot is Fanuc LR Mate 100iB [5], from the Laboratory of Applied Mechanics in Robotics, within the Department of Mechanical System Engineering from our university.

Given a robot with *n* degrees of freedom, the presence of rotation joints determines the increase in the complexity of the rotation sub-matrix ${}_0^n[R]$, part of the position-orientation matrix ${}_0^n[T]$, corresponding to the gripper of the robot, with respect to the fixed frame $\{0\}$.

2. MATLAB COMPUTATION BENEFITS

The commands and functions from the module *Symbolic Math Toolbox* within MATLAB [10, 11] are very useful in order to easily determine the equations of the geometric model of the considered robot. Thus, with the help of *syms* command or *sym()* function, one can define symbolic objects, identified just by their names, objects that can be used afterwards in complex symbolic computations.

The first step in geometric modelling of the robot is to define its mechanical structure. The geometric dimensions $l_i,\ i = 1 \div m$ and the generalized variables $q_i,\ i = 1 \div n$ will be defined as symbolic objects, and the geometric modelling algorithm [6, 7, 8, 9] will be applied on them. Once the equations of the geometric model are obtained, they can be easily numerically tailored by numerical substitutions, using the *subs()* function, obtaining various numerical simulations based on the determined symbolic equations. The *simple()* and *simplify()* functions allow the conversion of the obtained results into a more convenient shape.

3. THE 5R MODULAR, ARTICULATED INDUSTRIAL ROBOT

In order to illustrate the application of the symbolic computation in determining the geometric model, the mechanical structure of the 5R industrial robot has been chosen. This is a modular, articulated, industrial robot, whose kinematic diagram is shown in Fig. 1.

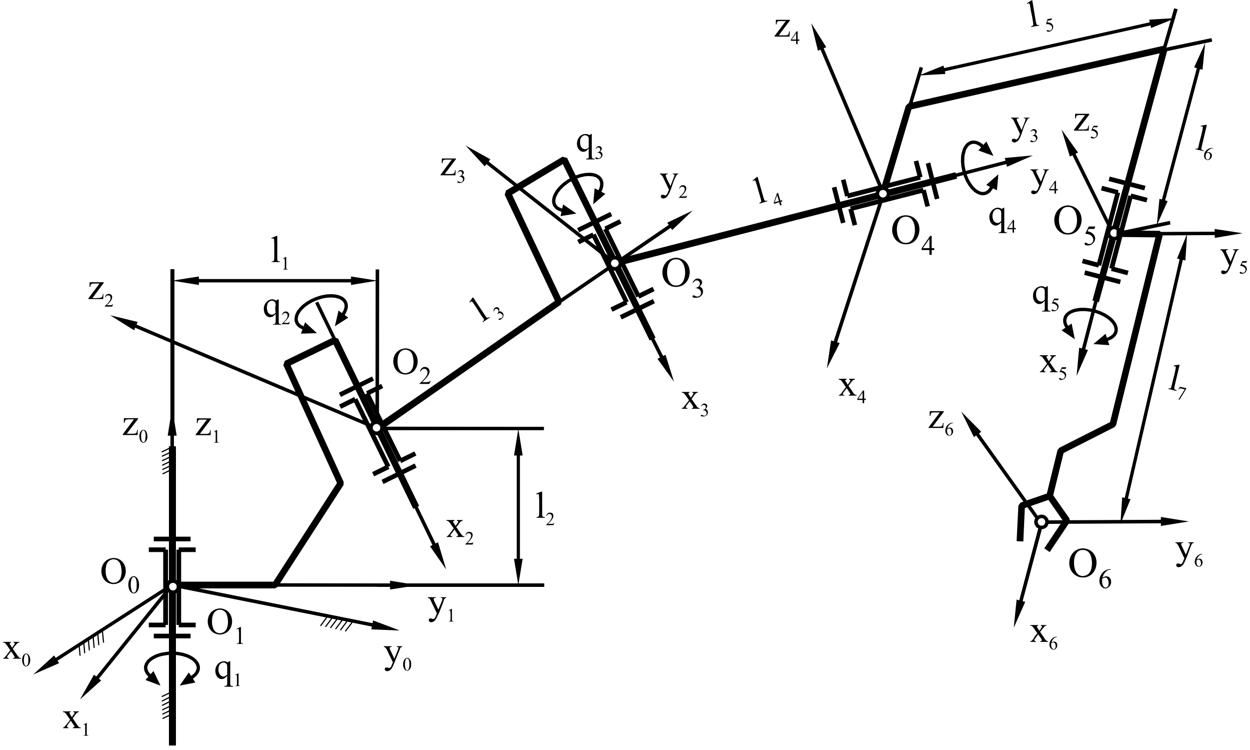

Fig. 1. The kinematic diagram of the 5R modular industrial robot

The above presented robot has five rotation joints, described as:

- the joint (1), making a rotation around the fixed vertical axis $O_1z_1 \equiv O_0z_0$ with the angle q_1
- the joint (2), rotating with the angle q_2 with respect to the axis O_2x_2
- the joint (3), having the generalized variable q_3, of rotation around the axis O_3x_3
- the joint (4), rotating with q_4, around the axis O_4y_4
- the joint (5), with the rotation angle q_5, around the axis O_5x_5.

The gripper is attached at the end of the joint (5) and it is oriented along the axis O_6x_6. The geometric dimensions of the robot are denoted by $l_1 ... l_7$.

4. PREPARING THE WORKING ENVIRONMENT

For the beginning, we must be sure that all the variables and defined functions from the MATLAB workspace are erased [1]. This can be done by performing *clear all*. Also, we need the results to be displayed as text in the Command window. The command *clc* clears the content of Command window. We also need the results to be written in a diary, as a text file. Let *RRRRR_geo.txt* be the name of the file. Because we do not intend the output to be overwritten to a possibly existing file with that name, we ensure that there is no file with the specified name:

```
clear all
clc
if exist('RRRRR_geo.txt')
    delete RRRRR_geo.txt
end
diary RRRRR_geo.txt
```

5. DEFINING THE MECHANICAL STRUCTURE OF THE ROBOT

In order to define the mechanical structure of the robot, the variables l_i, $i = 1 \div 7$ as well as the generalized coordinates q_i, $i = 1 \div 5$ are defined as symbolic objects:

```
syms l1 l2 l3 l4 l5 l6 l7 q1 q2 q3...
q4 q5 real
```

The position and orientation of the frames attached to the joints (i) as well as the gripper (6) are expressed by the layout matrices ${}^{i-1}_{i}[T], i = 1 \div n+1$. They contain the rotation sub-matrices ${}^{i-1}_{i}[R]$, expressing the orientation of the frame $\{i\}$ axes with respect to the frame $\{i\text{-}1\}$, as well as the position vectors ${}^{i-1}\overline{p}_i$, expressing the position of the origins (O_i) of the frames $\{i\}$ with respect to the origins (O_{i-1}) of the frames $\{i\text{-}1\}$.

Given the description of the mechanical structure, the expressions of these matrices are the following:

$$ {}^{0}_{1}[T] = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1) $$

$$ {}^{1}_{2}[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_2 & -sq_2 & l_1 \\ 0 & sq_2 & cq_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2) $$

$$ {}^{2}_{3}[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_3 & -sq_3 & l_3 \\ 0 & sq_3 & cq_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3) $$

$$ {}^{3}_{4}[T] = \begin{bmatrix} cq_4 & 0 & sq_4 & 0 \\ 0 & 1 & 0 & l_4 \\ -sq_4 & 0 & cq_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4) $$

$$ {}^{4}_{5}[T] = \begin{bmatrix} 1 & 0 & 0 & l_6 \\ 0 & cq_5 & -sq_5 & l_5 \\ 0 & sq_5 & cq_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5) $$

$$ {}^{5}_{6}[T] = \begin{bmatrix} 1 & 0 & 0 & l_7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6) $$

These matrices are expressed in MATLAB as follows:

```
T10=[c(q1) -s(q1) 0 0; s(q1) c(q1)...
0 0; 0 0 1 0;0 0 0 1]
T21=[1 0 0 0;0 c(q2) -s(q2) l1;0...
s(q2) c(q2) l2;0 0 0 1]
T32=[1 0 0 0;0 c(q3) -s(q3) l3;0...
s(q3) c(q3) 0;0 0 0 1]
T43=[c(q4) 0 s(q4) 0; 0 1 0 l4;...
-s(q4) 0 c(q4) 0; 0 0 0 1]
T54=[1 0 0 l6; 0 c(q5) -s(q5) l5;...
0 s(q5) c(q5) 0;0 0 0 1]
T65=sym(eye(4,3));
T65(:,4)=[l7; 0; 0; 1]
```

In the script above, *eye*(4,3) specifies the identity matrix with 4 lines and 3 columns, while the functions *c*() and *s*() are the shortcuts for *cos*() and *sin*().

6. DETERMINING THE EQUATIONS OF THE GEOMETRIC MODEL

By matrix multiplications of the type:

$$ {}^{0}_{i}[T] = {}^{0}_{i-1}[T] \cdot {}^{i-1}_{i}[T] \quad (7) $$

the position-orientation matrices of the frames $\{i\}$ attached to the joints (i), with respect to the fixed frame $\{0\}$, are determined.

Among these matrices, the most important is the matrix ${}^{0}_{6}[T]$. It expresses the position of the characteristic point of the gripper (which is the same with the origin O_6 of the frame $\{6\}$), by the position vector $\overline{p}_6$. The orientation of the axes of the frame $\{6\}$ with respect to the fixed frame $\{0\}$ is expressed by the elements of the rotation sub-matrix ${}^{0}_{6}[R]$, which are, by columns, the Cartesian components of the versors of the frame $\{6\}$ axes, with respect to the frame $\{0\}$.

In MATLAB, these matrix multiplications can be represented as:

```
T20=simplify(T10*T21)
T30=simplify(T20*T32)
T40=simplify(T30*T43)
T50=simplify(T40*T54)
T60=simplify(T50*T65)
```

The obtained results for the second joint can be rewritten as follows:

$$ {}_2^0[T]=\begin{bmatrix} cq_1 & -cq_2sq_1 & sq_1sq_2 & -l_1sq_1 \\ sq_1 & cq_1cq_2 & -cq_1sq_2 & l_1cq_1 \\ 0 & sq_2 & cq_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8) $$

For the third joint we have the following layout matrix (9), rotation matrix (10) with the elements (11-19) and the position vector (20):

$$ {}_3^0[T]=\begin{bmatrix} {}_3^0[R] & \bar{p}_3 \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix}, \quad (9) $$

$$ {}_3^0[R]=\begin{bmatrix} {}_3^0R_{1,1} & {}_3^0R_{1,2} & {}_3^0R_{1,3} \\ {}_3^0R_{2,1} & {}_3^0R_{2,2} & {}_3^0R_{2,3} \\ {}_3^0R_{3,1} & {}_3^0R_{3,2} & {}_3^0R_{3,3} \end{bmatrix}, \quad (10) $$

$$ {}_3^0R_{1,1}=cq_1, \quad (11) $$

$$ {}_3^0R_{1,2}=-c(q_2+q_3)\cdot sq_1, \quad (12) $$

$$ {}_3^0R_{1,3}=s(q_2+q_3)\cdot sq_1, \quad (13) $$

$$ {}_3^0R_{2,1}=sq_1, \quad (14) $$

$$ {}_3^0R_{2,2}=c(q_2+q_3)\cdot cq_1, \quad (15) $$

$$ {}_3^0R_{2,3}=-s(q_2+q_3)\cdot cq_1, \quad (16) $$

$$ {}_3^0R_{3,1}=0, \quad (17) $$

$$ {}_3^0R_{3,2}=s(q_2+q_3), \quad (18) $$

$$ {}_3^0R_{3,3}=c(q_2+q_3), \quad (19) $$

$$ \bar{p}_3=\begin{bmatrix} -sq_1\cdot(l_1+l_3cq_2) \\ cq_1\cdot(l_1+l_3cq_2) \\ l_2+l_3sq_2 \end{bmatrix}. \quad (20) $$

The fourth joint is featured by the following layout matrix (21), the rotation matrix (22) with the elements (23-31) and the position vector (32):

$$ {}_4^0[T]=\begin{bmatrix} {}_4^0[R] & \bar{p}_4 \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix}, \quad (21) $$

$$ {}_4^0[R]=\begin{bmatrix} {}_4^0R_{1,1} & {}_4^0R_{1,2} & {}_4^0R_{1,3} \\ {}_4^0R_{2,1} & {}_4^0R_{2,2} & {}_4^0R_{2,3} \\ {}_4^0R_{3,1} & {}_4^0R_{3,2} & {}_4^0R_{3,3} \end{bmatrix}, \quad (22) $$

$$ {}_4^0R_{1,1}=cq_1cq_4-s(q_2+q_3)\cdot sq_1sq_4, \quad (23) $$

$$ {}_4^0R_{1,2}=-c(q_2+q_3)\cdot sq_1, \quad (24) $$

$$ {}_4^0R_{1,3}=cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1, \quad (25) $$

$$ {}_4^0R_{2,1}=cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4, \quad (26) $$

$$ {}_4^0R_{2,2}=c(q_2+q_3)\cdot cq_1, \quad (27) $$

$$ {}_4^0R_{2,3}=sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4, \quad (28) $$

$$ {}_4^0R_{3,1}=-c(q_2+q_3)\cdot sq_4, \quad (29) $$

$$ {}_4^0R_{3,2}=s(q_2+q_3), \quad (30) $$

$$ {}_4^0R_{3,3}=c(q_2+q_3)\cdot cq_4, \quad (31) $$

$$ \bar{p}_4=\begin{bmatrix} -sq_1(l_1+l_3cq_2)-l_4c(q_2+q_3)\cdot sq_1 \\ cq_1(l_1+l_3cq_2)+l_4\cdot c(q_2+q_3)\cdot cq_1 \\ l_2+l_4s(q_2+q_3)+l_3sq_2 \end{bmatrix}. \quad (32) $$

The last joint has the following layout matrix (33), the rotation matrix (34) with the elements (35-43) and the position vector (44) with its explicit elements (45-47):

$$ {}_5^0[T]=\left[\begin{array}{ccc|c} & {}_5^0[R] & & \bar{p}_5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (33) $$

$$ {}_5^0[R]=\left[\begin{array}{c|c|c} {}_5^0R_{1,1} & {}_5^0R_{1,2} & {}_5^0R_{1,3} \\ \hline {}_5^0R_{2,1} & {}_5^0R_{2,2} & {}_5^0R_{2,3} \\ \hline {}_5^0R_{3,1} & {}_5^0R_{3,2} & {}_5^0R_{3,3} \end{array}\right], \quad (34) $$

$$ {}_5^0R_{1,1} = cq_1cq_4 - s(q_2+q_3)\cdot sq_1sq_4, \quad (35) $$

$$ \begin{aligned} {}_5^0R_{1,2} = {} & sq_5cq_1sq_4+s(q_2+q_3)\cdot \\ & cq_4sq_1 - c(q_2+q_3)\cdot cq_5sq_1 \end{aligned}, \quad (36) $$

$$ \begin{aligned} {}_5^0R_{1,3} = {} & cq_5(cq_1sq_4+s(q_2+q_3)\cdot \\ & cq_4sq_1)+c(q_2+q_3)\cdot sq_1sq_5 \end{aligned}, \quad (37) $$

$$ {}_5^0R_{2,1} = cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4 , \quad (38) $$

$$ \begin{aligned} {}_5^0R_{2,2} = {} & sq_5(sq_1sq_4 - s(q_2+q_3)\cdot \\ & cq_1cq_4) + c(q_2+q_3)\cdot cq_1cq_5 \end{aligned}, \quad (39) $$

$$ \begin{aligned} {}_5^0R_{2,3} = {} & cq_5(sq_1sq_4 - s(q_2+q_3)\cdot \\ & cq_1cq_4) - c(q_2+q_3)\cdot cq_1sq_5) \end{aligned}, \quad (40) $$

$$ {}_5^0R_{3,1} = -c(q_2+q_3)\cdot sq_4 , \quad (41) $$

$$ \begin{aligned} {}_5^0R_{3,2} = {} & s(q_2+q_3)\cdot cq_5+ \\ & c(q_2+q_3)\cdot cq_4sq_5 \end{aligned}, \quad (42) $$

$$ \begin{aligned} {}_5^0R_{3,3} = {} & c(q_2+q_3)\cdot cq_4cq_5 - \\ & s(q_2+q_3)\cdot sq_5 \end{aligned}, \quad (43) $$

$$ \bar{p}_5 = \begin{bmatrix} p_{5x} \\ p_{5y} \\ p_{5z} \end{bmatrix}, \quad (44) $$

$$ \begin{aligned} p_{5x} = {} & l_6\cdot(cq_1cq_4 - s(q_2+q_3)\cdot sq_1sq_4) - \\ & sq_1(l_1+l_3cq_2) - (l_4+l_5)c(q_2+q_3)\cdot sq_1 \end{aligned}, \quad (45) $$

$$ \begin{aligned} p_{5y} = {} & l_6\cdot(cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4)+ \\ & cq_1(l_1+l_3cq_2)+(l_4+l_5)\cdot c(q_2+q_3)\cdot cq_1 \end{aligned}, \quad (46) $$

$$ \begin{aligned} p_{5z} = {} & l_2+(l_4+l_5)\cdot s(q_2+q_3)+ \\ & l_3sq_2 - l_6\cdot c(q_2+q_3)\cdot sq_4 \end{aligned}. \quad (47) $$

Because the gripper has the same orientation as the fifth joint, we only present the components (50-52) of the position vector (49), part of the layout matrix (48):

$$ {}_6^0[T]=\left[\begin{array}{ccc|c} & {}_6^0[R] & & \bar{p}_6 \\ \hline 0 & 0 & 0 & 1 \end{array}\right], \quad (48) $$

$$ \bar{p}_6 = \begin{bmatrix} p_{6x} \\ p_{6y} \\ p_{6z} \end{bmatrix}, \quad (49) $$

$$ \begin{aligned} p_{6x} = {} & (l_6+l_7)\cdot(cq_1cq_4 - s(q_2+q_3)\cdot sq_1sq_4) - \\ & sq_1(l_1+l_3cq_2) - (l_4+l_5)\cdot c(q_2+q_3)\cdot sq_1 \end{aligned} \quad (50) $$

$$ \begin{aligned} p_{6y} = {} & (l_6+l_7)\cdot(cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4)+ \\ & cq_1(l_1+l_3cq_2)+(l_4+l_5)\cdot c(q_2+q_3)\cdot cq_1 \end{aligned} \quad (51) $$

$$ \begin{aligned} p_{6z} = {} & l_2+(l_4+l_5)\cdot s(q_2+q_3)+l_3sq_2 - \\ & (l_6+l_7)\cdot c(q_2+q_3)\cdot sq_4 \end{aligned}. \quad (52) $$

The equations of the geometric model of the robot are represented eventually by the expressions (49-52), depicting the position of the characteristic point of the gripper and by the expressions (34-43), representing the orientation of the axes of the frame {6}, with respect to the fixed frame {0}.

7. DATA SAVING

The obtained results will be saved in a MATLAB data file (with *.mat* extension) and they are intended to be passed as input data in the algorithm of kinematic modeling. This can be done by performing the *save* command. The suggested filename is *RRRRR_geo.mat*. Also, we need to close the diary text file, in order to write the content of the Command window to the file and to be accessible for reading.

```
save RRRRR_geo.mat
diary off
```

8. CONCLUSION

The script presented in this paper allows the user to generate the symbolic equations of the geometric model of the 5R (RRRRR) robot, further used in the kinematic and dynamic analysis of the robot.

It can be adapted to any robot with rotary joints, having 2-6 degrees of freedom, by changing the input data that define the geometry of the analyzed robot and it is useful in both research and teaching activities in the field of Robotics.

9. REFERENCES

1. Deteşan, O.A., *Cercetări privind modelarea, simularea şi construcţia miniroboţilor,* Ph.D. Thesis, U.T.C.N., Cluj-Napoca, 2007.
2. Deteşan, O.A., *The Definition of the Robot Mechanical Structure Using the Symbolic Computation in Matlab*. In: Acta Technica Napocensis, No.51, vol. IV, 2008, ISSN 1221-5872.
3. Deteşan, O.A., *The Implementation of the TRR Robots Modeling Using the Symbolic Computation in Matlab*. In: Acta Technica Napocensis, No.51, vol. IV, 2008, ISSN 1221-5872.
4. Deteşan, O.A., Ispas, Vrg., *The Kinematic Modelling of the Robot Mechanical Structure Using the Symbolic Computation in Matlab*, Annals of DAAAM for 2009 & Proceedings of 20th DAAAM International Symposium "Intelligent Manufacturing & Automation: Theory, Practice & Education" (ISI Proceedings), Viena, 2009, ISSN 1726-9679.
5. FANUC Robot Series, R-J3iB Mate Controller, LR Handling Tool, Operator's Manual, B81524EN/02, 2006.
6. Ispas, V., *Manipulatoare şi Roboţi Industriali.* Bucureşti. Editura Didactică şi Pedagogică, 2004, ISBN 973-30-1349-8.
7. Negrean, I., Vuşcan, I., Haiduc, N., *Robotics. Kinematic and Dynamic Modelling*, Bucureşti, Editura Didactică şi Pedagogică, 1998, ISBN 973-30-5309-8.
8. Negrean, I., *Kinematics and Dynamics of Robots. Modelling, Experiment, Accuracy*, Editura Didactică şi Pedagogică, 1999, ISBN 973-30-9313-0.
9. Negrean, I., Duca, A., Negrean, C., Kacso, K., *Mecanică avansată în robotică*, Cluj-Napoca, UP Press, 2008, ISBN 978-973-662-420-9.
10. Wilson, H.B., Turcotte, L.H., Halpern, D.: *Advanced Mathematics and Mechanics Applications Using MATLAB*, Boca Raton, New York, Washington D.C., Chapman & Hall/CRC, 2003, ISBN 1-58488-262-X.
11. *** *MATLAB, The Language of Technical Computing – Function Reference*, The MathWorks Inc., USA, 2004.

CONTRIBUŢII LA MODELAREA GEOMETRICĂ A ROBOŢILOR INDUSTRIALI SERIALI, ARTICULAŢI, MODULARI, CU CINCI GRADE DE LIBERTATE, FOLOSIND CALCULUL SIMBOLIC

Rezumat: Scopul lucrării de faţă este prezentarea modului în care poate fi utilizat calculul simbolic în MATLAB pentru determinarea ecuaţiilor modelului geometric al roboţilor industriali seriali articulaţi, modulari, cu cinci grade de libertate. În acest scop este creat un script MATLAB, folosind ca şi date de intrare matricele de poziţie-orientare care descriu geometria robotului considerat, datele de ieşire fiind chiar ecuaţiile modelului geometric, sub formă algebrică.

Florin Bugnar, Ph.D. student, Eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, bugnarf@hotmail.com, +40-64-401759, Cluj-Napoca, Romania.

Ovidiu-Aurelian Deteşan, Ph.D., Assoc. Prof., Eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, Ovidiu.Detesan@mep.utcluj.ro, +40-64-401667, Cluj-Napoca, Romania.

Viorel Ispas, Ph.D., Prof., Eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, Viorel.Ispas@mep.utcluj.ro, +40-64-401659, Cluj-Napoca, Romania.