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THE MODELING OF WORKING PROCESS OF THE SERIAL STRUCTURE FANUC

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Abstract: *In the paper is presented the working process modeling of the robot Fanuc LR Mate 100 iB, by using of 5-4-6 type polynomial interpolation functions. These higher order polynomial functions are used to study the jerk, specific to a robot with a complex mechanical structure, where is necessary to determine the higher order accelerations. In this case, the study extends to second order accelerations, called jerk in the literature [3]. This notion, represents first order absolute derivative with respect to time of the vector acceleration. Therefore, in the following analysis will determined the expressions of linear and angular jerk. The determination of control functions requires the mathematical modeling of the serial robot, presented in the first part of this paper.*

Key words: *exponential functions, motion trajectories, workflow, interpolation polynomial functions.*

1 THE MATHEMATICAL MODELING OF THE ROBOT

1.1 The direct geometrical modeling of the serial structure

Further, is realized the direct geometrical modeling of the serial structure *FANUC LR Mate 100 iB on the basis of algorithms specific to geometry , according to* [2], [1]. Also, throughout this part, will be studied the robot inverse geometrical model mentioned above.

To establish the direct geometrical model of the *FANUC LR Mate 100 iB* will be used the matrix exponential algorithm. The FANUC structure, has five degrees of freedom. By applying the matrix exponential algorithm, there is determined the position vector and the rotation matrix between the frames $\{0\} \rightarrow \{6\}$. These are describing the location (position and orientation) of end effector with respect to $\{0\}$ fixed frame attached to the fixed base of robot (Cartesian space). The rotation matrix and respectively the position vector are defined with the following expressions:

$$R_{i0} = \left\{ \exp\left\{ \sum_{j=1}^{i} \left\{ \overline{k}_j^{(0)} \times \right\} q_j \Delta_j \right\} \right\} \cdot R_{i0}^{(0)} = \\ = \prod_{j=1}^{i} \exp\left\{ \left\{ \overline{k}_j^{(0)} \times \right\} q_j \Delta_j \right\} \cdot R_{i0}^{(0)}; \quad ;(1)$$

$$\overline{p}_i = \sum_{j=1}^{i} \left\{ \exp\left\{ \sum_{k=0}^{j-1} \left\{ \overline{k}_k^{(0)} \times \right\} q_k \cdot \Delta_k \right\} \right\} \cdot \overline{b}_j \; ;(2)$$

$$\overline{b}_i = \left\{ I_3 \cdot q_i + \left\{ \overline{k}_i^{(0)} \times \right\} \left[1 - c\left(q_i \cdot \Delta_i\right) \right] + \\ + \overline{k}_i^{(0)} \cdot \overline{k}_i^{(0)T} \cdot \left[q_i - s\left(q_i \cdot \Delta_i\right) \right] \right\} \cdot \overline{v}_i^{(0)}; \quad ;(3)$$

In the previous expressions $\overline{k}_i = \{\overline{x}_i; \overline{y}_i; \overline{z}_i\}$ and $\overline{v}_i = \{\overline{p}_i \times\} \overline{k}_i \cdot \Delta_i + (1 - \Delta_i) \cdot \overline{k}_i$ are representing the screw parameters of the oriented axis $\{i\}$ around or along which are registered the generalized coordinates.

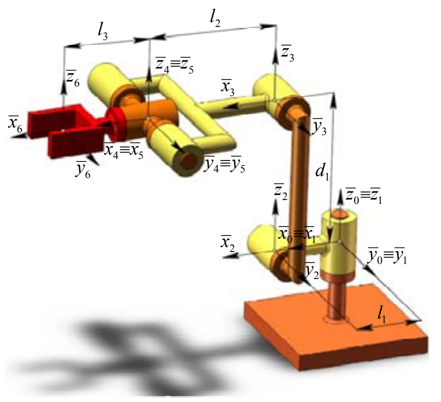


Table 1

The nominal geometry matrix

Elem.	*Link*	$\overline{k}_i^{(0)T}$			$\overline{p}_i^{(0)T}$			$\overline{v}_i^{T}$		
1	R	0	0	1	0	0	0	0	0	0
2	R	0	1	0	l_1	0	0	0	0	l_1
3	R	0	1	0	l_1	0	d_1	$-d_1$	0	l_1
4	R	0	1	0	$l_{1,2}$	0	d_1	$-d_1$	0	$l_{1,2}$
5	R	1	0	0	$l_{1,2}$	0	d_1	0	d_1	0
6	-	0	1	0	$l_{1,2,3}$	0	d_1	-	-	-

where $l_{1,2} = l_1 + l_2$; $l_{1,2,3} = l_1 + l_2 + l_3$;

Having the kinematic scheme of the 5R type robot, according to *Fig. 1,* respectively the nominal geometry matrix $M_{vn}^{(0)}$ (see *Table 1*), specific to initial configuration $\overline{\theta}^{(0)}$ of robot, there is determined the rotation matrix, respectively the position vector of the frame attached to the characteristic point $\{6\}$ with respect to fixed reference frame, using the expressions (1) and (2), resulting:

$${}^0_6[R] = \begin{bmatrix} cq_{2,3,4} \cdot cq_1 & sq_5 \cdot sq_{2,3,4} \cdot cq_1 - cq_5 \cdot sq_1 & cq_5 \cdot sq_{2,3,4} \cdot cq_1 + sq_5 \cdot sq_1 \\ cq_{2,3,4} \cdot sq_1 & sq_5 \cdot sq_{2,3,4} \cdot sq_1 + cq_5 \cdot cq_1 & cq_5 \cdot sq_{2,3,4} \cdot sq_1 - sq_5 \cdot cq_1 \\ -sq_{2,3,4} & cq_{2,3,4} \cdot sq_5 & cq_{2,3,4} \cdot cq_5 \end{bmatrix};$$

$$\overline{p}_6 = \begin{bmatrix} cq_1 \cdot \left(l_1 + d_1 \cdot sq_2 + l_2 \cdot cq_{2,3} + l_3 \cdot cq_{2,3,4}\right) \\ sq_1 \cdot \left(l_1 + d_1 \cdot sq_2 + l_2 \cdot cq_{2,3} + l_3 \cdot cq_{2,3,4}\right) \\ d_1 \cdot cq_2 - l_2 \cdot sq_{2,3} - l_3 \cdot sq_{2,3,4} \end{bmatrix}; \quad (5)$$

In order to establish the resultant column vector of the homogenous position and orientation parameters, it must be determined the Euler`s angle set, which expresses the orientation of the frame $\{6\}$ with respect to $\{0\}$ by using the rotation matrix ${}^0_6[R] = R(\alpha_z; \beta_y; \gamma_x)$:

$$-s\beta_y = -s\,q_{2,3,4} \Rightarrow \beta_y = q_{2,3,4};$$
$$\begin{cases} c\beta_y \cdot s\gamma_x = c\,q_{2,3,4} \cdot s\,q_5 \\ c\beta_y \cdot c\gamma_x = c\,q_{2,3,4} \cdot c\,q_5 \end{cases} \Rightarrow \gamma_x = q_5; \quad (6)$$
$$\begin{cases} c\alpha_z \cdot c\beta_y = cq_1 \cdot c\,q_{2,3,4} \\ s\alpha_z \cdot c\beta_y = sq_1 \cdot c\,q_{2,3,4} \end{cases} \Rightarrow \alpha_z = q_1.$$

From (6) are resulting the set of Euler angles which in the case of the articulated structure with 5 d.o.f, 5R is defined by using the expression: $\overline{\Omega} = \left[\alpha_z \;\; \beta_y \;\; \gamma_x\right]^T = \left[q_1 \;\; q_{2,3,4} \;\; q_5\right]^T$;

Thus, the column vector of the homogenous position and orientation parameters is obtained as:

$${}^0\overline{X} = \begin{bmatrix} cq_1 \cdot \left(l_1 + d_1 \cdot sq_2 + l_2 \cdot cq_{2,3} + l_3 \cdot cq_{2,3,4}\right) \\ sq_1 \cdot \left(l_1 + d_1 \cdot sq_2 + l_2 \cdot cq_{2,3} + l_3 \cdot cq_{2,3,4}\right) \\ d_1 \cdot cq_2 - l_2 \cdot sq_{2,3} - l_3 \cdot sq_{2,3,4} \\ \hline q_1 \\ q_{2,3,4} \\ q_5 \end{bmatrix}. \quad (7)$$

Remarks: Using as comparative study the two algorithms specific to direct geometry, there is a perfect identity of the results, so that each of the two algorithms have specific features regarding the direct geometrical modeling.

1.2 The Inverse geometry equations for the robot Fanuc

To establish the inverse geometrical equations, there is assumed that is known the position of the end effector $\{6\}$ with respect to the fixed reference frame, so the column vector of the homogeneous position and orientation parameters ${}^{0}\bar{X}$ is expressed by numerical values as:

$$ {}^{0}\bar{X} = \begin{bmatrix} p_x & p_y & p_z & \vdots & \beta_y & \gamma_x \end{bmatrix}^T . \quad (8) $$

Knowing the homogeneous vector parameters it can be determined locating matrix between the system attached to the end effector and the fixed reference system ${}_{6}^{0}[T]$ shown below:

$$ {}_{6}^{0}[T] = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & p_x \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & p_y \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \prod_{j=1}^{n} {}_{j}^{j-1}[T]; \quad (9) $$

The unknowns of this matrix equation are the generalized coordinates $q_i;\ i = 1 \to 5$. In order to determine these generalized coordinates there is applied an algebrical method, combined with a geometrical one. In keeping with (9) and multiple it at left side with ${}_{1}^{0}[T]^{-1}$ it is obtained:

$$ {}_{0}^{1}[T](q_1) \cdot {}_{6}^{0}[T] = {}_{2}^{1}[T] \cdot {}_{3}^{2}[T] \cdot {}_{4}^{3}[T] \cdot {}_{5}^{4}[T] \cdot {}_{6}^{5}[T]; \quad (10) $$

Developing the expression (10) resulting:

$$ {}_{0}^{1}[T](q_1) \cdot {}_{6}^{0}[T] = \begin{bmatrix} \mathrm{c}q_1 & \mathrm{s}q_1 & 0 & 0 \\ -\mathrm{s}q_1 & \mathrm{c}q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot {}_{6}^{0}[T] = \quad (11) $$

$$ = \begin{bmatrix} X & X & X & p_x \cdot \mathrm{c}q_1 + p_y \cdot \mathrm{s}q_1 \\ X & X & X & -p_x \cdot \mathrm{s}q_1 + p_y \cdot \mathrm{c}q_1 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}; $$

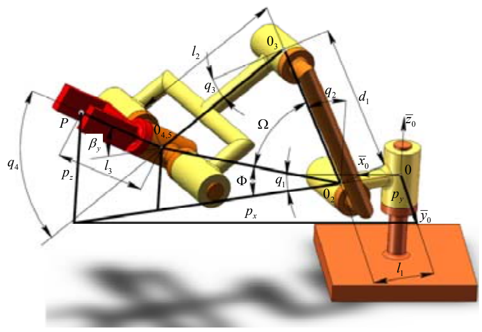
$$ {}_{6}^{1}[T] = \begin{bmatrix} cq_{2,3,4} & sq_5 \cdot sq_{2,3,4} & cq_5 \cdot sq_{2,3,4} & \begin{matrix} l_1 + d_1 \cdot sq_2 + \\ + l_2 \cdot cq_{2,3} + l_3 \cdot cq_{2,3,4} \end{matrix} \\ 0 & cq_5 & -sq_5 & 0 \\ -sq_{2,3,4} & sq_5 \cdot cq_{2,3,4} & cq_5 \cdot cq_{2,3,4} & \begin{matrix} d_1 \cdot cq_2 - l_2 \cdot sq_{2,3} - \\ - l_3 \cdot sq_{2,3,4} \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} . \quad (12) $$

To determine the q_1, by equating of elements $(3;4)$ from (11) and (12) is resulting the following equation, which by solving conducts to:

$-p_x \cdot \mathrm{s}q_1 + p_y \cdot \mathrm{c}q_1 = 0$; resulting

$$ q_1 = \mathrm{Atan\,2}\left(\frac{p_y}{\sqrt{p_x^2 + p_y^2}}; \frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right); \quad (13) $$

To establish the angles q_2, q_3 and q_4 is applied the geometrical method. There is considered the triangle $O_2O_3O_4$ (*Fig. 2*) where $O_2O_3 = l_2$, $O_3O_4 = l_3$. Applying the cosines theorem, there is determined the angle q_3 as:



$$ sq_3 = \frac{d_1^2 + l_2^2 - \left(p_x - \left(l_1 + l_3 \cdot \mathrm{c}\beta_y\right) \cdot cq_1\right)^2}{2 \cdot d_1 \cdot l_2} - \quad (14) $$

$$ - \frac{\left(p_y - \left(l_1 + l_3 \cdot \mathrm{c}\beta_y\right) \cdot sq_1\right)^2 + \left(p_z - l_3 \cdot \mathrm{s}\beta_y\right)^2}{2 \cdot d_1 \cdot l_2}; $$

$$ q_3 = \mathrm{Atan\,2}\left(sq_3; \sqrt{1 - s^2 q_3} \right); \quad (15) $$

From Fig. **2** is resulting the equation: $q_2 = \frac{\pi}{2} - \Phi - \Omega$; on which is obtained:

$$s\Phi = \frac{(p_z - l_3 \cdot s\varphi)}{\sqrt{(p_z - l_3 \cdot s\varphi)^2 + \left(\sqrt{p_x^2 + p_y^2} - l_1 - l_3 \cdot c\varphi\right)^2}};$$

$$c\Phi = \frac{\sqrt{p_x^2 + p_y^2} - l_1 - l_3 \cdot c\varphi}{\sqrt{(p_z - l_3 \cdot s\varphi)^2 + \left(\sqrt{p_x^2 + p_y^2} - l_1 - l_3 \cdot c\varphi\right)^2}};$$

$$q_2 = \frac{\pi}{2} - \text{Atan}\,2(s\Phi;\ c\Phi) - \text{Atan}\,2\left(\frac{l_2 \cdot cq_3}{\sqrt{(l_2 \cdot cq_3)^2 + (d_1 - l_2 \cdot sq_3)^2}};\ \frac{d_1 - l_2 \cdot sq_3}{\sqrt{(l_2 \cdot cq_3)^2 + (d_1 - l_2 \cdot sq_3)^2}}\right); \quad (16)$$

The generalized coordinates q_4 and q_5 are determined from direct geometry equations as:

$$q_4 = \beta_y - q_2 - q_3;\ q_5 = \gamma_x. \quad (17)$$

In this paragraph have been determined the generalized coordinates from each driving link by using the position and orientation parameters of the end-effector with respect to fixed coordinate system.

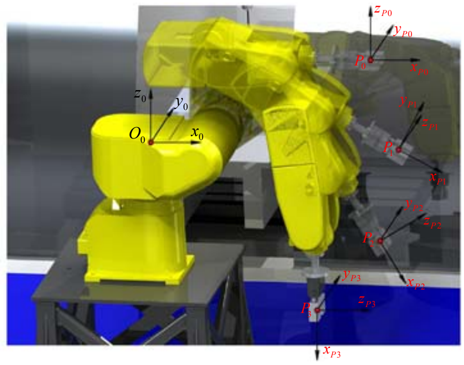
2 THE MODELING OF WORKING PROCESS

Further, will be presented the modeling of a working process for the serial structure *Fanuc LR Mate 100 iB.* by using the software (*SolidWorks*) there are determined the coordinations of the points which have to be covered by the robot during the workflow. These coordinations are presented as resulting from the *Table 2*.

Table 2

The representing of working process

Sequence 1



Seg. k	$^{(0)}X^T = (\bar{p}^T \quad \bar{\psi}^T)$					
	p_x	p_y	p_z	a_z	b_y	γ_x
	$\langle mm \rangle$			$\langle ° \rangle$		
0	595	0	250	0°	0°	0°
1	660,56	0	5,17	0°	30°	0°
2	599,91	0	-240,68	0°	60°	0°
3	428,36	0	-426,53	0°	90°	0°

In the expressions (13), (15), (16) and (17) are introduced the data from (*Table 2*), hence resulting the configuration table which contains the generalized coordinates from each driving link, corresponding to each point.

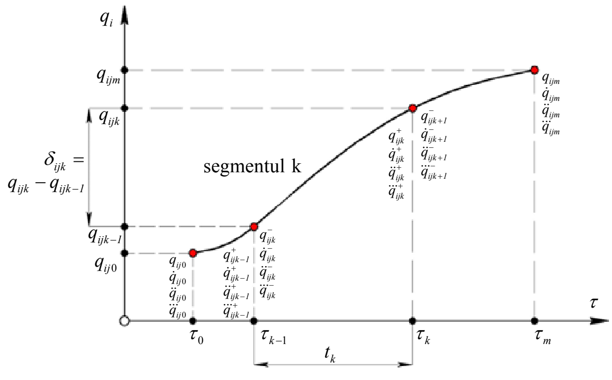
Table 3

Conf. k $1 \to 3$	Time $\tau \langle s \rangle$	Durat. t_{ik} $\langle s \rangle$	Generalized coordinate (robot)					
			q_{2jk} $\langle ° \rangle$	δ_{2jk} $\langle ° \rangle$	q_{3jk} $\langle ° \rangle$	δ_{3jk} $\langle ° \rangle$	q_{4jk} $\langle ° \rangle$	δ_{4jk} $\langle ° \rangle$
0	0	0	0	0	0	0	0	0
1	0,208	0,208	28,79	28,79	-1,2	-1,2	2,41	2,41
2	0,416	0,208	57,59	28,8	-2,41	-1,21	4,83	2,42
3	0,624	0,208	86,38	28,79	-3,62	-1,21	7,24	2,41

Where: $t_{ik} = \tau_{ik} - \tau_{ik-1}$, respectively $\delta_{ijk} = q_{ijk} - q_{ijk-1}$, for $\{i = 1 \to 5;\ j = 1 \to 2;\ k = 1 \to 6\}$

2.1 The Modeling of working process using $(5\text{-}4\text{-}6)$ type polynomial functions

The mechanical robotic systems, are parts of manufacturing systems that performing complex operations to achieve the proposed task. These systems, during operation are describing the trajectories of movement in the configuration space or in Cartesian space. So is needed a continuous control of the locating parameters (position and orientation), of the velocities, accelerations, and respectively of the generalized forces from every driving link from the mechanical system. In order to study the jerk, specific to a complex mechanical robot structure, is necessary to determine the higher order accelerations. In this case, the study is extended to second order accelerations, called jerk in the literature [3]. This notion represents the absolute first order derivative with respect to time of the vector acceleration (linear or angular). Therefore, in the following analysis will be found expressions of linear and angular jerk. Because of this aspect the modeling will be done by higher order polynomial functions, such as 5-4-6.



The input data for the 5-4-6 type polynomial functions are the generalized coordinates (q_{ij}) in kinetic link in each moment τ_k, $k = 0 \to m$ the velocities $(\dot{q}_{ij})$, the accelerations $(\ddot{q}_{ij})$ and jerk $(\dddot{q}_{ij})$ at the beginning and at the end of each sequence:

$$\begin{Bmatrix} (\tau_0) \to \{ q_{ij0};\ \dot{q}_{ij0};\ \ddot{q}_{ij0};\ \dddot{q}_{ij0}; \} \\ (\tau_k) \to \{ q_{ijk}; \}\ for\ k = 1 \to m-1 \\ (\tau_m) \to \{ q_{ijm};\ \dot{q}_{ijm};\ \ddot{q}_{ijm};\ \dddot{q}_{ijm}; \} \end{Bmatrix}. \quad (18)$$

On the first segment of the sequence, the interpolating polynoms for coordinates $q_{ij1}(t)$, velocities $\dot{q}_{ij1}(t)$, accelerations $\ddot{q}_{ij1}(t)$ and jerk $\dddot{q}_{ij1}(t)$ have the form presented below:

$$\begin{aligned} q_{ij1}(t) = a_{ij15} \cdot t^5 + a_{ij14} \cdot t^4 + a_{ij13} \cdot t^3 + \\ + a_{ij12} \cdot t^2 + a_{ij11} \cdot t + a_{ij10}; \end{aligned} \quad (19)$$

By using the input conditions at the time $t = 0$, $q_{ij1}(0) = q_{ij0}$, $v_{ij0}(0) = v_{ij0} = \dot{q}_{ij0}$, $a_{ij0}(0) = a_{ij0} = \ddot{q}_{ij0}$, and $\dot{a}_{ij0}(0) = \dot{a}_{ij0} = \dddot{q}_{ij0}$ are obtained directly the next coefficients: $a_{ij10} = q_{ij0}$, $a_{ij11} = \dot{q}_{ij0} \cdot t_1$, $a_{ij12} = \ddot{q}_{ij0} \cdot t_1^2/2$ and $a_{ij13} = \dddot{q}_{ij0} \cdot t_1^3/6$.

For intermediary segments, the interpolating is realized by the functions:

$$q_{ijk}(t) = a_{ijk4} \cdot t^4 + a_{ijk3} \cdot t^3 + a_{ijk2} \cdot t^2 + a_{ijk1} \cdot t + a_{ijk0}; \quad k = 2 \to m-1. \quad (20)$$

On the intermediary segments, the determination of the polynomial coefficients is realized by applying restrictive conditions specific to intermediary segment of the trajectory. For this, the normalized time variable is introduced by values $t = \{0;1\}$. Therefore, for $t = 0$, there are applied the specific conditions to τ_k, represented by expressions:

$$\begin{aligned} q_{ijk}(0) = q_{ijk-1} \equiv q_{ijk-1}(1);\ \dot{q}_{ijk}^{-} = \dot{q}_{ijk-1}^{+}; \\ \ddot{q}_{ijk}^{-} = \ddot{q}_{ijk-1}^{+};\ \dddot{q}_{ijk}^{-} = \dddot{q}_{ijk-1}^{+}. \end{aligned} \quad (21)$$

For $t = 1$ the restrictive conditions can be written:

$$q_{ijk}(1) = q_{ijk};\ \delta_{ijk} = q_{ijk} - q_{ijk-1};\ \dot{q}_{ijk}^{+} = \dot{q}_{ijk+1}^{-};\ \ddot{q}_{ijk}^{+} = \ddot{q}_{ijk+1}^{-};\ \dddot{q}_{ijk}^{+} = \dddot{q}_{ijk+1}^{-}.$$

After applying the previous conditions, there is obtained the polynomial coefficient $a_{ijk0} = q_{ijk-1}$ where $k = 2 \to m-1$. The interpolation of the last segment is realized by the following functions:

$$q_{ijm}\left(\bar{t}\right) = a_{ijm6}\cdot\bar{t}^{6} + a_{ijm5}\cdot\bar{t}^{5} + a_{ijm4}\cdot\bar{t}^{4} + a_{ijm3}\cdot\bar{t}^{3} + a_{ijm2}\cdot\bar{t}^{2} + a_{ijm1}\cdot\bar{t} + a_{ijm0} \quad (22)$$

In the expressions which are characterizing the interpolation functions, on the last segment the normalized time variable is $\bar{t} = t - 1$. The polynomial coefficients resulted directly from the restrictive conditions are characterized by expressions: $q_{ijm}(0) = q_{ijm}$, $v_{ijm}(0) = v_{ijm} = \dot{q}_{ijm}$, $a_{ijm}(0) = a_{ijm} = \ddot{q}_{ijm}$, and $\dot{a}_{ijm}(0) = \dot{a}_{ijm} = \dddot{q}_{ijm}$, resulting the polynomial coefficients determined directly at $\bar{t} = 0$: $a_{ijm0} = q_{ijm}$, $a_{ijm1} = \dot{q}_{ijm}\cdot t_m$, $a_{ijm2} = \ddot{q}_{ijm}\cdot t_m^2/2$, and $a_{ijm3} = \dddot{q}_{ijm}\cdot t_m^3/6$.

After applying the continuity conditions for intermediate segments, is resulting the following system of equations consisting of three major parts. For the first and the second segment are resulting the equations

$$\begin{aligned}
&a_{ij14} + a_{ij15} = \delta_{ij1} - \dot{q}_{ij0}\cdot t_1 - \ddot{q}_{ij0}\cdot t_1^2/2 - \dddot{q}_{ij0}\cdot t_1^3/6;\\
&\frac{4}{t_1}\cdot a_{ij14} + \frac{5}{t_1}\cdot a_{ij15} - \frac{1}{t_2}\cdot a_{ij21} = -\dot{q}_{ij0} - \ddot{q}_{ij0}\cdot t_1 - \dddot{q}_{ij0}\cdot t_1^2/2;\\
&\frac{12}{t_1^2}\cdot a_{ij14} + \frac{20}{t_1^2}\cdot a_{ij15} - \frac{2}{t_2^2}\cdot a_{ij22} = -\ddot{q}_{ij0} - \dddot{q}_{ij0}\cdot t_1;\\
&\frac{24}{t_1^3}\cdot a_{ij14} + \frac{60}{t_1^3}\cdot a_{ij15} - \frac{6}{t_2^3}\cdot a_{ij23} = -\dddot{q}_{ij0}.
\end{aligned} \quad (23)$$

For intermediary segments $k = 2 \rightarrow m-2$, using the continuity conditions, results:

$$\begin{aligned}
&a_{ijk1} + a_{ijk2} + a_{ijk3} + a_{ijk4} = \delta_{ijk};\\
&\frac{1}{t_k}\cdot a_{ijk1} + \frac{2}{t_k}\cdot a_{ijk2} + \frac{3}{t_k}\cdot a_{ijk3} + \frac{4}{t_k}\cdot a_{ijk4} - \frac{1}{t_{k+1}}\cdot a_{ij(k+1)1} = 0;\\
&\frac{2}{t_k^2}\cdot a_{ijk2} + \frac{6}{t_k^2}\cdot a_{ijk3} + \frac{12}{t_k^2}\cdot a_{ijk4} - \frac{2}{t_{k+1}^2}\cdot a_{ij(k+1)2} = 0;\\
&\frac{6}{t_k^3}\cdot a_{ijk3} + \frac{24}{t_k^3}\cdot a_{ijk4} - \frac{6}{t_{k+1}^2}\cdot a_{ij(k+1)3} = 0.
\end{aligned} \quad (24)$$

Using the continuity conditions on the last but one and on the last segment, there is obtained the system:

$$\begin{aligned}
&a_{ij(m-1)1} + a_{ij(m-1)2} + a_{ij(m-1)3} + a_{ij(m-1)4} = \delta_{ij(m-1)};\\
&\frac{1}{t_{m-1}}\cdot a_{ij(m-1)1} + \frac{2}{t_{m-1}}\cdot a_{ij(m-1)2} + \frac{3}{t_{m-1}}\cdot a_{ij(m-1)3} + \frac{4}{t_{m-1}}\cdot a_{ij(m-1)4} +\\
&+\frac{4}{t_m}\cdot a_{ijm4} - \frac{5}{t_m}\cdot a_{ijm5} + \frac{6}{t_m}\cdot a_{ijm6} = \dot{q}_{ijm} - \ddot{q}_{ijm}\cdot t_m + \dddot{q}_{ijm}\cdot t_m^2/2;\\
&\frac{2}{t_{m-1}^2}\cdot a_{ij(m-1)2} + \frac{6}{t_{m-1}^2}\cdot a_{ij(m-1)3} + \frac{12}{t_{m-1}^2}\cdot a_{ij(m-1)4} -\\
&-\frac{12}{t_m^2}\cdot a_{ijm4} + \frac{20}{t_m^2}\cdot a_{ijm5} - \frac{30}{t_m^2}\cdot a_{ijm6} = \ddot{q}_{ijm} - \dddot{q}_{ijm}\cdot t_m;\\
&\frac{6}{t_{m-1}^3}\cdot a_{ij(m-1)3} + \frac{24}{t_{m-1}^3}\cdot a_{ij(m-1)4} + \frac{24}{t_m^3}\cdot a_{ijm4} - \frac{60}{t_m^3}\cdot a_{ijm5} + \frac{120}{t_m^3}\cdot a_{ijm6} = \dddot{q}_{ijm};\\
&a_{ijm4} - a_{ijm5} + a_{ijm6} = -\delta_{ijm} + \dot{q}_{ijm}\cdot t_m - \ddot{q}_{ij0}\cdot t_1^2/2 + \dddot{q}_{ij0}\cdot t_1^3/6.
\end{aligned}$$

Using the restrictive and continuity conditions in every points of the trajectory there will be established the polynomial coefficients, some by direct application, as shown in the previous theory, others from the calculation using systems of equations. In this sense, it been applied MuPad software. Polynomial coefficients directly determined by calculation are presented in the table below:

Table 4

Polynomial coefficients

Segment	***Link***	Polynomial coefficients ***established directly and by matrix calculus***						
		a_{ik6}	a_{ik5}	a_{ik4}	a_{ik3}	a_{ik2}	a_{ik1}	a_{ik0}
1.	2.	0.00	-41.08	69.87	0.00	0.00	0.00	0.00
	3.	0.00	1.63	-2.83	0.00	0.00	0.00	0.00
	4.	0.00	-3.44	5.85	0.00	0.00	0.00	0.00
2.	2.	0.00	0.00	77.66	-131.34	8.40	74.08	28.79
	3.	0.00	0.00	-2.29	4.95	-0.70	-3.17	-1.20
	4.	0.00	0.00	6.49	-10.99	0.71	6.20	2.41
3.	2.	-177.52	-462.69	-313.96	0.00	0.00	0.00	86.38
	3.	0.00	0.00	1.46	-4.21	0.42	1.13	-2.41
	4.	-14.84	-38.68	-26.25	0.00	0.00	0.00	7.24

The real interpolation polynomials are obtained by replacing in normalized polynomial interpolation functions the variable t, with $\tau - \tau_{k-1}/t_k$ presented in the following table:

Table 5

Polynomial interpolation functions

Seg.	***Link***	***Expressions for angular coordinates, velocities, accelerations and jerk***

Seg.	Link	*Expressions for angular coordinates, velocities, accelerations and jerk*
1	2	$-105526,01\cdot\tau^5+37330,6\cdot\tau^4$
	3	$4175,16\cdot\tau^5-1509,54\cdot\tau^4$
	4	$-8829,36\cdot\tau^5+3124,06\cdot\tau^4$
2	2	$41492,19\cdot\tau^4-49117,04\cdot\tau^3+$ $+20072,56\cdot\tau^2-3112,59\cdot\tau+172,12$
	3	$-1223,87\cdot\tau^4+1568,68\cdot\tau^3-$ $-677,29\cdot\tau^2+106,94\cdot\tau-5,96$
	4	$3468,65\cdot\tau^4-4106,62\cdot\tau^3+$ $+1678,53\cdot\tau^2-260,3\cdot\tau+14,39$
3	2	$-2192115,2\cdot\tau^6+7018837,43\cdot\tau^5-$ $-9263157,66\cdot\tau^4+6443564,41\cdot\tau^3-$ $-2489638,56\cdot\tau^2+506438,95\cdot\tau-42321,3$
	3	$778,46\cdot\tau^4-1763,19\cdot\tau^3+$ $+1401,79\cdot\tau^2-469,65\cdot\tau+53,99$
	4	$-183255,93\cdot\tau^6+586751,48\cdot\tau^5-$ $-774358,71\cdot\tau^4+538646,03\cdot\tau^3-$ $-208117,34\cdot\tau^2+42334,57\cdot\tau-3537,72$

2.2 The graphical representation of kinematical parameters

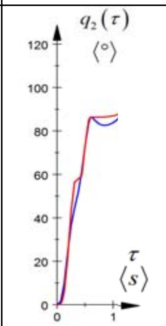
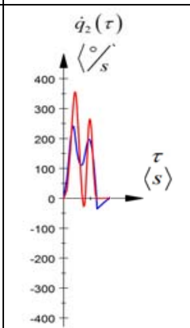
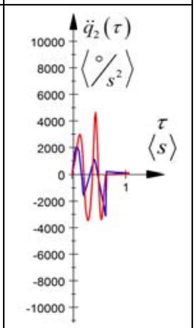
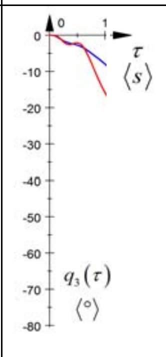
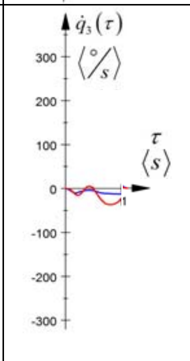
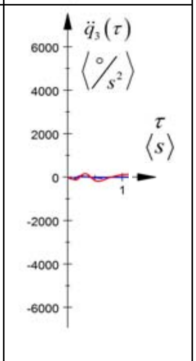
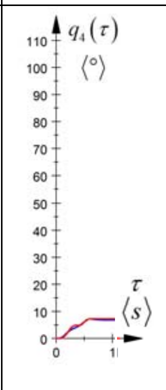
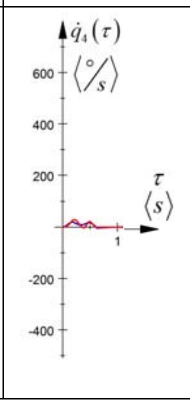
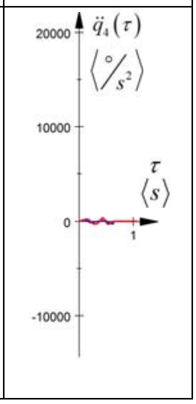
The variation laws of kinematic parameters mentioned above and which are expressing the relative movement of each kinetic link were established as shown in the above aspects, such as the 5-4-6 type polynomial functions. According to the same aspects, described above, the curves from the graphs are represented by red color. The representation with higher degree polynomial functions aimed at highlighting of angular jerk (first-order derivatives with respect to time of the angular accelerations), which are a feature of the sudden movement of the mechanical structure of the considered robot.

3 CONCLUSIONS

In the paper has presented the application of theoretical models of direct and inverse geometry for an industrial robot, concerning the achieving of a working process. Thus were established the kinematic control functions for coordinates, velocities, accelerations and generalized jerk, their

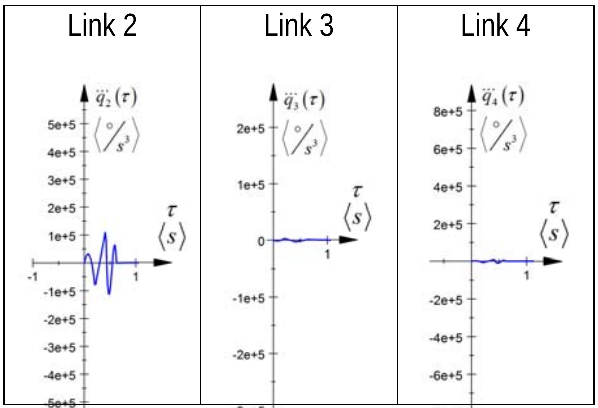
Table 6

Representation of the kinematical parameters

Link	Angular coordinate	Angular velocity	Angular acceleration
2			
3			
4			

Jerk for each kinetic link is presented in the following table:

calculation being performed using higher order polynomial functions, which led to an increased precision in the calculations. Based on these polynomial functions, there been presented graphically the variation of kinematic parameters.



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MODELAREA PROCESULUI DE LUCRU AL ROBOTULUI CU STRUCTURĂ SERIALĂ TIP FANUC

Rezumat: În lucrare este prezentată modelarea unui proces de lucru al robotului *Fanuc LR Mate 100 iB,* utilizând funcţiile polinomiale de interpolare de tipul 5-4-6. Aceste funcţii polinomiale de ordin superior se utilizează pentru a studia mişcarea bruscă, specifică unui robot cu o structură mecanică complexă, unde este necesară determinarea acceleraţiilor de ordin superior. În acest caz, studiul se extinde asupra acceleraţiilor de ordinul doi (*supraacceleraţii*), denumită *jerk* în literatura de specialitate. Această noţiune reprezintă derivata absolută de ordinul întâi în raport cu timpul a vectorului acceleraţie (liniară sau unghiulară). De aceea, în analiza care urmează se vor regăsi expresiile supraacceleraţiei liniare şi unghiulare. Determinarea funcţiilor de comandă necesită modelarea matematică a robotului serial, prezentată în prima parte a lucrării.

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