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GRAPHO-ANALYTICAL STUDY ON THE RELATION BETWEEN DOUBLE-ROTATION AND AXONOMETRY

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Abstract: *In design activities the graphical representations need to be as suggestive as possible and easily achievable. Axonometry employed as a method of intuitively representing the objects can be affected applying classic methods of descriptive geometry, change of projection planes and rotation. The present paper aims at analysing the possibilities of using a method of compound angle, equivalent to two successive rotations for finding the axonometric images of some solids, considered components of some parts in machine building technology.*
Key words: *projection transformation methods, compound angle, rotation, axonometry.*

1. INTRODUCTION

The double rotation employed as transformation method of projections in descriptive geometry enables, by simple constructions, to solve metric problems, to find distances and angles, as well as projections of some solids whose edges or vertices occupy special positions.

In [1], developing the method of the compound angle [2], applying two successive rotations, the projections of some lines and planes which are to occupy imposed positions within a design, have been determined.

The present paper aims at developing an alternative for the construction of the axonometric image of a solid, by using the double rotation to which its orthogonal projections are subjected.

2. FUNDAMENTALS CONSIDERATIONS

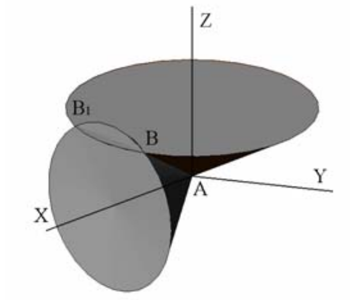
The method of the compound angle [2] or the double rotation, applied for finding the projections of some segments of line forming known angles with the projection planes is based on the fact that, the locus of the extremity *B* of a segment *AB* of known length *l*, having the extremity *A* fixed, situated on the line Ox, is a spherical surface with the center in *A* and radius *AB*. The segment *AB*, in this case, is the common generatrix of two cones having the common vertex *A* and bases lying in plane [*N*], respectively, plane [*F*] (Fig. 1-a).

One considers a segment *AM* = *AB* = *l*, included in the vertical plane [*V*], having the extremity *A* on axis *Ox* which forms the known angle β with the horizontal projection plane [*H*] (Fig. 1-b). By level rotation, around vertical axis *Z* passing through point *A*, the segment *AB*, generates a rotation cone of axis *Z* and whose generatrix all forms the angle β with plane [*H*]. The extremity *M* of segment *AM*, thus, describes, in its level plane, a circle with center lying on the rotation axis *Z* and radius *AM*.

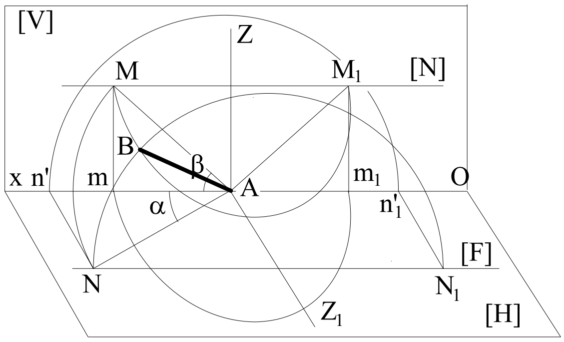
A second segment *AN* = *AB* = *l*, included in plane [*H*], having the extremity *A* on axis *Ox* and forming the known angle α with plane [*V*], generates in its turn, by a front rotation around the end axis Z_1, a rotation cone whose generatrix forms angle α with plane [*V*]. By rotation the extremity point *N* describes the circle of radius *AN* in the front plane (Fig. 1-b).

From the composition of the two rotations, the extremity *B* of segment *AB,* will lie at the intersection of two circumferences on the surface of the sphere with center in *A* and radius *AB* (Fig. 2-a).

In epure (Fig. 2-a), one considers segment *am*, contained in plane [*V*] with equal length to the length of the segment *a'm'* = *AB* = *l* forming angle *β* with the horizontal plane [*H*] and segment *an* = *AB* = *l*, respectively, contained in the horizontal plane [*H*] which forms angle α with plane [*V*].



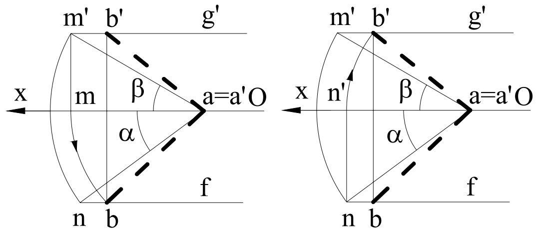
a)



b)

Fig. 1. Double rotation - compound angle method [2].

The level rotation of point *M*(*m*,*m*'), form at the intersection of circumferences *a'm'* with the frontal of point *n*, the point *b*, which is the horizontal projection of point *B* looked for. Its vertical projection *b'*, lies on the horizontal of point *M*. Point *B*(*b*,*b'*) is the extremity of segment *AB* looked for and can occupy more positions in accordance with the initial conditions of the problem. The horizontal rotated projection of point *B* can be any of the points of arc with diameter MM_1 (Fig.1-b). The directions of *AB* segment projections will be parallel to *ab*, respectively *a'b'*, whatever would be the extremity point *A* in space. A similar result is obtained by application of front rotation (Fig. 2-b).



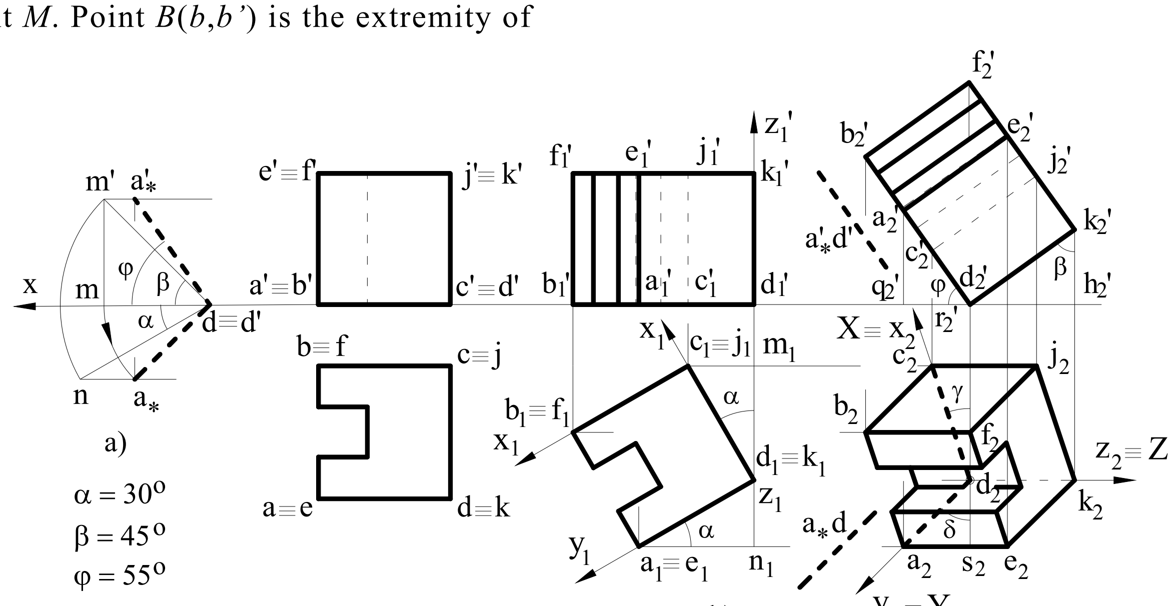
a) b)

Fig.2. Finding, in epure, of the directions of segment *AB* projections *ab* and *a'b'*.

3. ESTABLISHING THE RELATION BETWEEN DOUBLE ROTATION AND AXONOMETRY

3.1. By graphical method

Based on the above considerations, for establishing the relations between double rotation and axonometry, the paper presents the application of double rotation to a given machine part, knowing that one of its edges must form the angles α with plane [*V*] respectively *β* with plane [*H*] (Fig. 3).



b)

Fig. 3. Determining the rotated projection of a solid by double rotation.

The considered part, of cubic form, has one of the bases lying in the horizontal projection plane, and edge AD forms angles α and β with the vertical respectively horizontal projection planes. The preliminary auxiliary construction enables the determination of edge AD projection directions, subsequent the two successive rotations, namely a_2d_2 $a_* d$, and $a_2'd_2'$ $a_*'d'$ from Figure 3-a.

After the first level rotation around the vertical edge D_1K_1 with angle α, the edge DA becomes the level line $D_1A_1(d_1a_1, d_1'a_1')$. The second front rotation is effected around the end line passing through vertex D_2 of plane [H], until the vertical projection of edge A_2D_2, becomes $a_2'd_2'$ $a_*'d'$. There results the rotation angle $\varphi = 55^0$.

The horizontal projection, $a_2b_2c_2d_2f_2j_2k_2$, obtained may be considered as axonometric image of the solid in the system of axes $c_2x_2 \equiv X$, $c_2y_2 \equiv Y$, $c_2z_2 \equiv Z$ [2].

3.2. By analytical method

For the case under consideration, the rotation angles $\alpha = 30^0$, respectively $\varphi = 55^0$, resulting from the auxiliary graphical construction (Fig.3-a), within the paper, the deforming coefficients were calculated u, v, w, in relation to axonometric axes $c_2x_2 \equiv X$, $c_2y_2 \equiv Y$, $c_2z_2 \equiv Z$, graphically determined [3]:

a) *the deformation coefficients*

- in relation with axis X: $u = \dfrac{d_2c_2}{dc}$; $dc = l$

From $\Delta d_2c_2m_2$:

$$b_2c_2 = \sqrt{(m_2d_2)^2 + (m_2c_2)^2}$$

From $\Delta r_2'c_2'c_2'$: $r_2'c_2' = d_2'c_2'\cos\varphi$,

where $d_2'c_2' = d_1'c_1' = c_1m_1$.

So, $c_2m_2 = d_2'c_2'\cos\varphi = d_1c_1 \sin\alpha .\cos\varphi$ and $d_2m_2 = d_1m_1 = d_1c_1\sin\alpha$ and

$$d_2c_2 = d_1c_1\sqrt{\cos^2\alpha + \sin^2\alpha\cos^2\varphi} =$$
$$= l\sqrt{\cos^2\alpha + \sin^2\alpha\cos^2\varphi}$$

whence, it results:

$$u = \frac{d_2c_2}{dc} = \frac{l\sqrt{\cos^2\alpha + \sin^2\alpha\cos^2\varphi}}{l};$$

$$u = \sqrt{\cos^2\alpha + \sin^2\alpha\cos^2\varphi} \qquad (2.1)$$

Substituting $u = \sqrt{\cos^2 30 + \cos^2 30\cos^2 55}$, it results: $u = 0{,}9123 \approx 0{,}91$.

- in relation with axis Y: $v = \dfrac{d_2a_2}{da}$; $da = l$.

$$d_2a_2 = \sqrt{(s_2a_2)^2 + (s_2d_2)^2};$$

$s_2a_2 = d_2'q_2'$; $s_2d_2 = d_1n_1$

From $\Delta d_2'a_2'q_2'$:

$d_2'q_2' = d_2'a_2'\cos\varphi$,

$d_2'a_2' = d_1'a_1' = a_1n_1 = l\cos\alpha$;

From $\Delta d_1a_1n_1$: $a_1n_1 = c_1a_1\cos\alpha = l\cos\alpha$,

$d_1n_1 = d_1a_1\sin\alpha = l\sin\alpha$; $s_2d_2 = l\sin\alpha$

$s_2a_2 = d_2'a_2'\cos\varphi = l\cos\alpha\cos\varphi$

$$d_2a_2 = l\sqrt{\sin^2\alpha + \cos^2\alpha\cos^2\varphi}$$

$$v = \frac{d_2a_2}{da} = \frac{l\sqrt{\sin^2\alpha + \cos^2\alpha\cos^2\varphi}}{l}$$

$$v = \frac{d_2a_2}{da} = \sqrt{\sin^2\alpha + \cos^2\alpha\cos^2\varphi} \qquad (2.2)$$

Substituting $v = \sqrt{\sin^2 30 + \cos^2 30\cos^2 55}$, it results: $v = 0{,}7048 \approx 0{,}71$.

- in relation with axis Z: $w = \dfrac{d_2k_2}{dk}$; $dk = l$.

$\sin\beta = \dfrac{d_2'h_2'}{d_2'k_2'} = \dfrac{d_2k_2}{l}$; $w = \dfrac{l\sin\varphi}{l}$; then

$$w = \sin\varphi \qquad (2.3)$$

so $w = \sin 55^0 = 0{,}8191 \approx 0{,}82$.

Applying the basic relation of axonometry projection [4]:

$$u^2 + v^2 + w^2 = 2 \qquad (2.4)$$

and substituting the values of determined deformation coefficients, it result:

$u^2 + v^2 + w^2 =$

$= 0{,}9123^2 + 0{,}7048^2 + 0{,}8191^2 =$ **2,0046**,

with a difference of **0,0046** with respect to the relation given by the classic axonometry.

The approximated values of deformation coefficients verify the basis relation of axonometry, namely:

$u^2 + v^2 + w^2 = 0{,}91^2 + 0{,}71^2 + 0{,}82^2 =$ **2**.

b) *the angles of axonometric axes X,Y,Z*

From Figure 3, it results:

$\angle XOZ = \pi/2 + \gamma$; $\angle YOZ = \pi/2 + \delta$;

$\angle XOY = 2\pi + \gamma + \delta$.

For values $\alpha = 30^0$ and $\varphi = 55^0$, we obtain:

$$\sin\gamma = \frac{c_2m_2}{c_2d_2} = \frac{l\sin 30^0\cos 55^0}{l\sqrt{\cos^2 30^0 + \sin^2 30^0\cos^2 55^0}}$$

$\sin\gamma = 0{,}315153$, iar $\gamma = 18{,}37^0 \approx 18^0 22'$.

$$\sin\delta = \frac{a_2 s_2}{a_2 d_2} = \frac{l\cos 30^0 \cos 55^0}{l\sqrt{\sin^2 30^0 + \cos^2 30^0 \cos^2 55^0}}$$

$\sin\delta = 0{,}705593$ iar $\delta = 44{,}87^0 \approx 44^0 52'$.

It approximates $\gamma = 18^0$ and $\delta = 45^0$, results confirmed by the values of the angles obtained and by graphical methods, as well, in Figure 3. With these values, the angles between the axonometric axes result:

$< XOZ = 90^0 + 18^0 = 108^0$

$< YOZ = 90^0 + 45^0 = 135^0$

$< XOY = 180^0 - (18^0 + 45^0) = 117^0$.

The values graphically and analytically obtained, given in Table 1, values comparable with those from literature, correspound to an trimetric axonometric representation.

The axonometric image obtained for the considered machine part is shown in Figure 4.

Table 1. The deformation coefficients and the angles between axes – literature / determined.

Rotation angles		Type of axonometry		
$\alpha = 30^0$ $\varphi = 55^0$		Izometrie [4]	Trimetrie [4]	Trimetrie determined
Def. coeff.	u	0,82	0,8	**0,91**
	v	0,82	0,6	**0,71**
	w	0,82	0,9	**0,82**
< axes	XOZ	120^0	105^0	**108^0**
	YOZ	120^0	120^0	**135^0**
	XOY	120^0	135^0	**117^0**

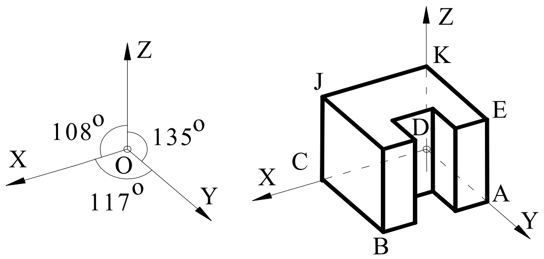


Fig.4. The system of axes and the axonometric image obtained for the machine part considerated.

4. CONCLUSIONS

Employing the double rotation we have graphically and then analitycally determined the axonometric image of a prismatic machine part. Thus, the deformation coefficients have been calculated, respectively the angles between the axonometric axes. The values graphically obtained are confirmed by those obtained by calculations and they verify the bazic relation of axonometry. The method used presents a double advantage, since by a relatively simple and occurate graphical construction allows positioning required of the machine part in space, and from the axonometric image, intuitive, can easily deduce the geometric configuration of the part.

5. REFERENCES

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STUDIU GRAFO-ANALITIC ASUPRA RELAȚIEI DINTRE DUBLA ROTAȚIE ŞI AXONOMETRIE

Rezumat: În activitatea de proiectare este necesar ca reprezentările grafice să fie cât mai sugestive dar şi cât mai uşor de realizat. Reprezentarea axonometrică utilizată ca metodă de reprezentare intuitivă, se poate realiza utilizând metodele clasice ale geometriei descriptive, schimbarea planelor de proiecție, rotația sau rabaterea. Lucrarea de față îşi propune un studiu asupra posibilității de utilizare a metodei unghiului compus echivalentă cu două rotații succesive, la determinarea imaginii axonometrice a unor corpuri solide, considerate ca părți componente ale unor piese întâlnite în construcția de maşini.

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