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METHODS OF TRACKING AND CORRECTING THE TRAJECTORIES USING THE PROXIMITY TRANSDUCERS (PART 1)

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Abstract: *In this article will be studied the problem of tracking trajectories (unknown) located in the plane yOz, at a given distance d_0 . Throughout the trajectory, the end-effector, equipped with proximity transducers, must solve two problems:identification of the shape of the trajectory and the positioning of the end-effector so that the distance to the trajectory travelled to be d_0 (known).*

Key words: *tracking, trajectory, end-effector, proximity, transducer, mobile system, coordinates, displacement*

1. A METHOD OF TRACKING THE TRAJECTORIES.

1.1. Specify the shape of the trajectory

In the following will be studied the problem of tracking trajectories (unknown) located in the plane yOz, at a given distance d_0. Throughout the trajectory, the end-effector, equipped with proximity transducers, must solve two problems:-identification of the shape of the trajectory and the positioning of the end-effector so that the distance to the trajectory travelled to be d_0 (known).

The following are chosen:

$\{S_0\}$ as fixed reference system of the path;
$\{S_1\}$ as fixed reference system of incline sector of the path;
$\{S\}$ as mobile reference system of end-effector at starting point;
$\{S_t^1\}$ as reference system of end-effector at t_1 point;
$\{S_t^2\}$ as reference system of end-effector at t_2 point;
$\{S_t^3\}$ as reference system of end-effector after the rotation of the path with α angle;
$\{S_t^4\}$ as reference system of end-effector after the translation along the axis $O_t^3{\cdot}Z_t^3$ with dz;
$\{S_t^5\}, \{S_t^{5'}\}$ as positions of mobile system of end-effector;
$A \rightarrow y_A$ as initial position of proximity transducer in relation to system $\{S\}$;
$A_1 \rightarrow a_x$ as shifted position of proximity transducer together with the mobile system when the distance $d_x \neq d_0$;
$y_A = vt_1$ as the movement of the mobile system $\{S\}$ in $\{S_t^1\}$ position.

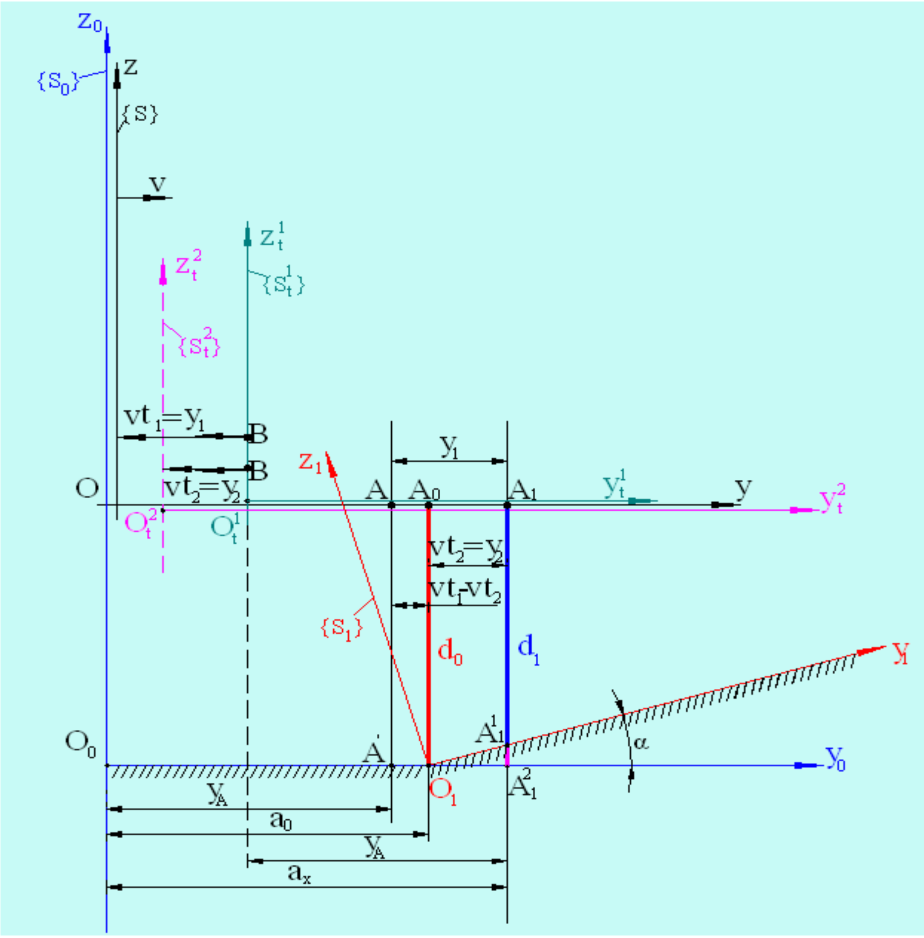

Fig 1.1. The movement of mobile system with the proximity transducer from point A to point A_1

It is considered a movement of the mobile system together with the proximity transducer from position *A* to position A_1.

Initially, the proximity transducer from position *A* is set in y_A position towards the mobile system {S} which has the axis *Oz* superimposed on axis O_0S_0. The distance y_A is constant. The movement of mobile system is registered by ttransducer from position *B*.

The distance of y_1 is

$$y_1 = v \cdot t_1 \tag{1}$$

The measured value of y_1 is known and also the value of v and t_1 resulting from this is:

$$t_1 = \frac{y_1}{v} \tag{2}$$

According to the figure 1.1 the relations can be written:

$$y_A + v \cdot t_1 = y_A + y_1 = a_x \tag{3}$$

$$a_0 + O_1A_1^2 = a_x \tag{4}$$

The result a_x can be calculatedbecause all factors are known. In equation (4) are two unknown values: a_0 and $O_1A_1^2$. According to fig.1.1 it can be observed that:

$$A_1^1A_1^2 = d_0 - d_1 \tag{5}$$

Both terms from the right member of equation are known (value of d_1 is known) and from this is resulting $A_1^1A_1^2$. Will shift the mobile system {S} in the negative sense of the axis $O_t^1y_t^1$ until the distance $d_0 - d_1$ will be null $d_0 - d_1 = 0$. Therefore, the distance $O_1A_1^2$ can not be used because the position of point O_1 and distance a_0 are unknown.

If the mobile system {S} is shifted from initial position to a point where the distance d_x measured by the transducer from point A, is lower in value than d_0:

$$d_x < d_0 \tag{6}$$

Assuming that the mobile system moved in position $\{S_1\}$ in which, the transducer from point A measures the distance d_1, this will give to the system a movement in the opposite direction, so the distance $A_1^1A_1^2 = d_0 - d_1$ shall be null. Maintaining the same constant speed we can write the relation:

$$v \cdot t_2 = y_2 = O_1A_1^2, \qquad (7)$$

t_2- is the measured time until $A_1^1A_1^2$ is null, $A_1^1A_1^2 = 0$. From the right triangle $O_1A_1^1A_1^2$ we obtain:

$$tg\alpha = \frac{A_1^1A_1^2}{O_1A_1^2} = \frac{d_0 - d_1}{a_x - a_0} \qquad (8)$$

1.2. Getting over the path

Initially, the mobile system {S} is in a position in which the axis Oz coincides with axis O_0z_0 and axis Oy is parallel with axis O_0y_0 at the distance d_0. The transducer from A is at the distance y_A on the axis O_y of the mobile system {S}. This position of the transducer A is maintained towards the mobile system regardless of its position (fig.1.1).

The proximity transducer B from the initial position indicates a null distance. The both transducers measure in the perpendicular direction. To the mobile system {S} is given a displacement with a constant speed v. Being ***v·t*** the displacement that is situated in the interval $O - O_1^t$. After this displacement, the mobile system arrives into the position $\{S_1^t\}$. The transformation matrix from the system {S} to the system $\{S_1^t\}$ is:

$$S_S^{S_1^t} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (9)$$

Towards the system $\{S_1^t\}$ the position of point A_1 is:

$$x_{A_1} = 0;\ y_{A_1} = y_A;\ z_{A_1} = 0. \qquad (10)$$

Here is the position of point A towards the systems {S} and $\{S_0\}$.

Towards the system {S}, the position is:

$$A_S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y_A \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ y_A + v \cdot t \\ 0 \\ 1 \end{bmatrix} \qquad (11)$$

Towards the system $\{S_0\}$ the position of point A is given by the next matrix relation:

$$A_{S_0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y_A \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ y_A + v \cdot t \\ d_0 \\ 1 \end{bmatrix} \qquad (12)$$

The relation (12) is valid for the horizontal path O_0O_1 when $a_x \leq O_0O_1$ (fig.1.1).

This way the transducer A is positioned and transducer B will specify which is the course path.

Suppose that the transducer from A reached A_0 position, mobile system will be: $\{S_t^2\} \rightarrow O_t^2y_t^2z_t^2$, (fig.1.2).

In this position a rotation with an angle ***α*** is given to the mobile system $\{S_t^2\}$ until arrives in position $\{S_t^3\} \rightarrow (O_t^3y_t^3z_t^3)$ having the axes parallel with the system $O_1z_1y_1$ of the incline section of the path.

Point A_0 will get to point A_3 and the transducer from A_3 will measure the distance $dz_1 > d_0$:

$$dz_1 = A_3A_5^1 = d_0 + dz \qquad (13)$$

In order to obtain d_0 and to be measured from O_1 (at the beginning of the incline path) will assume the following:

- a translation $[S_3^4]$ of the system $[S_t^3]$ along the $O_t^3z_t^3$ so the $[S_t^3]$ becomes $[S_t^4]$. Point A_3 will become A_4 (fig.1.2). Distance measured by the transducer A (A_4) will be: $A_4A_5^1 = d_0$.

- because this distance is not measured in point A_7 which corresponds with point O_1, a displacement $O_t^4O_t^5$ will be given to the system $\{S_t^4\}$, and the system $\{S_t^4\}$ becomes system $\{S_t^5\} \rightarrow O_t^5y_t^5z_t^5$ and point A_4 reach A_7. To this transformation corresponds matrix $[S_4^5]$. We note this with $y_3=v\cdot t_3$ (t_3 is unknown).

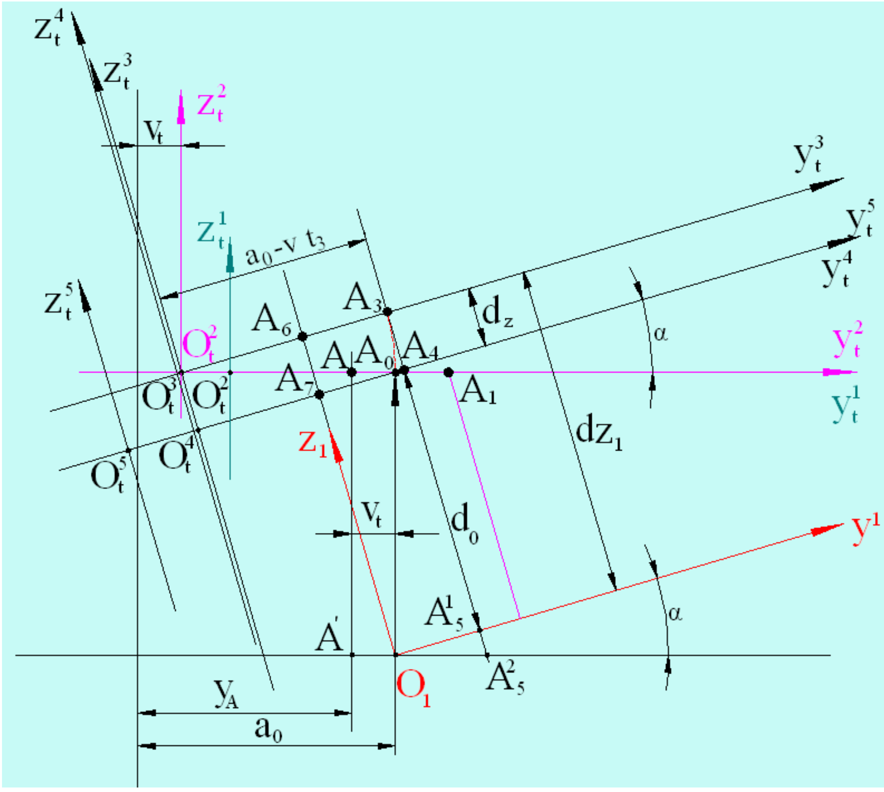

Fig 1.2. Successive positions of the transducer placed at the point of A

$$A_3O_t^3 = A_4O_t^4 = A_7O_t^5 = y_A \qquad (14)$$

The transducer position does not change towards the mobile system. For the transformations mentioned we have:

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (15)$$

Coordinates of the point A_3 towards the immobile system $\{S_0\}$ will be:

$$A_0^3 = [T_0^2]\cdot[R_2^3]\cdot[A_3^3] =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v\cdot t \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y_A \\ 0 \\ 1 \end{bmatrix} = \qquad (16)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & v\cdot t \\ 0 & s\alpha & c\alpha & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y_A \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ y_A c\alpha + v\cdot t \\ y_A s\alpha + d_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_{A_3}^0 \\ y_{A_3}^0 \\ z_{A_3}^0 \\ 1 \end{bmatrix}$$

For the translation of the system $\{S_t^3\}$ into the position $\{S_t^4\}$ we have the matrix:

$$T_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (17)$$

Towards the system $\{S_t^4\}$ position of point A_3 is point A_4 given by the matrix relation:

$$\begin{bmatrix} A_{4x} \\ A_{4y} \\ A_{4z} \\ 1 \end{bmatrix} = [T_3^4]\cdot \begin{bmatrix} A_{3x} \\ A_{3y} \\ A_{3z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ A_y \\ 0 \\ 1 \end{bmatrix} = \qquad (18)$$

$$= \begin{bmatrix} 0 \\ A_y \\ dz \\ 1 \end{bmatrix}$$

For the translation from the system $\{S_t^4\}$ to system $\{S_t^5\}$ the matrix relation is:

$$T_4^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(19)

Position of point A_4 towards the system $\{S_t^5\}$ is given by the following:

$$\begin{bmatrix} A^5_{x_4} \\ A^5_{y_4} \\ A^5_{z_4} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A^4_{x_4} \\ A^4_{y_4} \\ A^4_{z_4} \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & v \cdot t_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ y_A \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ y_A + v \cdot t_3 \\ 0 \\ 1 \end{bmatrix} \quad (20)$$

As a result, point A_4 will reach A_7 having the coordinates $x_{A_7} = 0$; $y_{A_7} = y_A$ in the system $\{S_t^5\}$ (the position of the transducer from point A does not change towards the mobile system).

The mobile reference system $\{S_t^5\}$ is parallel with the immobile system $\{S_t^1\}$ of the incline section of the path, so the transducer from A_7 will measure the distance d_0.

There are three stages for tracking the path:

-First stage describes the path which is unknown by the transducer from A and B;

-The second stage describes the movement on the horizontal path;

-The third stage, describing the movement of the incline section of the path.

Mobile system positions will be presented below, through a succession of images:

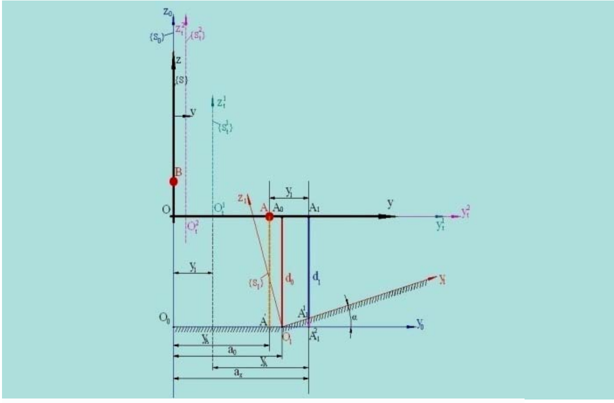

Fig 1.3. The initial position of the mobile system

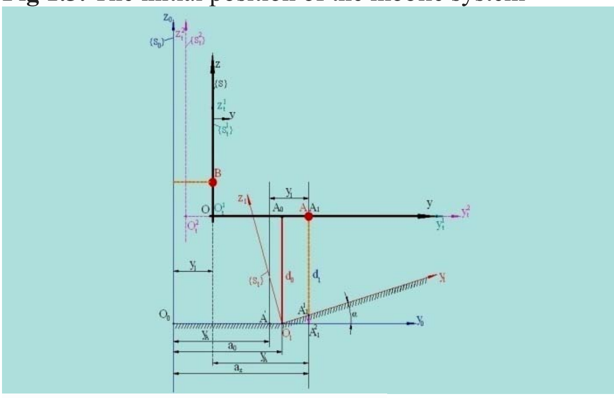

Fig.1.4. Mobile system position, for $A=A_1$ and $\{S\} = \{S_t^1\}$

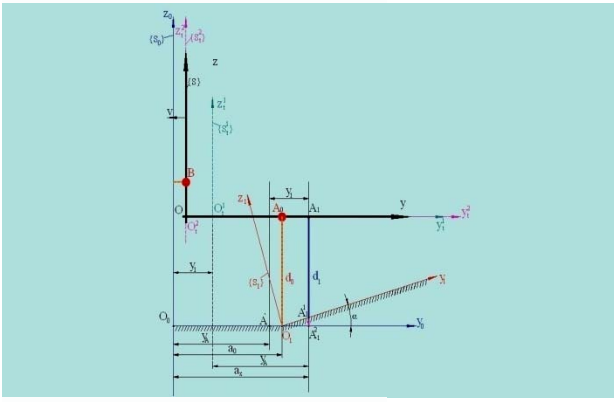

Fig.1.5. Mobile system position, for $A=A_0$ and $\{S\} = \{S_t^2\}$

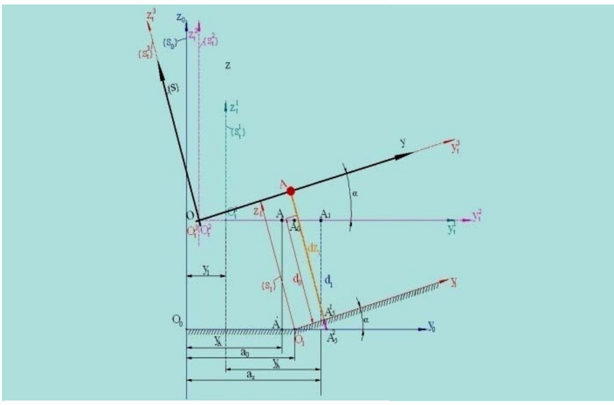

Fig.1.6. Mobile system position, for $A=A_3$ and $\{S\} = \{S_t^3\}$

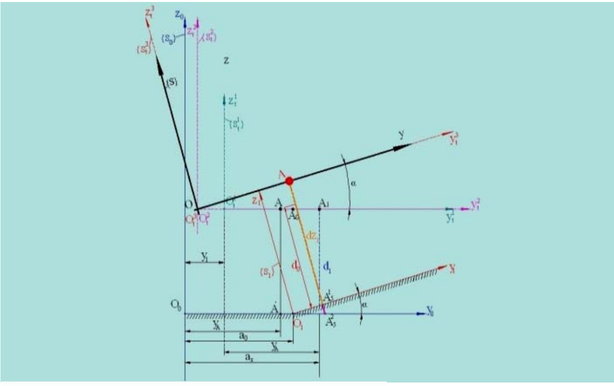

Fig.1.7. Mobile system position, for A=A_4 and $\{S\}=\{S_t^4\}$

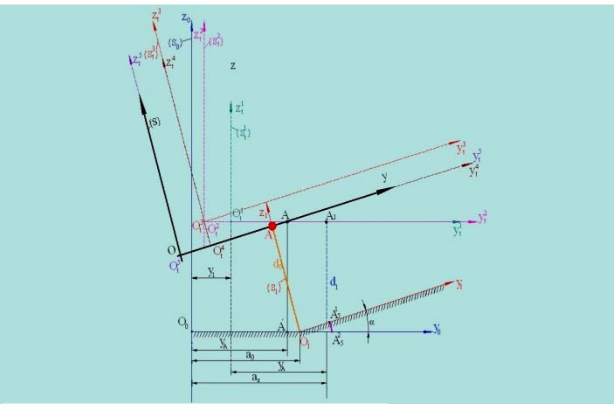

Fig.1.8. Mobile system position, for A=A_5 and $\{S\}=\{S_t^5\}$

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Metode de urmărire şi corectare s traiectoriilor cu ajutourl traductoarilor de proximitate (partea I-a)

Rezumat: În această lucrare este studiată problema urmăririi unei traiectorii (necunoscută) situată în planul yOz, la o distanţă dată d_0. În drumul parcurs, dispozitivul de prehensiune, dotat cu traductori de proximitate, trebuie să rezolve două probleme: - identificarea formei traiectoriei; - poziţionarea dispozitivului de prehensiune astfel încât distanţa faţă de traiectoria parcursă să fie d_0 (cunoscută).

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