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MATHEMATICAL MODELING OF THE LINEAR OSCILOMOTOR USED AT BORE SUPERFINISHING

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Abstract: *Bores superfinishing with the linear oscilomotor helps us obtain Ra roughness of processed surfaces under 0.4 μm. In the paper is briefly presented a linear oscilomotor used for superfinishing bores and its mathematical modeling, from which reveals the connection between the friction coefficient μ and roughness R, connection that expressed graphically is line type.*
Key words: *Superfinishing, signal, oscilomotor, roughness, friction.*

1. SHORT PRESENTATION OF THE LINEAR OSCILO-MOTOR

The oscilo-motor Figure 1 snaps into the lathe tool place, in the carriage of the parallel lathe using the stand (15) which can be observed in Figure 2. Movement that occurs in the coil (2) by its excitation is transmitted through the springs (18) which are fixed in the holder (21) of the upper lever (1) operating the abrasive tool (25) and by this we can obtain the vibration movement necessary for superfinishing the surfaces.

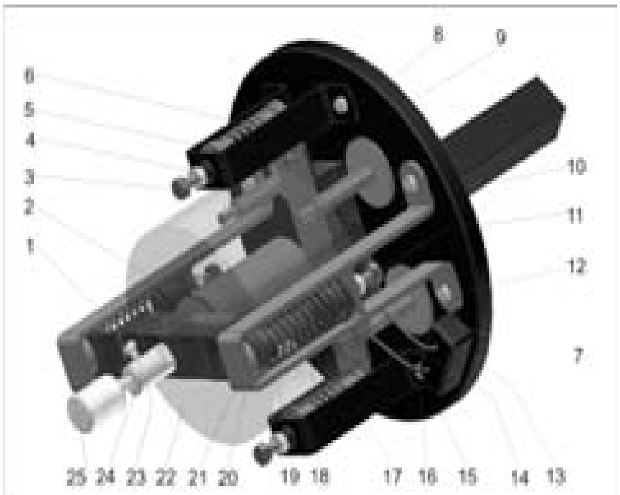

Fig. 1. Linear oscilo-motor

2. MATHEMATICAL MODELING OF THE LINEAR OSCILOMOTOR

If we consider the oscilomotor a mechanical system in translational motion without friction, we have the following situation:

2.1. Mechanical system in translational movement without friction

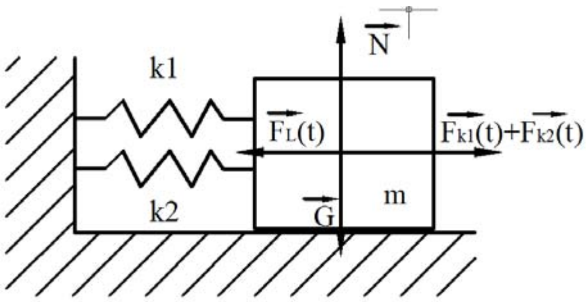

Fig. 2. Mechanical system in translational movement without friction

$$\vec{F}_R(t) = \vec{F}_L(t) + \vec{F}_{K_1}(t) + \vec{F}_{K_2}(t) + \vec{G} + \vec{N} \quad (1)$$

the equilibrium equation of forces

$\vec{F}_R$ – resultant force

$\vec{F}_L$ – deformed force (excitation)

$\vec{F}_{K_1}, \vec{F}_{K_2}$ – elastic forces

$\vec{G}$ – weight

$\vec{N}$ – normal pressing

$$\begin{cases} (O_X: \ F_R(t) = F_L(t) - F_{K_1}(t) - F_{K_2}(t) \\ (O_Z: \ -G + N = \phi \end{cases} \quad (2)$$

$$\left. \begin{aligned} & m.a(t) = F_L(t) - k_1.x(t) - k_2.x(t) \\ & a(t) = \frac{d^2.x(t)}{dt^2} \end{aligned} \right\} \Rightarrow$$

$$m.\frac{d^2.x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) \quad (3)$$

equilibrium equation on the x axis

⇓ Laplace transform

$$m.s^2.x(s) = F_L(s) - (k_1 + k_2).x(s) \quad (4)$$

equation in the complex domain s (Laplace)

$$m.s^2.x(s) + (k_1 + k_2).x(s) = F_L(s) \quad (5)$$

$$[m.s^2 + (k_1 + k_2)].x(s) = F_L(s) \quad (6)$$

transfer function for the mechanical part :

$$\frac{X(s)}{F_L(s)} = \frac{1}{m.s^2 + (k_1 + k_2)} = H(s) \quad (7)$$

In case that the oscilomotor receives a microstep type command, the excitation signal is sine, and in case that the oscilomotor is controlled by a full step, the excitation signal is step type. The two types of signal are listed below, and were generated with Octave.org software.

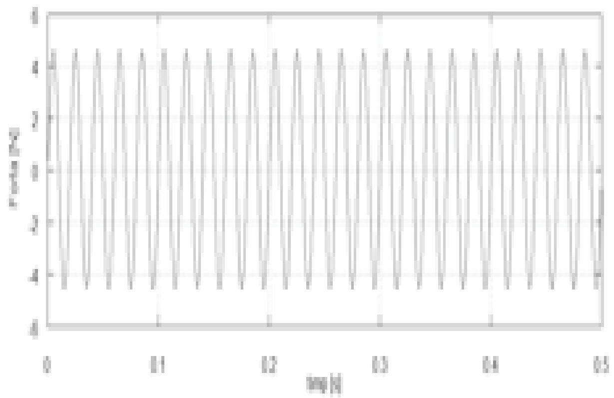

Fig. 3. Sine excitation signal

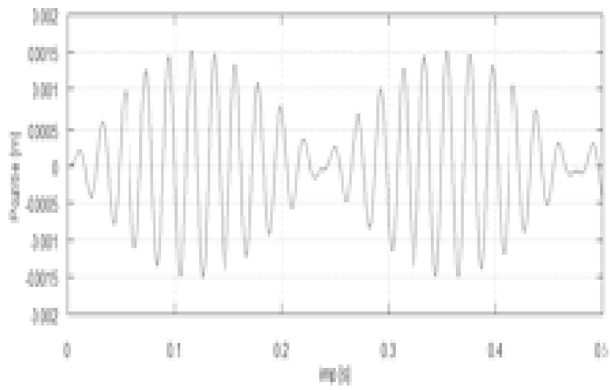

Fig. 4. Oscilomotor response to sine

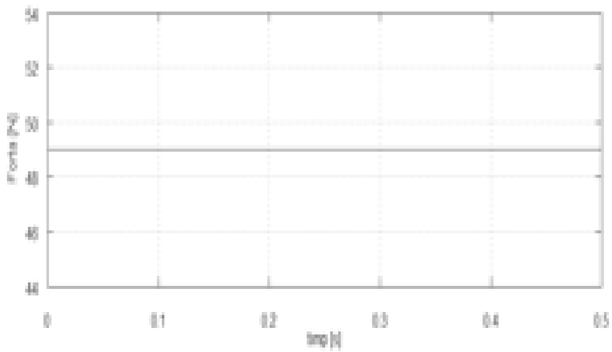

Fig. 5. Step excitation signal

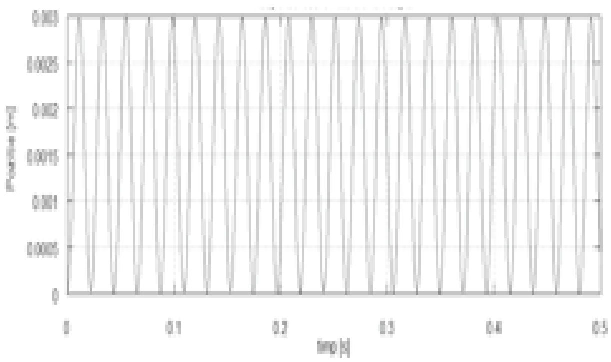

Fig. 6. Oscilomotor response to step signal

2.2. Mechanical system in translational movement with friction

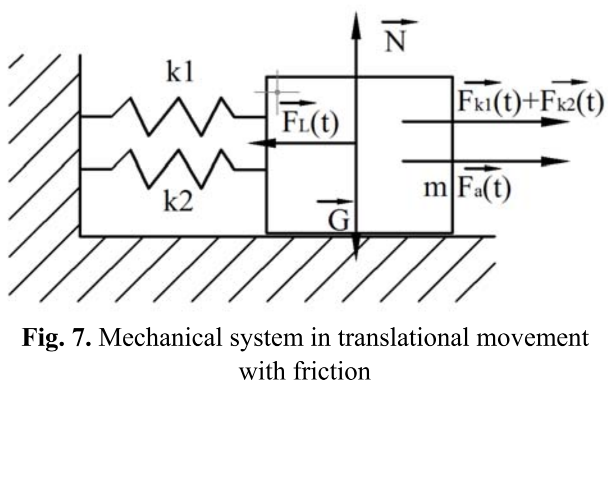

Fig. 7. Mechanical system in translational movement with friction

$$m.a(t) = F_L(t) - (F_{K_1}(t) + F_{K_2}(t)) - F_f(t) \quad (8)$$

$F_f(t) -$ friction force

$F_f(t) = \mu(t).N$

$\mu(t) -$ nonlinear

$$m.a(t) = F_L(t) - F_{K_1}(t) - F_{K_2}(t) - F_f(t) \quad (9)$$

$$m.a(t) = F_L(t) - k_1.x(t) - k_2.x(t) - \mu(t).N \quad (10)$$

$$m.\frac{d^2.x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) - \mu(t).N \quad (11)$$

$$F_L(t) = m.\frac{d^2x(t)}{dt^2} + (k_1 + k_2).x(t) + \frac{k_{\mu R}}{R(t)}.N \quad (12)$$

2.3. Determination of the linear dependence of roughness for each in the case of translational motion with friction

At superfinishing, we have three distinct slopes for roughness decrease, so, for a slope (Fig. 8) we have:

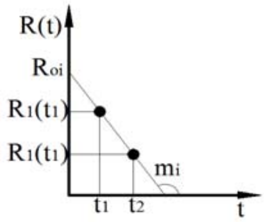

Fig. 8. R dependence grapf of t

$$\begin{cases} R_1(t_1) = m_1.t_1 + n_1 \\ R_1(t_2) = m_1.t_2 + n_1 \end{cases} \quad (13)$$

$$R_i(t) = m_i.t + n_i \quad (14)$$

R_i segment equation

$$n_1 = R_1(t_2) - m_1.t_2 \quad (15)$$

$$R_1(t_1) = m_1.t_1 + R_1(t_2) - m_1.t_2 \quad (16)$$

$$R_1(t_1) - R_1(t_2) = m_1.(t_1 - t_2) \Rightarrow$$

$$m_1 = \frac{R_1(t_1) - R_1(t_2)}{t_1 - t_2} \quad (17)$$

$$n_1 = R_1(t_2) - \frac{R_1(t_1) - R_1(t_2)}{t_1 - t_2}.t_2 \quad (18)$$

$$n_1 = \frac{R_1(t_2).t_1 - R_1(t_2).t_2 - R_1(t_1).t_2 + R_1(t_2).t_2}{t_1 - t_2} \quad (19)$$

$$R_i(t) = \frac{R_i(t_{i1}) - R_i(t_{i2})}{t_{i1} - t_{i2}}.t + \frac{R_i(t_{i2}).t_{i1} - R_i(t_{i1}).t_{i2}}{t_{i1} - t_{i2}} \quad (20)$$

$R_i(t) = m_i.t + n_i$

$m_i -$ proportionality constant between time and roughness (slope)

$n_i -$ global initial roughness (not range)

The mathematical relationship of the roughness time dependence is:

$$\frac{m.d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + (k_1 + k_2).x(t) = $$
$$= F_L(t) + \mu.N|L(t) \quad (21)$$

$$m.s^2x(s) + c.s.x(s) + (k_1 + k_2).x(s) =$$
$$= F_L(s) + \frac{\mu.N}{s} \quad (22)$$

$$[m.s^2 + c.s + (k_1 + k_2)].x(s) = s\frac{F_L(s)}{s} +$$
$$+\frac{\mu.N}{s} \quad (23)$$

2.3. Determining the relationship between μ and R

$$\mu(t) \approx \frac{1}{R(t)} \quad (24)$$

$\mu -$ friction coefficient

$$\mu(t) = \frac{k_{\mu R}}{R(t)} \quad (25)$$

$k_{\mu R} -$ proportionality constant between the inverse roughness and friction coefficient

$k_{\mu R} -$ the same for all intervals

kμ determination:

$$\left.\begin{matrix} \mu(t_1) \\ R(t_1) \end{matrix}\right\} \Rightarrow k_{\mu R} = \mu(t_1).R(t_1) \quad (26)$$

$$F_f = \mu.N \quad (27)$$

$$\left.\begin{matrix} F_f - \text{dynamometer} \\ N - \text{pressing} \end{matrix}\right\} \Rightarrow \mu = \frac{F_f}{N} \quad (28)$$

From (11) and (25) $\Rightarrow$ Determining the link in: x(t), $F_L(t)$ si $R_i(t)$

$$m.\frac{d^2x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) - \mu(t).N \quad (29)$$

$$m.\frac{d^2x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) - \frac{k_{\mu R}.N}{R(t)} \quad (30)$$

Version 1:

- equation is valid for the period of processing if:

$$R(t) = \begin{cases} k_1.\int_0^{t_1} |x(\tau)|d\tau + r_{01}, t \in [0, t_1) \\ k_2.\int_{t_1}^{t_2} |x(\tau)|d\tau + r_{02}, t \in [t_1, t_2) \\ k_3.\int_{t_2}^{t_3} |x(\tau)|d\tau + r_{03}, t \in [t_2, t_3) \end{cases} \quad (31)$$

Version 2:

- equation is considered only on intervals

$$m.\frac{d^2x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) - \frac{k_{\mu R}.N}{R_i(t)} \quad (32)$$

$$m.\frac{d^2x(t)}{dt^2} = F_L(t) - (k_1 + k_2).x(t) - \frac{k_{\mu R}.N}{k_i.\frac{4a}{T} + r_{01}} + $$
$$+c\frac{dx(t)}{dt} \quad (33)$$

$$k_i\frac{4a}{T}.m.t.\frac{d^2x(t)}{dt^2} + m.r_{01}.\frac{d^2x(t)}{dt^2} + k_i\frac{4a}{T}.(k_1 + k_2).t.x(t) +$$
$$+r_{01}.(k_1 + k_2).x(t) = k_i\frac{4a}{T}.t.F_L(t) + r_{01}.F_L(t) - k_{\mu R}.N \quad (34)$$

$$L\left\{k_i.\frac{4a}{T}.m.t.\frac{d^2x(t)}{dt^2}\right\} + m.r_{01}.s^2.x(s) - k_i.\frac{4a}{T}(k_1 + k_2).x'(s) +$$
$$+r_{01}(k_1 + k_2).x(s) = k_i.\frac{4a}{T}.F_L'(s) + r_{01}.F_L(s) - \frac{k_{\mu R}.N}{s} \quad (35)$$

3. CONCLUSION

The oscilomotor, whether it receives a micro step type command, sine excitation signal, or whole step, step type excitation signal, there is a connection between the friction coefficient and roughness, which is expressed mathematically by the relationship (30), that can be expressed throughout the processing time, or only on intervals where the slope of the roughness variation is constant.

4. REFERENCES

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Modelarea matematică a oscilomotorului liniar utilizat la vibronetezirea alezajelor

Vibronetezirea alezajelor cu ajutorul oscilomotorului liniar, ne ajută să obţinem rugozităţi Ra a suprafeţelor prelucrate, sub 0.4 μm. În lucrare este prezentat pe scurt un oscilomotor liniar utilizat la vibronetezirea alezajelor, precum şi modelarea matematică a acestuia, din care se evidenţiază legătura între coeficientul de frecare μ şi rugozitatea R, legătură care exprimată grafic este de tip dreaptă.

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