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METHODS OF TRACKING AND CORRECTING THE TRAJECTORIES USING THE PROXIMITY TRANSDUCERS (PART 2)

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Abstract: *In this article will be studied the problem of correcting the trajectories (unknown) located in the plane yOz, at a given distance d_0 . Throughout the trajectory, the end-effector, equipped with proximity transducers, must solve two problems:identification of the shape of the trajectory and the positioning of the end-effector so that the distance to the trajectory travelled to be d_0 (known).*

Key words: *Correcting, trajectory, end-effector, proximity, transducer, mobile system, coordinates, displacement*

1. TRAJECTORY CORRECTION POSSIBILITIES

Being known position of the system attached to the trajectory, and of the mobile system position in relation to this trajectory, must have correct mobile system position, linked to the end-effector so that it will follow a portion of the trajectory at a fixed distance d0.

1.1. First variant

In the first variant, a rotation with angle α around the system origin O_t^0 and then a translation dz=d-d_0 along the axis $O_t^1 z_t^1$ is given to the mobile system $\{S_t\}$, followed by a translation along the axis $O_t^1 y_t^1$ (figure 1.1), so the transducer P reached in final position P3.

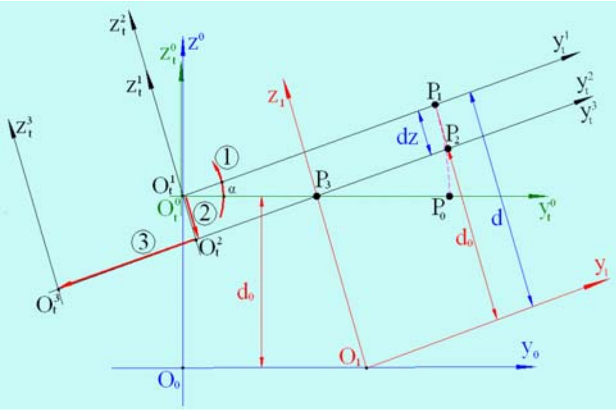

Fig. 1.1 Mobile system movement at the first trajectory correction variant

The successive positions of the mobile system will be presented in the following figures:

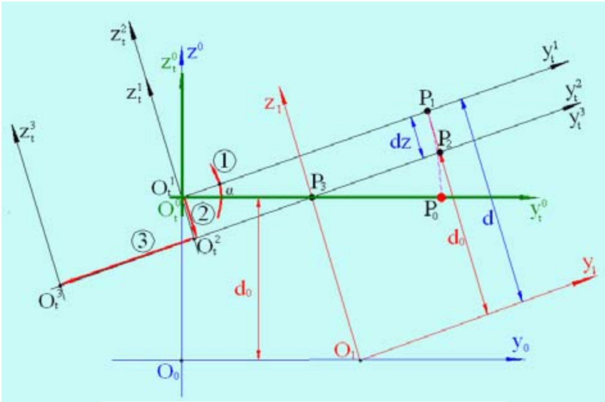

Fig.1.2. Position of mobile system (1)

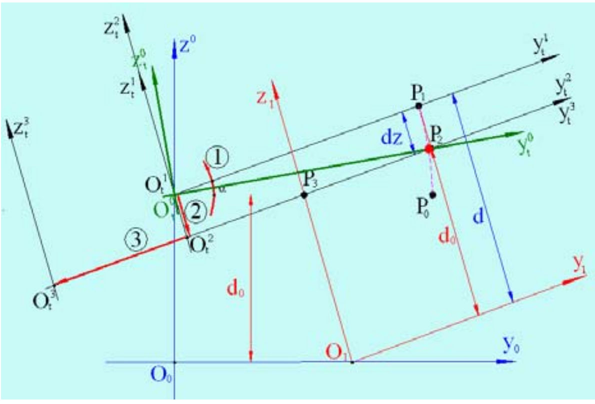

Fig.1.3. Position of mobile system (2)

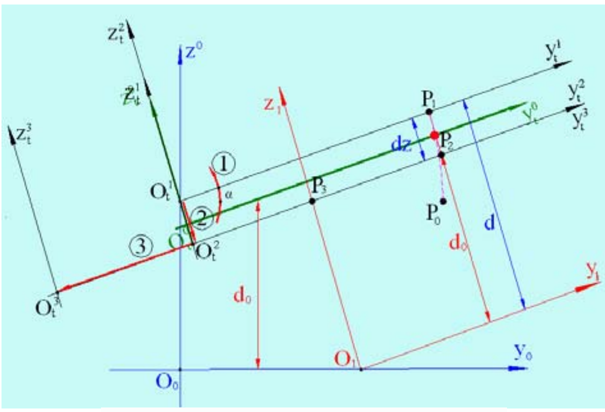

Fig.1.4. Position of mobile system (3)

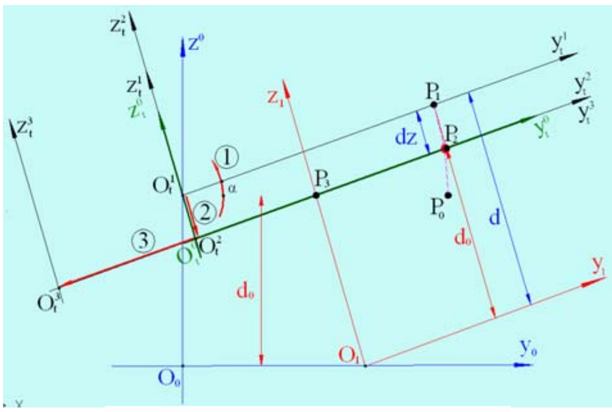

Fig.1.5. Position of mobile system (4)

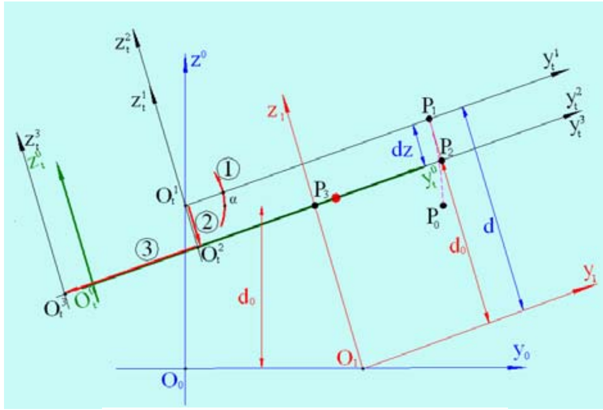

Fig.1.6. Position of mobile system (5)

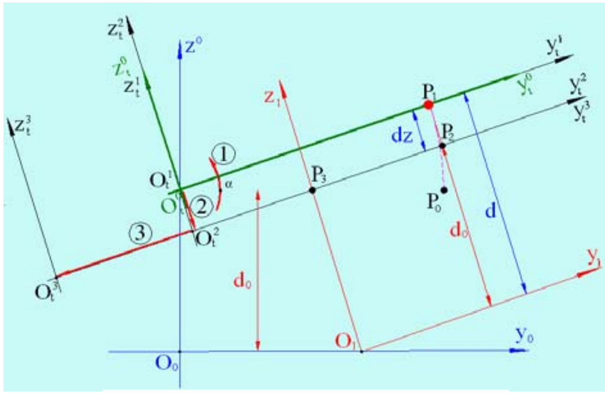

Fig.1.7. Position of mobile system (6)

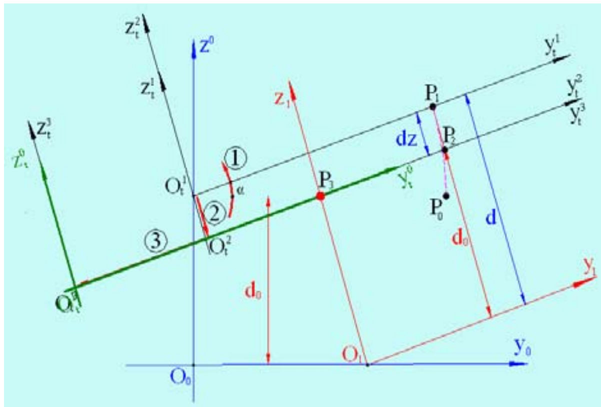

Fig.1.8. Position of mobile system (7)

1.2. The second variant

In version 2, a translation along the axis $O_t^0y_t^0$, followed by a translation along the axis $O_t^1z_t^1$ with a distance which will be determined, and then by a rotation of angle around the axis $O_t^2x_t^2$ (fig. 1.9) is given to the mobile system $\{S_t\}$ at zero time.To specify the translations and rotation, will proceed in the following way :-
Moving, according to the movement noted by 1, the transducer Q_0, which measures distance

P_1P_2, P_2 point being the point of intersection of the axes O_1z_1 and. $O_t^0y_t^0$. It get so translation along the axis $O_t^0y_t^0$. P_1P_2 distance can be calculated with relation:

$$P_2R_2 + P_1R_2 = d_0 tg\alpha + P_1R_2 \qquad (1)$$

P_1R_2 segment can be expressed as:

$$P_1R_2 = y^* - (a_2 - a_1). \qquad (2)$$

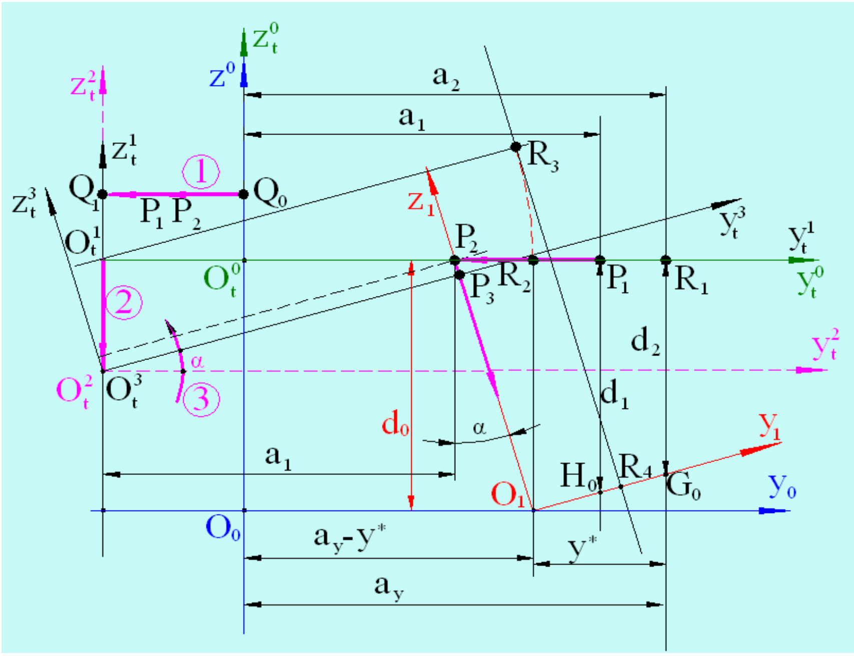

Fig 1.9. The movement of mobile system in version 2 of the trajectory correction.

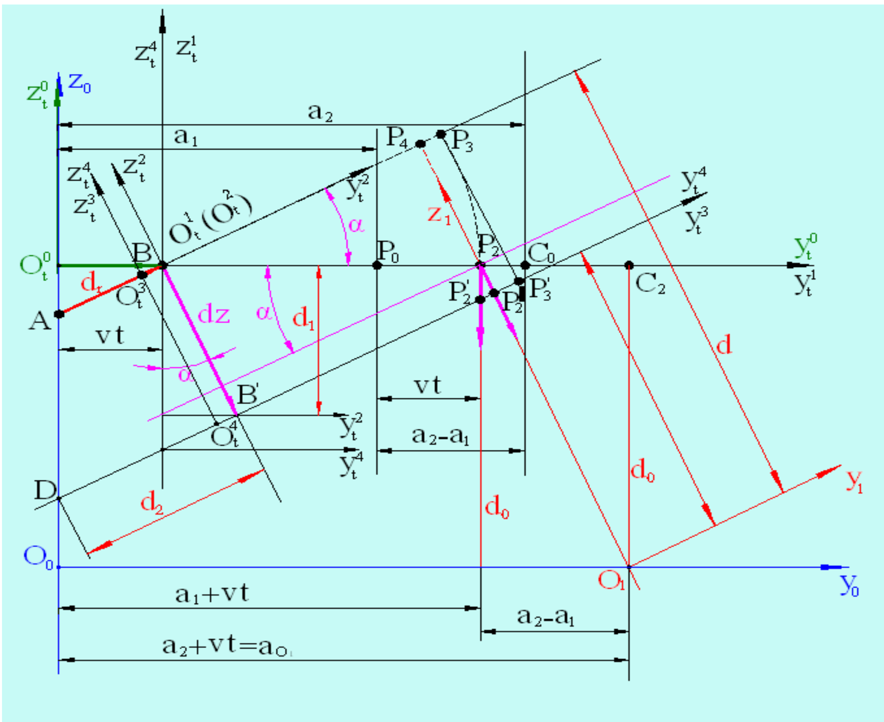

Fig 1.10. The movement of mobile system in version 3 of the trajectory correction.

P_3 position is determined by:

$$P_2P_3 = \frac{d_0}{c\alpha} - d_0 = d_0(\frac{d}{c\alpha} - 1) \qquad (3)$$

It requires that the distance measured from the transducer from P_2 to be d_0 . Thus we obtain:

$$P_3O_1 = d_0 ;\; P_2O_1 = \frac{d_0}{c\alpha} . \qquad (4)$$

Through the P_3 goes a parallel line to the axis $O_t^1z_t^1$ in the point O_t^2 and $O_t^3z_t^3$ axis is identical with $O_t^2P_3$ axis.

1.3. The third variant

Are given a_{o_1}, a_1 and a_2 distances. In initial position, the proximity transducers are P_0 and C_0 at the distances a_1 and a_2 to the system $\{S_t^0\}$ (figure 1.10).

A motion expressed as:

$$a_{o_1} - a_1 = P_0P_2 = C_0C_2 \qquad (5)$$

is given to the mobile system.

The point P_0 will arrive in P_2, and the point C_0 in C_2. Through the point P_2 goes a parallel line to O_1y_1, which will intersect the axis $O_t^1z_t^1$ at the point O_t^2. It is noted that the displacement $O_t^2O_t^1$ is:

$$O_t^2O_t^1 = d_1 = a_1 tg\alpha \qquad (6)$$

Therefore, the method has the advantage that movement of the mobile system $\{S_t^0\}$ is done in one direction (from left to right) on the axis $O_t^0y_t^0$, and the rotation aret automatically obtained. The transducer from P_2, after the rotation with angle α of the system $\{S_t^2\}$ reaches P_3, and to return on O_1z_1 axis direction, the system $\{S_t^2\}$ will translate in the negative sense of the axis $O_t^2y_t^2$ with the value(a_1-$a_1\cos\alpha$), and point P_3 becomes P_4 . Transducer arrived in P4 will measure a distance:

$$d > d_o \qquad (7)$$

To correct, the system $\{S_t^3\}$ will move along the axis $O_t^3z_t^3$ with the distance:

$$dz = d - d_0 \qquad (8)$$

1.4. The displacement of mobile system

To follow the path (now known) at the distance d_0 the next method will be used (fig. 1.10):

-A displacement is given to the mobile system $\{S_t^n\}$:

$$a \le a_{O_2} - a_2 \qquad (9)$$

If equality, the transducer from point P reach point P_2.

-Is given to the mobile system $\{S_t^n\}$ a rotation with angle α. Transducer from P_2 measures the distance d.

About the rotation with the angle α it has to be considered the following:

-The transducer from B which initially was measuring the distance a_{01}-a_2 and it was fixed in O_t^1, will measure, after the rotation with angle α, the next distance:

$$d_r = \frac{a_{O_1} - a_2}{c\alpha} \qquad (10)$$

This rotation is necessary, which is also the rotation of the transducer from point P_2, to measure the distance P_2P_2”.

Two translations of the system are possible now:

a) One of the negative translation is made along the axis $O_t^2z_t^2$ having the value dz; the transducer B reach C. Then a negative translation along the axis $B'y_t^3$ with a value (a_1-$a_1$cos), **or**
b) One of the negative translation is made along the axis By_t^2 having the value (a_1-$a_1\cos\alpha$),;. Then a negative translation along the axis $O_t^2z_t^2$ with

a value dz; the transducer B reach B’, **and**

c) The other translation is along axis $O_t^4B \equiv Bz_t^1$ that is equal with:

$$\frac{dz}{c\alpha} = O_t^4B \qquad (11)$$

The proximity transducer from $O_t^3 \equiv B'$ will measure a distance B’D equal with:

$$d_2 = dz \cdot tg\alpha + \frac{v \cdot t}{c\alpha} \qquad (12)$$

Moving then point O_t^2 with a distance dz, becomes O_t^3; Through O_t^3 goes a parallel line to the axis O_1y_1, and P_2 point becomes P_2''. The movement of the end-effector will be on the $O_t^3z_t^3$ axis at the distance d_0 to the trajectory.

In conclusion, the order of operations for correction is: a translation along the axis $O_t^0y_t^0$, followed by a rotation of angle α around the point $B \equiv O_t^1 \equiv O_t^2$ and continued with a translation along the axis O_t^3B or O_t^4B.

This itinerary will be shown in the following figures, the transducers being colored in red

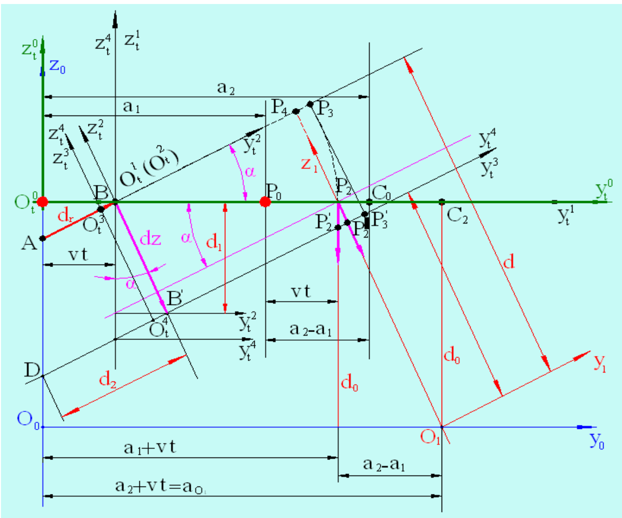

Fig.1.11. Position of mobile system (1)

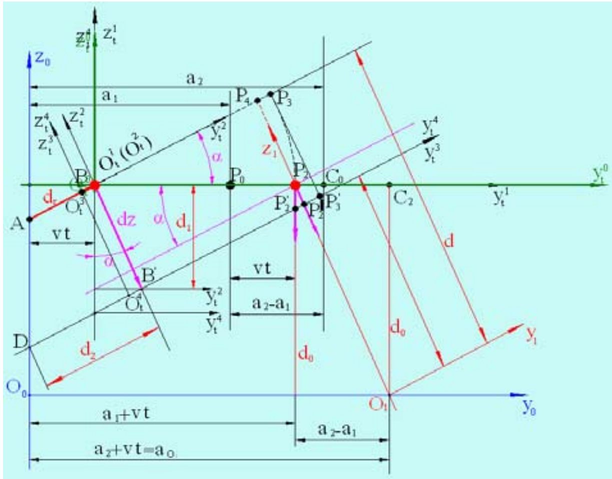

Fig.1.12. Position of mobile system (2)

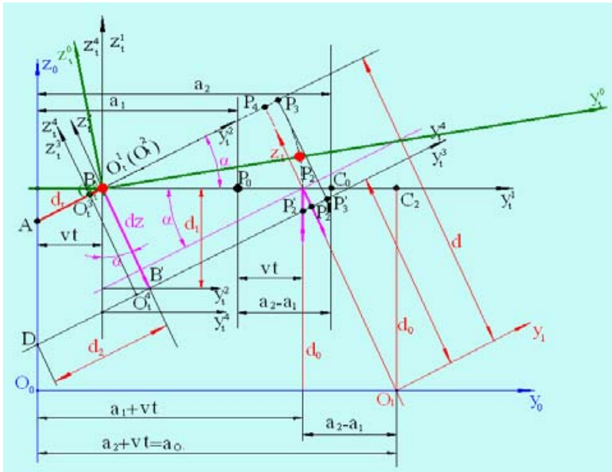

Fig.1.13. Position of mobile system (3)

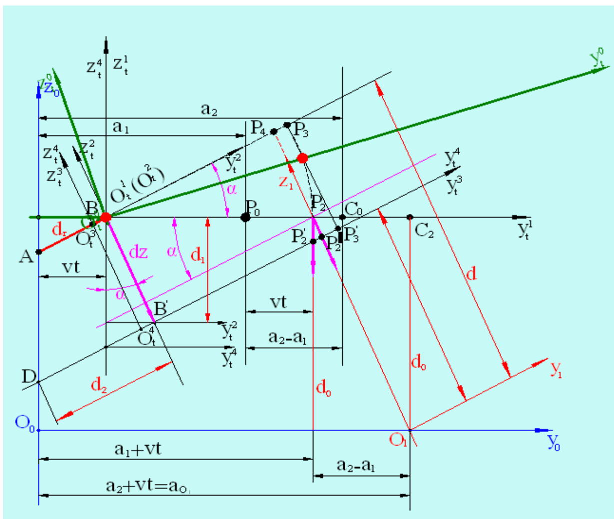

Fig.1.14. Position of mobile system (4)

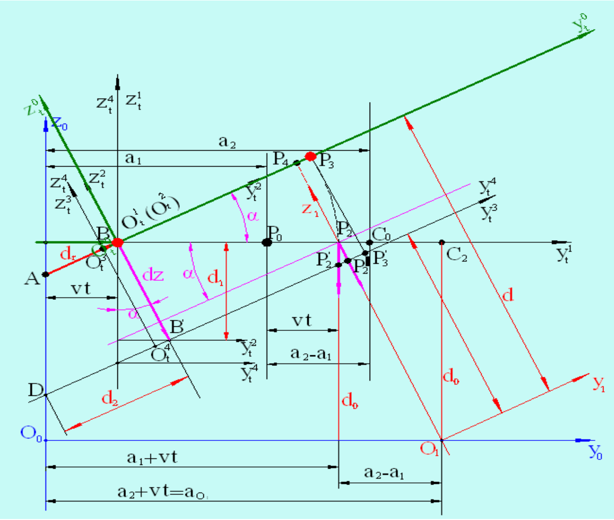

Fig.1.15. Position of mobile system (5)

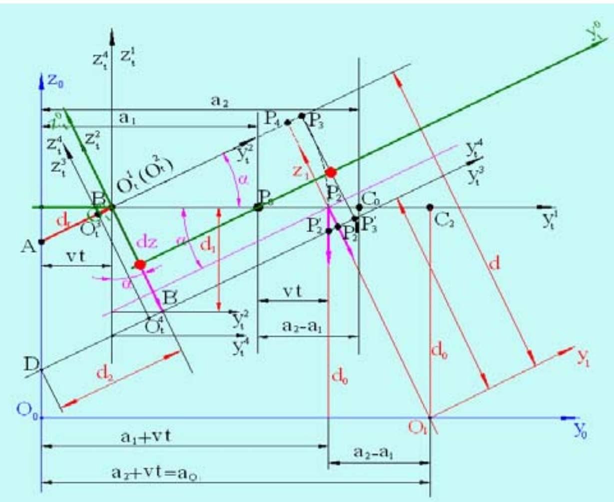

Fig.1.16. Position of mobile system (6)

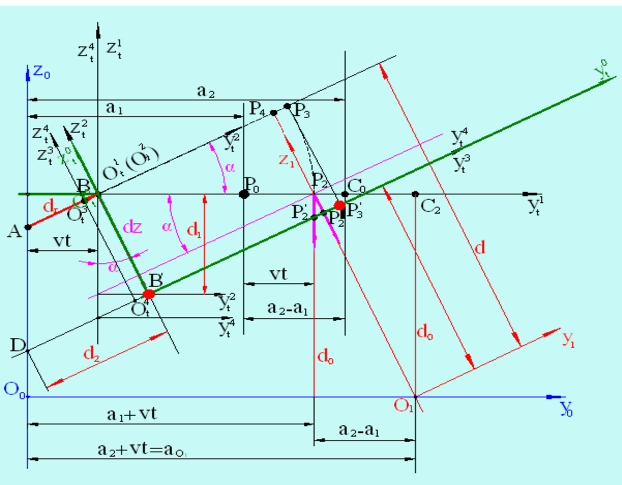

Fig.1.17. Position of mobile system (7)

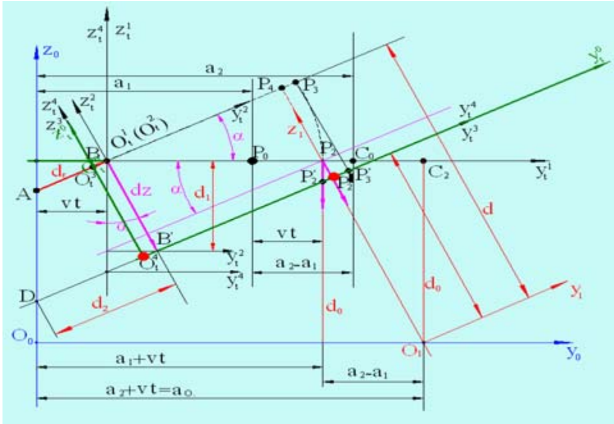

Fig.1.18. Position of mobile system (8)

2.THE MATRIX FORM OF FOLLOWING THE TRACK

For the horizontal section of the track a translation of the mobile system $\{S_t^n\}$ corresponds from position $\{S_t^0\}$ which coincides with the immobile system $\{S_0\}$ to position $\{S_t^1\}$. The proximity transducer from B will measure the distance y_t. When this distance will be equal with:

$$y_t = a_{O_1} - a_2 = v \cdot t \tag{13}$$

then the transducer from P arrives in P_2. The movement speed is constant in its value, resulting:

$$t = \frac{a_{O_1} - a_2}{v} \tag{14}$$

The corresponding matrix of the displacement of the system $\{S_t^n\}$ towards the system $\{S_t^0\}$ is:

$$[M_{t^n}^{t^0}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & vt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Towards the system $\{S_0\}$ the corresponding matrix is:

$$[M_{t^n}^{0}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & vt \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

From position $\{S_t^1\}$ is given to the system a translation d_1 according with relation (5) and the system arrives to $\{S_t^2\}$ position The corresponding matrix of this displacement is:

$$[M_{t^2}^{t^1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

Towards the immobile system $\{S_0\}$ the position of system $\{S_t^2\}$ is given by matrix:

$$[M_{t^2}^{0}] = [M_{t^1}^{0}][M_{t^2}^{t^1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$
$$\cdot\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_{O_1}-a_2 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_{O_1}-a_2 \\ 0 & 0 & 1 & d_0-d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

From position $\{S_t^2\}$, a rotation with angle α is given to the system so it can reach the position $\{S_t^3\}$. The matrix which positions the system $\{S_t^3\}$ towards the immobile system $\{S_0\}$ is:

$$[M_{t^3}^{0}] = [M_{t^2}^{0}][M_{t^3}^{t^2}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_{O_1}-a_2 \\ 0 & 0 & 1 & d_0-d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$
$$\cdot\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & a_{O_1}-a_2 \\ 0 & s\alpha & c\alpha & d_0-d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (19)$$

In this position the transducer from P2 will measure the distance d and to correct the system $\{S_t^3\}$ will make a translation along the axis $O_t^2 z_t^2$ which is equal with:

$$(d-d_0)c\alpha \quad (20)$$

reaching the position $\{S_t^4\}$.

The position of the system $\{S_t^4\}$ towards the system $\{S_0\}$ is given by the matrix:

$$[M_{t^4}^{0}] = [M_{t^3}^{0}][M_{t^4}^{t^3}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & a_{O_1}-a_2 \\ 0 & s\alpha & c\alpha & d_0-d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$
$$\cdot\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{d-d_0}{c\alpha} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \quad (21)$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & s\alpha\frac{d-d_0}{c\alpha}+a_{O_1}-a_2 \\ 0 & s\alpha & c\alpha & -c\alpha\frac{d-d_0}{c\alpha}+d_0-d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3. CONCLUSIONS

Using a method of straight lines parallel it was determined the shape of the trajectory, as well as the positioning of the end-effector, fitted with the proximity transducers, so that its distance from the travelled trajectory to be d_0 (known).

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Metode de urmărire şi corectare s traiectoriilor cu ajutourl traductoarilor de proximitate (partea II-a)

Rezumat: În această lucrare este studiată problema corectării unei traiectorii (necunoscută) situată în planul yOz, la o distanță dată d_0. În drumul parcurs, dispozitivul de prehensiune, dotat cu traductori de proximitate, trebuie să rezolve două probleme:

- identificarea formei traiectoriei;
- poziționarea dispozitivului de prehensiune astfel încât distanța față de traiectoria parcursă să fie d_0 (cunoscută).

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