



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics

Vol. 55, Issue IV, 2012

## THE DYNAMICS OF A NEW RECONFIGURABLE PARALLEL ROBOT WITH SIX DEGREES OF FREEDOM

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**Abstract:** The paper presents a new parallel reconfigurable robot with six degrees of freedom and three guiding kinematic chains of the platform. A reconfigurable robot can change its structure in order to quickly adjust to production capacity and functionality. Using the virtual work method, this paper presents the workspace dynamics of the parallel reconfigurable robot. Some simulation results and some conclusions are presented at the end. **Key words:** reconfiguration, parallel robot, dynamic, virtual.

### 1. INTRODUCTION

Robotic system have been developed in every field where a further progress was constricted due the human limitation in terms of speed, precision, fatigue, repeatability, strength, etc. Parallel robots have a series of advantages in comparison with the serial one, like: high stiffness, high accelerations and speeds, high precision and a modular simple construction. [1]

A reconfigurable robot can change its structure in order to quickly adjust to production capacity and functionality.

Main characteristics: Modularity; Scalability; Convertibility; Ease of diagnosis.

The reconfiguration of a robot can be achieved by means of:

- Joints position and characteristics modification;
- Robot configuration changing;
- Geometrical parameters adjustment.

Coupling - uncoupling mechanisms are important elements in the field of reconfigurable robots. The structures will be simulated and analyzed using computers in order to coupling - uncoupling elements and possible collisions between the robot elements. [2]

A new approach for the analysis of a family of parallel reconfigurable robots is proposed by Gogu in [3]. The parallel structure, called

Isoglidon-TaRb, can have up to five degrees of freedom which are a combination of maximum 3 independent translations and maximum 2 rotations.

Other parallel reconfigurable machines are developed by Negri in [4] and several authors in [1], [5], [6], [7].

The dynamic models of parallel robots are usually developed following one of two approaches: the Newton Euler or the Lagrange methods. The Lagrange method describes the dynamics of a mechanical system from the concepts of work and energy [8], [9], [10], [11].

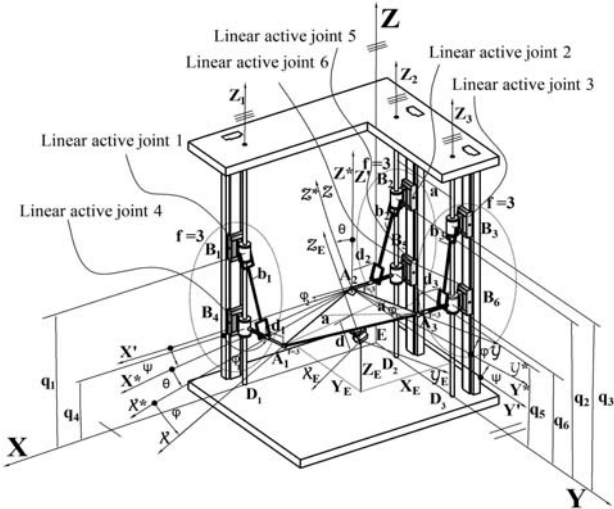
The parallel reconfigurable robot whose dynamic model is here presented is contained in the pending patent [12]. In order to continue the work presented in [13] where the kinematics model is established, this paper presents simulation results and task space dynamics.

The paper is organized as follows: Section 2 is dedicated to the description and design of the parallel reconfigurable robot with six degrees of freedom. The task space dynamics is the subject of the Section 3. In Section 4 and 5 numerical results and some conclusions are presented.

### 2. DESIGN CONSIDERATION

Figure 1 presents a six degrees of freedom parallel robot with three kinematic chains of the guiding platform. With the help of fasteners the

robot can be reconfigured in one with five, four, three or two degrees of freedom [14]. On the fixed frame three guides are fixed on which the six linear active joints slide. The robot has three identical kinematic chains: fork arm and slide, rotation couple and two linear motors on each of the three guides.



**Fig. 1.** Reconfigurable parallel robot with  $M=6$  d.o.f and three guiding kinematics chains of the platform

Mobile platform (end-effector) in this case is a right angle triangle defined by  $A_1, A_2, A_3$  related to the three kinematic chain structures via three spherical joints. Figure 2 illustrates a detail of the end-effector, along with the mobile coordinate systems attached to it. The three kinematic chains use fork-slide joint type. Movement and orientation are performed using six linear active joint mounted on three guides of the frame.

The geometrical parameters of the robot are represented by  $a, d_i, e_i, b_i$  where  $a$  - the cathetes length of the right angle triangle,  $d_i$  - the slider length,  $b_i$  - the fork slider,  $e_i$  - distances between point O and the frame guides, and the coordinates of point  $E(X_E, Y_E, Z_E)$ .

### 3. DYNAMICS

#### 3.1 Kinematics

Starting from the positional equations bellow obtained through the geometrical model

[13, 15], expressed in function by the coordinates of the end-effector  $X_E, Y_E, Z_E, \psi, \theta, \varphi$  and the active coordinates of the robot  $q_i, q_{i+3}$  the relations for the positional nonlinear equations are:

$$\begin{cases} F_i = -q_i + AA + \\ + \sqrt{b_i^2 - \left[ \left( \sqrt{b_i^2 - (q_i - AA)^2} + d_i \right)^2 - d_i \right]^2} \\ F_{i+3} = -q_{i+3} + Z_E + (x_{A_i} - x_E)(-s\theta c\varphi) + \\ + (y_{A_i} - y_E)s\theta s\varphi + (z_{A_i} - z_E)c\theta \\ \text{where } AA = Z_E + (x_{A_i} - x_E)(-s\theta c\varphi) + \\ + (y_{A_i} - y_E)s\theta s\varphi + (z_{A_i} - z_E)c\theta, i = 1, 2, 3 \end{cases} \quad (1)$$

The six positional nonlinear equations in implicit form are:

$$\begin{aligned} F_1(q_1, X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv & \left[ X_E + (c\psi c\theta c\varphi - s\psi s\varphi)(x_{A_1} - x_E) + \right. \\ & \left. + (-c\psi c\theta s\varphi - s\psi c\varphi)(y_{A_1} - y_E) + \right. \\ & \left. + c\psi s\theta(z_{A_1} - z_E) - X_{B_1} \right]^2 + \\ & \left[ Y_E + (s\psi c\theta c\varphi + c\psi s\varphi)(x_{A_1} - x_E) + \right. \\ & \left. + (-s\psi c\theta s\varphi + c\psi c\varphi)(y_{A_1} - y_E) + \right. \\ & \left. + s\psi s\theta(z_{A_1} - z_E) - Y_{B_1} \right]^2 - \\ & - \left\{ b_1^2 - \left\{ q_1 - \begin{bmatrix} Z_E + \\ + (-s\theta c\varphi)(x_{A_1} - x_E) \\ + s\theta s\varphi(y_{A_1} - y_E) + \\ + c\theta(z_{A_1} - z_E) \end{bmatrix} \right\}^2 \right\}^{\frac{1}{2}} + d_1 = 0 \\ F_2(q_2, X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv & \left[ X_E + (c\psi c\theta c\varphi - s\psi s\varphi)(x_{A_2} - x_E) + \right. \\ & \left. + (-c\psi c\theta s\varphi - s\psi c\varphi)(y_{A_2} - y_E) + \right. \\ & \left. + c\psi s\theta(z_{A_2} - z_E) - X_{B_2} \right]^2 + \\ & \left[ Y_E + (s\psi c\theta c\varphi + c\psi s\varphi)(x_{A_2} - x_E) + \right. \\ & \left. + (-s\psi c\theta s\varphi + c\psi c\varphi)(y_{A_2} - y_E) + \right. \\ & \left. + s\psi s\theta(z_{A_2} - z_E) - Y_{B_2} \right]^2 - \\ & - \left\{ b_2^2 - \left\{ q_2 - \begin{bmatrix} Z_E + \\ + (-s\theta c\varphi)(x_{A_2} - x_E) + \\ + s\theta s\varphi(y_{A_2} - y_E) + \\ + c\theta(z_{A_2} - z_E) \end{bmatrix} \right\}^2 \right\}^{\frac{1}{2}} + d_2 = 0 \end{aligned}$$

$$F_3(q_3, X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv \left[ \begin{array}{l} X_E + (c\psi c\theta c\varphi - s\psi s\varphi)(x_{A_3} - x_E) + \\ + (-c\psi c\theta s\varphi - s\psi c\varphi)(y_{A_3} - y_E) + \\ + c\psi s\theta(z_{A_3} - z_E) - X_{B_3} \end{array} \right]^2 + \left[ \begin{array}{l} Y_E + (s\psi c\theta c\varphi + c\psi s\varphi)(x_{A_3} - x_E) + \\ + (-s\psi c\theta s\varphi + c\psi c\varphi)(y_{A_3} - y_E) + \\ + s\psi s\theta(z_{A_3} - z_E) - Y_{B_3} \end{array} \right]^2 - \left[ \left[ \left[ \left[ \begin{array}{l} Z_E + \\ + (-s\theta c\varphi)(x_{A_3} - x_E) + \\ + s\theta s\varphi(y_{A_3} - y_E) + \\ + c\theta(z_{A_3} - z_E) \end{array} \right] \right]^2 \right]^{\frac{1}{2}} + d_3 \right]^2 = 0$$

$$F_4(q_4, Z_E, \theta, \varphi) \equiv -q_4 + Z_E + (-s\theta c\varphi) \cdot (x_{A_1} - x_E) + s\theta s\varphi(y_{A_1} - y_E) + c\theta(z_{A_1} - z_E) = 0$$

$$F_5(q_5, Z_E, \theta, \varphi) \equiv -q_5 + Z_E + (-s\theta c\varphi) \cdot (x_{A_2} - x_E) + s\theta s\varphi(y_{A_2} - y_E) + c\theta(z_{A_2} - z_E) = 0$$

$$F_6(q_6, Z_E, \theta, \varphi) \equiv -q_6 + Z_E + (-s\theta c\varphi) \cdot (x_{A_3} - x_E) + s\theta s\varphi(y_{A_3} - y_E) + c\theta(z_{A_3} - z_E) = 0$$

(2)

### 3.2 Task space dynamics

For the inverse dynamic the generalized coordinates of the end-effector are given and the forces and torques for the driving linear joints are computed.

$$\begin{aligned} X_E &= X_E(t), Y_E = Y_E(t), Z_E = Z_E(t), \\ \psi &= \psi(t), \theta = \theta(t), \varphi = \varphi(t), \\ \tau_1 &= F_1, \tau_2 = F_2, \tau_3 = F_3, \\ \tau_4 &= F_4, \tau_5 = F_5, \tau_6 = F_6 \end{aligned} \quad (3)$$

The mobile elements of the robot are replaced with an equivalent system of lumped masses  $m_1^*, \dots, m_{29}^*$  in points  $M_i$ ,  $i=1, \dots, 29$  with the coordinates  $X_{M_i}, Y_{M_i}, Z_{M_i}$ ,  $i=1, \dots, 29$ .

With the following notations:

$q = [q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6]^T$  the vector of the linear active joints coordinates;

$\delta q = [\delta q_1 \ \delta q_2 \ \delta q_3 \ \delta q_4 \ \delta q_5 \ \delta q_6]^T$  the vector of virtual displacements of the active joints positions;

$\dot{q} = [\dot{q}_1 \ \dot{q}_2 \ \dot{q}_3 \ \dot{q}_4 \ \dot{q}_5 \ \dot{q}_6]^T$  the vector of active linear joints speeds;

$\ddot{q} = [\ddot{q}_1 \ \ddot{q}_2 \ \ddot{q}_3 \ \ddot{q}_4 \ \ddot{q}_5 \ \ddot{q}_6]^T$  the vector of active linear joints accelerations;

$X_P = [X_E \ Y_E \ Z_E \ \psi \ \theta \ \varphi]^T$  the vector of the generalized coordinates of the end-effector;

$X_{M_i} = [X_i \ Y_i \ Z_i]^T$ ,  $i=1, 2, \dots, 29$  the vector of the Cartesian coordinates of points  $M_i$  of the lumped masses  $m_i^*$  dynamic equivalent with the system of  $N$  mobile elements;

$\dot{X}_P = [\dot{X}_E \ \dot{Y}_E \ \dot{Z}_E \ \dot{\psi} \ \dot{\theta} \ \dot{\varphi}]^T$  the vector of velocities for the point E of the mobile platform;

$\ddot{X}_P = [\ddot{X}_E \ \ddot{Y}_E \ \ddot{Z}_E \ \ddot{\psi} \ \ddot{\theta} \ \ddot{\varphi}]^T$  the vector of accelerations of the point E;

$\dot{X}_{M_i} = [\dot{X}_i \ \dot{Y}_i \ \dot{Z}_i]^T$  the vector of speeds for the points  $M_i$ ;

$\tau = [\tau_1 \ \tau_2 \ \tau_3 \ \tau_4 \ \tau_5 \ \tau_6]^T \equiv [F_1 \ F_2 \ F_3 \ F_4 \ F_5 \ F_6]^T$  the vector of the active forces.

The virtual work principle was used in order to obtain the dynamic model for the robot with six degrees of freedom.

$$\delta W \equiv \delta q^T \tau + \sum_{i=1}^{29} \delta X_{M_i}^T (T_i^{Tr} + T_i^g) = 0 \quad (4)$$

Principle that states the fact that the sum of virtual torques of the forces, the gravity forces  $m_i g$ ,  $i=1, \dots, 29$  and of the inertia forces  $m_i a_i$ ,  $i=1, \dots, 29$  equals to zero for the virtual displacements compatible with the constraints; where  $\delta q^T \tau$  is the virtual work of all actuating forces and moments and  $\sum_{i=1}^{29} \delta X_{M_i}^T (T_i^{Tr} + T_i^g)$  is the virtual work of inertia forces and gravitation forces corresponding to the equivalent system (with 29 lumped masses).

$$\delta q^T \tau = [\delta q_1 \quad \delta q_2 \quad \delta q_3 \quad \delta q_4 \quad \delta q_5 \quad \delta q_6] \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \tau_1 \cdot \delta q_1 + \tau_2 \cdot \delta q_2 + \dots + \tau_6 \cdot \delta q_6 \tag{5}$$

The matrix of inertia forces is given by the equation:

$$T_i^{Tr} = \begin{bmatrix} -m_i^* \ddot{X}_i \\ -m_i^* \ddot{Y}_i \\ -m_i^* \ddot{Z}_i \end{bmatrix} = - \begin{bmatrix} m_i^* & 0 & 0 \\ 0 & m_i^* & 0 \\ 0 & 0 & m_i^* \end{bmatrix} \begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i \end{bmatrix} \tag{6}$$

The matrix of the gravitation forces:

$$T_i^g = \begin{bmatrix} 0 \\ 0 \\ -m_i \cdot g \end{bmatrix} \tag{7}$$

The coordinates of points  $M_i$  having the coordinates  $X_{M_i}, Y_{M_i}, Z_{M_i}$ , can be expressed in respect to the generalized coordinates of the end-effector  $X_E, Y_E, Z_E, \psi, \theta, \varphi$ .

$$\begin{cases} X_i = X_i(X_E, Y_E, Z_E, \psi, \theta, \varphi) \\ Y_i = Y_i(X_E, Y_E, Z_E, \psi, \theta, \varphi) \\ Z_i = Z_i(X_E, Y_E, Z_E, \psi, \theta, \varphi) \end{cases}, i=1, 2, \dots, 29 \tag{8}$$

Deriving with respect to time it results:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{Z}_i \end{bmatrix} = \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial X_i}{\partial Y_E} & \frac{\partial X_i}{\partial Z_E} & \frac{\partial X_i}{\partial \psi} & \frac{\partial X_i}{\partial \theta} & \frac{\partial X_i}{\partial \varphi} \\ \frac{\partial Y_i}{\partial X_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Y_i}{\partial \varphi} \\ \frac{\partial Z_i}{\partial X_E} & \frac{\partial Z_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Z_E} & \frac{\partial Z_i}{\partial \psi} & \frac{\partial Z_i}{\partial \theta} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} \tag{9}$$

$i=1, 2, 3.$

With the following notations:

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = X_{M_i}, \begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{Z}_i \end{bmatrix} = \dot{X}_{M_i},$$

$$J_i = \frac{\partial X_{M_i}}{\partial X_P} = \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial X_i}{\partial Y_E} & \frac{\partial X_i}{\partial Z_E} & \frac{\partial X_i}{\partial \psi} & \frac{\partial X_i}{\partial \theta} & \frac{\partial X_i}{\partial \varphi} \\ \frac{\partial Y_i}{\partial X_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Y_i}{\partial \varphi} \\ \frac{\partial Z_i}{\partial X_E} & \frac{\partial Z_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Z_E} & \frac{\partial Z_i}{\partial \psi} & \frac{\partial Z_i}{\partial \theta} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix} \tag{10}$$

$i=1,2,\dots,29.$

Using the notation (10) equation (9) takes the following form:

$$\begin{aligned} \dot{X}_{M_i} &= J_i \cdot \dot{X}_P \\ \ddot{X}_{M_i} &= \frac{\partial X_{M_i}}{\partial X_P} \ddot{X}_P, i=1,2,3 \end{aligned} \tag{11}$$

After the derivation of the above equation it results:

$$\ddot{X}_{M_i} = J_i \ddot{X}_P + \dot{J} \dot{X}_P, i=1,2,3 \tag{12}$$

The next notations can be made:

$$\ddot{X}_{M_i} = \begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i \end{bmatrix}, \ddot{X}_P = \begin{bmatrix} \ddot{X}_E \\ \ddot{Y}_E \\ \ddot{Z}_E \\ \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix}$$

$$J_i = \frac{d}{dt} \left( \frac{\partial X_{M_i}}{\partial X_P} \right) = \begin{bmatrix} \frac{d}{dt} \left( \frac{\partial X_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Y_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Z_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \varphi} \right) \end{bmatrix} \tag{13}$$

Equation (12) takes the form:

$$\begin{bmatrix} \ddot{X}_i \\ \ddot{Y}_i \\ \ddot{Z}_i \end{bmatrix} = \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial X_i}{\partial Y_E} & \frac{\partial X_i}{\partial Z_E} & \frac{\partial X_i}{\partial \psi} & \frac{\partial X_i}{\partial \theta} & \frac{\partial X_i}{\partial \varphi} \\ \frac{\partial Y_i}{\partial X_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Y_i}{\partial \varphi} \\ \frac{\partial Z_i}{\partial X_E} & \frac{\partial Z_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Z_E} & \frac{\partial Z_i}{\partial \psi} & \frac{\partial Z_i}{\partial \theta} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix} \begin{bmatrix} \ddot{X}_E \\ \ddot{Y}_E \\ \ddot{Z}_E \\ \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} +$$

$$\begin{bmatrix} \frac{d}{dt} \left( \frac{\partial X_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Y_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Z_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \varphi} \right) \end{bmatrix} \begin{bmatrix} \dot{X}_E \\ Y_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} \quad (14)$$

The derivatives in respect to time from the (14) equation are:

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial X_E} \right) = \frac{\partial^2 X_i}{\partial X_E^2} \dot{X}_E + \frac{\partial^2 X_i}{\partial X_E \partial Y_E} \dot{Y}_E + \frac{\partial^2 X_i}{\partial X_E \partial Z_E} \dot{Z}_E + \frac{\partial^2 X_i}{\partial X_E \partial \psi} \dot{\psi} + \frac{\partial^2 X_i}{\partial X_E \partial \theta} \dot{\theta} + \frac{\partial^2 X_i}{\partial X_E \partial \varphi} \dot{\varphi} \quad (15)$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial Y_E} \right) = \frac{\partial^2 X_i}{\partial Y_E \partial X_E} \dot{X}_E + \frac{\partial^2 X_i}{\partial Y_E^2} \dot{Y}_E + \frac{\partial^2 X_i}{\partial Y_E \partial Z_E} \dot{Z}_E + \frac{\partial^2 X_i}{\partial Y_E \partial \psi} \dot{\psi} + \frac{\partial^2 X_i}{\partial Y_E \partial \theta} \dot{\theta} + \frac{\partial^2 X_i}{\partial Y_E \partial \varphi} \dot{\varphi} \quad (16)$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial Z_E} \right) = \frac{\partial^2 X_i}{\partial Z_E \partial X_E} \dot{X}_E + \frac{\partial^2 X_i}{\partial Z_E \partial Y_E} \dot{Y}_E + \frac{\partial^2 X_i}{\partial Z_E^2} \dot{Z}_E + \frac{\partial^2 X_i}{\partial Z_E \partial \psi} \dot{\psi} + \frac{\partial^2 X_i}{\partial Z_E \partial \theta} \dot{\theta} + \frac{\partial^2 X_i}{\partial Z_E \partial \varphi} \dot{\varphi} \quad (17)$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial \psi} \right) = \frac{\partial^2 X_i}{\partial \psi \partial X_E} \dot{X}_E + \frac{\partial^2 X_i}{\partial \psi \partial Y_E} \dot{Y}_E + \frac{\partial^2 X_i}{\partial \psi \partial Z_E} \dot{Z}_E + \frac{\partial^2 X_i}{\partial \psi^2} \dot{\psi} + \frac{\partial^2 X_i}{\partial \psi \partial \theta} \dot{\theta} + \frac{\partial^2 X_i}{\partial \psi \partial \varphi} \dot{\varphi} \quad (18)$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial \theta} \right) = \frac{\partial^2 X_i}{\partial \theta \partial X_E} \dot{X}_E + \frac{\partial^2 X_i}{\partial \theta \partial Y_E} \dot{Y}_E + \frac{\partial^2 X_i}{\partial \theta \partial Z_E} \dot{Z}_E + \frac{\partial^2 X_i}{\partial \theta \partial \psi} \dot{\psi} + \frac{\partial^2 X_i}{\partial \theta^2} \dot{\theta} + \frac{\partial^2 X_i}{\partial \theta \partial \varphi} \dot{\varphi} \quad (19)$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial \varphi} \right) = \frac{\partial^2 X_i}{\partial \varphi \partial X_E} \dot{X}_E + \frac{\partial^2 X_i}{\partial \varphi \partial Y_E} \dot{Y}_E + \frac{\partial^2 X_i}{\partial \varphi \partial Z_E} \dot{Z}_E + \frac{\partial^2 X_i}{\partial \varphi \partial \psi} \dot{\psi} + \frac{\partial^2 X_i}{\partial \varphi \partial \theta} \dot{\theta} + \frac{\partial^2 X_i}{\partial \varphi^2} \dot{\varphi} \quad (20)$$

For the other derivatives in respect to time of the coordinates  $Y_i, Z_i, i=1, \dots, 29$  can be computed in a similar way.

The vector of virtual displacement of points  $M_i, i=1, \dots, 29$  have the expression:

$$\begin{bmatrix} \delta X_{M_i} \\ \delta Y_{M_i} \\ \delta Z_{M_i} \end{bmatrix} = \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial X_i}{\partial Y_E} & \frac{\partial X_i}{\partial Z_E} & \frac{\partial X_i}{\partial \psi} & \frac{\partial X_i}{\partial \theta} & \frac{\partial X_i}{\partial \varphi} \\ \frac{\partial Y_i}{\partial X_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Y_i}{\partial \varphi} \\ \frac{\partial Z_i}{\partial X_E} & \frac{\partial Z_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Z_E} & \frac{\partial Z_i}{\partial \psi} & \frac{\partial Z_i}{\partial \theta} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix} \begin{bmatrix} \delta X_E \\ \delta Y_E \\ \delta Z_E \\ \delta \psi \\ \delta \theta \\ \delta \varphi \end{bmatrix} \quad (21)$$

$i=1, \dots, 29$ .  
Relation rewritten under the compact form:  
 $\delta X_{M_i} = J_i \cdot \delta X_P$  (22)

Resulting so for the relation above:  
 $\delta X_{M_i}^T = \delta X_P^T \cdot J_i^T; i=1, 2, \dots, 29$  (23)

$$\begin{bmatrix} \delta X_i & \delta Y_i & \delta Z_i \end{bmatrix} = \begin{bmatrix} \delta X_E & \delta Y_E & \delta Z_E & \delta \psi & \delta \theta & \delta \varphi \end{bmatrix} \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial Y_i}{\partial X_E} & \frac{\partial Z_i}{\partial X_E} \\ \frac{\partial X_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Y_E} \\ \frac{\partial X_i}{\partial Z_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Z_i}{\partial Z_E} \\ \frac{\partial X_i}{\partial \psi} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Z_i}{\partial \psi} \\ \frac{\partial X_i}{\partial \theta} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Z_i}{\partial \theta} \\ \frac{\partial X_i}{\partial \varphi} & \frac{\partial Y_i}{\partial \varphi} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix} \quad (24)$$

$i=1, 2, \dots, 29$ .  
Using equation above the expression of the virtual work principle takes the form:

$$\delta W \equiv \delta q^T \tau + \sum_{i=1}^{29} \delta X_P^T J_i^T (T_i^{Tr} + T_i^s) = 0 \quad (25)$$

$$\begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ \delta q_4 \\ \delta q_5 \\ \delta q_6 \end{bmatrix} = \begin{bmatrix} \frac{\partial q_1}{\partial X_E} & \frac{\partial q_1}{\partial Y_E} & \frac{\partial q_1}{\partial Z_E} & \frac{\partial q_1}{\partial \psi} & \frac{\partial q_1}{\partial \theta} & \frac{\partial q_1}{\partial \varphi} \\ \frac{\partial q_2}{\partial X_E} & \frac{\partial q_2}{\partial Y_E} & \frac{\partial q_2}{\partial Z_E} & \frac{\partial q_2}{\partial \psi} & \frac{\partial q_2}{\partial \theta} & \frac{\partial q_2}{\partial \varphi} \\ \frac{\partial q_3}{\partial X_E} & \frac{\partial q_3}{\partial Y_E} & \frac{\partial q_3}{\partial Z_E} & \frac{\partial q_3}{\partial \psi} & \frac{\partial q_3}{\partial \theta} & \frac{\partial q_3}{\partial \varphi} \\ \frac{\partial q_4}{\partial X_E} & \frac{\partial q_4}{\partial Y_E} & \frac{\partial q_4}{\partial Z_E} & \frac{\partial q_4}{\partial \psi} & \frac{\partial q_4}{\partial \theta} & \frac{\partial q_4}{\partial \varphi} \\ \frac{\partial q_5}{\partial X_E} & \frac{\partial q_5}{\partial Y_E} & \frac{\partial q_5}{\partial Z_E} & \frac{\partial q_5}{\partial \psi} & \frac{\partial q_5}{\partial \theta} & \frac{\partial q_5}{\partial \varphi} \\ \frac{\partial q_6}{\partial X_E} & \frac{\partial q_6}{\partial Y_E} & \frac{\partial q_6}{\partial Z_E} & \frac{\partial q_6}{\partial \psi} & \frac{\partial q_6}{\partial \theta} & \frac{\partial q_6}{\partial \varphi} \end{bmatrix} \begin{bmatrix} \delta X_E \\ \delta Y_E \\ \delta Z_E \\ \delta \psi \\ \delta \theta \\ \delta \varphi \end{bmatrix} \quad (26)$$

With the following notations:

$$\delta q = \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ \delta q_4 \\ \delta q_5 \\ \delta q_6 \end{bmatrix} J_q = \frac{\partial q}{\partial X_P} = \begin{bmatrix} \frac{\partial q_1}{\partial X_E} & \frac{\partial q_1}{\partial Y_E} & \frac{\partial q_1}{\partial Z_E} & \frac{\partial q_1}{\partial \psi} & \frac{\partial q_1}{\partial \theta} & \frac{\partial q_1}{\partial \varphi} \\ \frac{\partial q_2}{\partial X_E} & \frac{\partial q_2}{\partial Y_E} & \frac{\partial q_2}{\partial Z_E} & \frac{\partial q_2}{\partial \psi} & \frac{\partial q_2}{\partial \theta} & \frac{\partial q_2}{\partial \varphi} \\ \frac{\partial q_3}{\partial X_E} & \frac{\partial q_3}{\partial Y_E} & \frac{\partial q_3}{\partial Z_E} & \frac{\partial q_3}{\partial \psi} & \frac{\partial q_3}{\partial \theta} & \frac{\partial q_3}{\partial \varphi} \\ \frac{\partial q_4}{\partial X_E} & \frac{\partial q_4}{\partial Y_E} & \frac{\partial q_4}{\partial Z_E} & \frac{\partial q_4}{\partial \psi} & \frac{\partial q_4}{\partial \theta} & \frac{\partial q_4}{\partial \varphi} \\ \frac{\partial q_5}{\partial X_E} & \frac{\partial q_5}{\partial Y_E} & \frac{\partial q_5}{\partial Z_E} & \frac{\partial q_5}{\partial \psi} & \frac{\partial q_5}{\partial \theta} & \frac{\partial q_5}{\partial \varphi} \\ \frac{\partial q_6}{\partial X_E} & \frac{\partial q_6}{\partial Y_E} & \frac{\partial q_6}{\partial Z_E} & \frac{\partial q_6}{\partial \psi} & \frac{\partial q_6}{\partial \theta} & \frac{\partial q_6}{\partial \varphi} \end{bmatrix} \quad (27)$$

Using the notations above (27) becomes:

$$\delta q = J_q \delta X_P \quad (28)$$

It results:

$$\delta q^T = \delta X_P^T J_q^T \quad (29)$$

Using (27) and (28) the expression of the virtual work becomes:

$$\delta X_P^T J_q^T \tau + \sum_{i=1}^{29} \delta X_P^T J_i^T (T_i^{Tr} + T_i^s) = 0 \quad (30)$$

It results:

$$J_q^T \tau + \sum_{i=1}^{29} J_i^T (T_i^{Tr} + T_i^s) = 0 \quad (31)$$

If (31) is left multiplied with  $(J_q^T)^{-1}$ , it results:

$$\tau = -(J_q^T)^{-1} \sum_{i=1}^{29} J_i^T (T_i^{Tr} + T_i^s) \quad (32)$$

Using the relations (6) and (7) the equation (32) results:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} \frac{\partial q_1}{\partial X_E} & \frac{\partial q_2}{\partial X_E} & \frac{\partial q_3}{\partial X_E} & \frac{\partial q_4}{\partial X_E} & \frac{\partial q_5}{\partial X_E} & \frac{\partial q_6}{\partial X_E} \\ \frac{\partial q_1}{\partial Y_E} & \frac{\partial q_2}{\partial Y_E} & \frac{\partial q_3}{\partial Y_E} & \frac{\partial q_4}{\partial Y_E} & \frac{\partial q_5}{\partial Y_E} & \frac{\partial q_6}{\partial Y_E} \\ \frac{\partial q_1}{\partial Z_E} & \frac{\partial q_2}{\partial Z_E} & \frac{\partial q_3}{\partial Z_E} & \frac{\partial q_4}{\partial Z_E} & \frac{\partial q_5}{\partial Z_E} & \frac{\partial q_6}{\partial Z_E} \\ \frac{\partial q_1}{\partial \psi} & \frac{\partial q_2}{\partial \psi} & \frac{\partial q_3}{\partial \psi} & \frac{\partial q_4}{\partial \psi} & \frac{\partial q_5}{\partial \psi} & \frac{\partial q_6}{\partial \psi} \\ \frac{\partial q_1}{\partial \theta} & \frac{\partial q_2}{\partial \theta} & \frac{\partial q_3}{\partial \theta} & \frac{\partial q_4}{\partial \theta} & \frac{\partial q_5}{\partial \theta} & \frac{\partial q_6}{\partial \theta} \\ \frac{\partial q_1}{\partial \varphi} & \frac{\partial q_2}{\partial \varphi} & \frac{\partial q_3}{\partial \varphi} & \frac{\partial q_4}{\partial \varphi} & \frac{\partial q_5}{\partial \varphi} & \frac{\partial q_6}{\partial \varphi} \end{bmatrix}^{-1} \sum_{i=1}^{29} \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial Y_i}{\partial X_E} & \frac{\partial Z_i}{\partial X_E} \\ \frac{\partial X_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Y_E} \\ \frac{\partial X_i}{\partial Z_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Z_i}{\partial Z_E} \\ \frac{\partial X_i}{\partial \psi} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Z_i}{\partial \psi} \\ \frac{\partial X_i}{\partial \theta} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Z_i}{\partial \theta} \\ \frac{\partial X_i}{\partial \varphi} & \frac{\partial Y_i}{\partial \varphi} & \frac{\partial Z_i}{\partial \varphi} \end{bmatrix}$$

$$\begin{bmatrix} m_i^* & 0 & 0 \\ 0 & m_i^* & 0 \\ 0 & 0 & m_i^* \end{bmatrix} \cdot \left( \begin{bmatrix} \frac{\partial X_i}{\partial X_E} & \frac{\partial X_i}{\partial Y_E} & \frac{\partial X_i}{\partial Z_E} & \frac{\partial X_i}{\partial \psi} & \frac{\partial X_i}{\partial \theta} & \frac{\partial X_i}{\partial \varphi} \\ \frac{\partial Y_i}{\partial X_E} & \frac{\partial Y_i}{\partial Y_E} & \frac{\partial Y_i}{\partial Z_E} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Y_i}{\partial \varphi} \\ \frac{\partial Z_i}{\partial X_E} & \frac{\partial Z_i}{\partial Y_E} & \frac{\partial Z_i}{\partial Z_E} & \frac{\partial Z_i}{\partial \psi} & \frac{\partial Z_i}{\partial \theta} & \frac{\partial Z_i}{\partial \varphi} \\ \frac{\partial X_i}{\partial \psi} & \frac{\partial Y_i}{\partial \psi} & \frac{\partial Z_i}{\partial \psi} & \ddots & \ddots & \ddots \\ \frac{\partial X_i}{\partial \theta} & \frac{\partial Y_i}{\partial \theta} & \frac{\partial Z_i}{\partial \theta} & \ddots & \ddots & \ddots \\ \frac{\partial X_i}{\partial \varphi} & \frac{\partial Y_i}{\partial \varphi} & \frac{\partial Z_i}{\partial \varphi} & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \ddot{X}_E \\ \ddot{Y}_E \\ \ddot{Z}_E \\ \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} \frac{d}{dt} \left( \frac{\partial X_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial X_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Y_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial Z_i}{\partial X_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Y_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial Z_E} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \varphi} \right) \\ \frac{d}{dt} \left( \frac{\partial X_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \psi} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \psi} \right) & \ddots & \ddots & \ddots \\ \frac{d}{dt} \left( \frac{\partial X_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \theta} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \theta} \right) & \ddots & \ddots & \ddots \\ \frac{d}{dt} \left( \frac{\partial X_i}{\partial \varphi} \right) & \frac{d}{dt} \left( \frac{\partial Y_i}{\partial \varphi} \right) & \frac{d}{dt} \left( \frac{\partial Z_i}{\partial \varphi} \right) & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \right) \quad (33)$$

In order to obtain the 29 lumped masses and their coordinates with respect to the fixed coordinate system of the robot, we assume that the linear motors have the masses  $m_i$  and  $m_{i+3}$ ,  $i=1,2,3$ .

As in [16], simplifying hypothesis were used to obtain the equivalent system represented by the mass points.

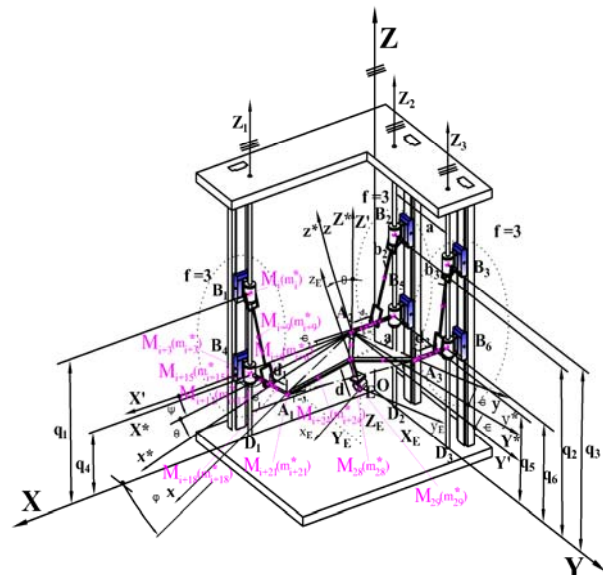


Fig. 3. Mass distribution for one kinematic chain

So, for a homogeneous bar with mass  $m$  and length  $l$ , the dynamic equivalent system points is formed by the masses  $m_A, m_B, m_C$  - the ends of the bar, respectively the bar center of gravity C [17]:

$$m_A = \frac{1}{6} m, m_B = \frac{1}{6} m, m_C = \frac{2}{3} m \quad (34)$$

The lumped masses  $m_i^*$   $i = 1 \div 3$ ; will be next determined. The lumped masses from  $M_1, M_2, M_3$  will be:

$$\begin{cases} m_1^* = m_1 + \frac{m_7}{6} \\ m_2^* = m_2 + \frac{m_8}{6} \\ m_3^* = m_3 + \frac{m_9}{6} \end{cases} \quad (35)$$

The coordinates of lumped masses  $m_1^*, m_2^*, m_3^*$  dynamic equivalent with 1,2,3 body masses  $m_1, m_2, m_3$  will be:

$$\begin{cases} X_1 = e_1, Y_1 = 0, Z_1 = q_1 \\ X_2 = 0, Y_2 = e_2, Z_2 = q_2 \\ X_3 = 0, Y_3 = e_3, Z_3 = q_3 \end{cases} \quad (36)$$

The masses lumped in  $B_4, B_5, B_6$  of the actuation elements that move along the  $D_1Z_1, D_2Z_2, D_3Z_3$  of masses  $m_4, m_5, m_6$  are:

$$\begin{cases} X_4 = e_1, Y_4 = 0, Z_4 = q_4 \\ X_5 = 0, Y_5 = e_2, Z_5 = q_5 \\ X_6 = 0, Y_6 = e_3, Z_6 = q_6 \end{cases} \quad (37)$$

Where:

$$\begin{cases} q_1 = Z_{A1} + \sqrt{b_1^2 - (q_1^* - d_1)^2} \\ q_2 = Z_{A2} + \sqrt{b_2^2 - (q_2^* - d_2)^2} \\ q_3 = Z_{A3} + \sqrt{b_3^2 - (q_3^* - d_3)^2} \\ q_4 = Z_{A1} \\ q_5 = Z_{A2} \\ q_6 = Z_{A3} \end{cases} \quad (38)$$

The lumped masses  $m_7^*, m_8^*, m_9^*$  of auxiliary arms will be determined with these relations:

$$\begin{cases} m_7^* = \frac{m_7}{6} + \frac{m_{13}}{6} \\ m_8^* = \frac{m_8}{6} + \frac{m_{14}}{6} \\ m_9^* = \frac{m_9}{6} + \frac{m_{15}}{6} \end{cases} \quad (39)$$

The coordinates of  $M_7, M_8, M_9$  points are:

$$\begin{cases} X_7 = e_1 + (q_1^* - d_1) \cdot c\varphi_1, Y_7 = (q_1^* - d_1) \cdot s\varphi_1, Z_7 = q_4 \\ X_8 = (q_2^* - d_2) \cdot c\varphi_2, Y_8 = e_2 + (q_2^* - d_2) \cdot s\varphi_2, Z_8 = q_5 \\ X_9 = (q_3^* - d_3) \cdot c\varphi_3, Y_9 = e_3 + (q_3^* - d_3) \cdot s\varphi_3, Z_9 = q_6 \end{cases} \quad (40)$$

The next lumped mass point, centers corresponding homogeneous bar is:

$$\begin{cases} m_{10}^* = \frac{2}{3} m_7 \\ m_{11}^* = \frac{2}{3} m_8 \\ m_{12}^* = \frac{2}{3} m_9 \end{cases} \quad (41)$$

$$\begin{cases} X_{10} = \frac{X_1 + X_7}{2}, Y_{10} = \frac{Y_1 + Y_7}{2}, Z_{10} = \frac{Z_1 + Z_7}{2} \\ X_{11} = \frac{X_2 + X_8}{2}, Y_{11} = \frac{Y_2 + Y_8}{2}, Z_{11} = \frac{Z_2 + Z_8}{2} \\ X_{12} = \frac{X_3 + X_9}{2}, Y_{12} = \frac{Y_3 + Y_9}{2}, Z_{12} = \frac{Z_3 + Z_9}{2} \end{cases} \quad (42)$$

Piston shirt will be considered as a homogeneous bar of masses  $m_{10}, m_{11}, m_{12}$  with centers of gravity in points  $M_{16}, M_{17}, M_{19}$  of  $l_1, l_2, l_3$  lengths.

The lumped masses in  $M_{i+12}$  are:

$$\begin{cases} m_{13}^* = \frac{m_{10}}{6} \\ m_{14}^* = \frac{m_{11}}{6} \\ m_{15}^* = \frac{m_{12}}{6} \end{cases} \quad (43)$$

Coordinates of points  $M_{i+12}$  are:

$$\begin{cases} X_{13} = l_1 \cdot c\varphi_1, Y_{13} = l_1 \cdot s\varphi_1, Z_{13} = q_4 \\ X_{14} = l_2 \cdot c\varphi_2, Y_{14} = l_2 \cdot s\varphi_2, Z_{14} = q_5 \\ X_{15} = l_3 \cdot c\varphi_3, Y_{15} = l_3 \cdot s\varphi_3, Z_{15} = q_6 \end{cases} \quad (44)$$

The lumped masses of piston shirt center are:

$$\begin{cases} m_{16}^* = \frac{2}{3} m_{10} \\ m_{17}^* = \frac{2}{3} m_{11} \\ m_{18}^* = \frac{2}{3} m_{12} \end{cases} \quad (45)$$

Coordinates of points  $M_{i+15}$  are:

$$\begin{cases} X_{16} = \frac{X_4 + X_{13}}{2}, Y_{16} = \frac{Y_4 + Y_{13}}{2}, Z_{16} = \frac{Z_4 + Z_{13}}{2} \\ X_{17} = \frac{X_5 + X_{14}}{2}, Y_{17} = \frac{Y_5 + Y_{14}}{2}, Z_{17} = \frac{Z_5 + Z_{14}}{2} \\ X_{18} = \frac{X_6 + X_{15}}{2}, Y_{18} = \frac{Y_6 + Y_{15}}{2}, Z_{18} = \frac{Z_6 + Z_{15}}{2} \end{cases} \quad (46)$$

The piston will be considered a homogeneous bar, dynamically determined by the lumped masses  $M_{i+6}, M_{i+18}, M_{i+21}$ . The  $M_{i+18}$  points corresponding to the centers of gravity of the piston and  $M_{i+21} \equiv A_i$ .

$$\begin{cases} m_{19}^* = \frac{2}{3}m_{13} \\ m_{20}^* = \frac{2}{3}m_{14} \\ m_{21}^* = \frac{2}{3}m_{15} \end{cases} \quad (47)$$

$$\begin{cases} X_{19} = \frac{X_7 + X_{22}}{2}, Y_{19} = \frac{Y_7 + Y_{22}}{2}, Z_{19} = \frac{Z_7 + Z_{22}}{2} \\ X_{20} = \frac{X_8 + X_{23}}{2}, Y_{20} = \frac{Y_8 + Y_{23}}{2}, Z_{20} = \frac{Z_8 + Z_{23}}{2} \\ X_{21} = \frac{X_9 + X_{24}}{2}, Y_{21} = \frac{Y_9 + Y_{24}}{2}, Z_{21} = \frac{Z_9 + Z_{24}}{2} \end{cases} \quad (48)$$

For ease of the dynamic model the mobile platform is now made of homogeneous rods whose ends form the default triangle.

$$\begin{cases} x_E = \frac{a}{3} \\ y_E = \frac{a}{3} \\ z_E = -d \end{cases} \quad (49)$$

The 3 points of lumped masses  $m_{22}^*, m_{23}^*, m_{24}^*$  from points  $M_{22} \equiv A_1, M_{23} \equiv A_2, M_{24} \equiv A_3$  determines the following relationships:

$$\begin{cases} m_{22}^* = \frac{m_{13}}{6} + \frac{m_{bp1}}{6} \\ m_{23}^* = \frac{m_{14}}{6} + \frac{m_{bp2}}{6} \\ m_{24}^* = \frac{m_{15}}{6} + \frac{m_{bp3}}{6} \end{cases} \quad (50)$$

Coordinates of points  $M_{i+21} \equiv A_i; i=1,2,3$  will result from relation:

$$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\psi c\theta\varphi - s\psi s\theta\varphi & -c\psi c\theta\varphi - s\psi c\theta\varphi & c\psi s\theta \\ s\psi c\theta\varphi + c\psi s\theta\varphi & -s\psi c\theta\varphi + c\psi c\theta\varphi & -s\psi s\theta \\ -s\theta\varphi & s\theta\varphi & c\theta \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix} \quad (51)$$

$i=1,2,3.$

The lumped masses  $M_{i+24}$  will be determined:

$$\begin{cases} m_{25}^* = \frac{2}{3}m_{bp1} \\ m_{26}^* = \frac{2}{3}m_{bp2} \\ m_{27}^* = \frac{2}{3}m_{bp3} \end{cases} \quad (52)$$

$$\begin{cases} X_{25} = \frac{X_{22} + X_{28}}{2}, Y_{25} = \frac{Y_{22} + Y_{28}}{2}, Z_{25} = \frac{Z_{22} + Z_{28}}{2} \\ X_{26} = \frac{X_{23} + X_{28}}{2}, Y_{26} = \frac{Y_{23} + Y_{28}}{2}, Z_{26} = \frac{Z_{23} + Z_{28}}{2} \\ X_{27} = \frac{X_{24} + X_{28}}{2}, Y_{27} = \frac{Y_{24} + Y_{28}}{2}, Z_{27} = \frac{Z_{24} + Z_{28}}{2} \end{cases} \quad (53)$$

The lumped mass  $M_{28}$  corresponding to the center of gravity of the end-effector will be determined:

$$m_{28}^* = \frac{1}{6}m_{16} \quad (54)$$

Where:

$$\begin{bmatrix} X_{M_{28}} \\ Y_{M_{28}} \\ Z_{M_{28}} \end{bmatrix} = \begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} + \begin{bmatrix} c\psi c\theta\varphi - s\psi s\theta\varphi & -c\psi c\theta\varphi - s\psi c\theta\varphi & c\psi s\theta \\ s\psi c\theta\varphi + c\psi s\theta\varphi & -s\psi c\theta\varphi + c\psi c\theta\varphi & -s\psi s\theta \\ -s\theta\varphi & s\theta\varphi & c\theta \end{bmatrix} \begin{bmatrix} x_{M_{28}} - x_{A_i} \\ y_{M_{28}} - y_{A_i} \\ z_{M_{28}} - z_{A_i} \end{bmatrix} \quad (55)$$

In point  $M_{29}$  it's concentrated the in module's clamping mass and the object's manipulated mass:

$$m_{29}^* = \frac{1}{6}m_{dp} + m_{ob} \quad (56)$$

Where:

$$\begin{bmatrix} X_{M_{29}} \\ Y_{M_{29}} \\ Z_{M_{29}} \end{bmatrix} = \begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} + \begin{bmatrix} c\psi c\theta\varphi - s\psi s\theta\varphi & -c\psi c\theta\varphi - s\psi c\theta\varphi & c\psi s\theta \\ s\psi c\theta\varphi + c\psi s\theta\varphi & -s\psi c\theta\varphi + c\psi c\theta\varphi & -s\psi s\theta \\ -s\theta\varphi & s\theta\varphi & c\theta \end{bmatrix} \begin{bmatrix} x_{M_{29}} - x_{A_i} \\ y_{M_{29}} - y_{A_i} \\ z_{M_{29}} - z_{A_i} \end{bmatrix} \quad (57)$$

#### 4. NUMERICAL RESULTS

A dynamic simulation was made using the software package Matlab and the obtained expression of the torque vector.

The established geometric parameters for this simulation are:

$$\begin{cases} p = 100 \\ h = 40 \\ d = 58 \\ e = 109 \\ x_{A_1} = p, y_{A_1} = 0, z_{A_1} = 0 \\ x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0 \\ x_{A_3} = 0, y_{A_3} = p, z_{A_3} = 0 \\ x_E = \frac{p}{3}, y_E = \frac{p}{3}, z_E = -h \end{cases} \quad (58)$$

The results of a linear trajectory of the end-effector with the following data are presented in the next figure:

- the starting point is A(415 465 500)
- the end point is B(550 550 600)
- the maximum speed of the end effector was chosen to be 5[mm/s]
- the maximum acceleration of the end-effector was chosen to be 10[mm/s]

Other constant used for the simulation are:

$$m_1 = \dots = m_6 = 1.616kg \text{ - the masses of}$$

the linear motor;  $m_7 = m_8 = m_9 = 0.063kg$  - the

mass of the auxiliary arms;

$m_{13} = m_{14} = m_{15} = 0.0150kg$  - the mass of the

piston;  $m_{16} = 0.074835kg$  - the mass of the

mobile platform:  $m_{dp} = 0.25522kg$

,  $m_{obm} = 0.05kg$  - the mass of the gripper and

the manipulated mass.

## 5. CONCLUSIONS

This paper presents a new reconfigurable parallel robot with six degrees and three kinematic chains for guiding platform. The platform is driven by six linear active joint disposed on the three guides, having reduced weight and size allowing high speeds and accelerations.

The task space dynamics of the reconfigurable parallel robot with six degrees of freedom was solved.

Using Matlab programming environment the graphical visualization for the simulation results was presented.

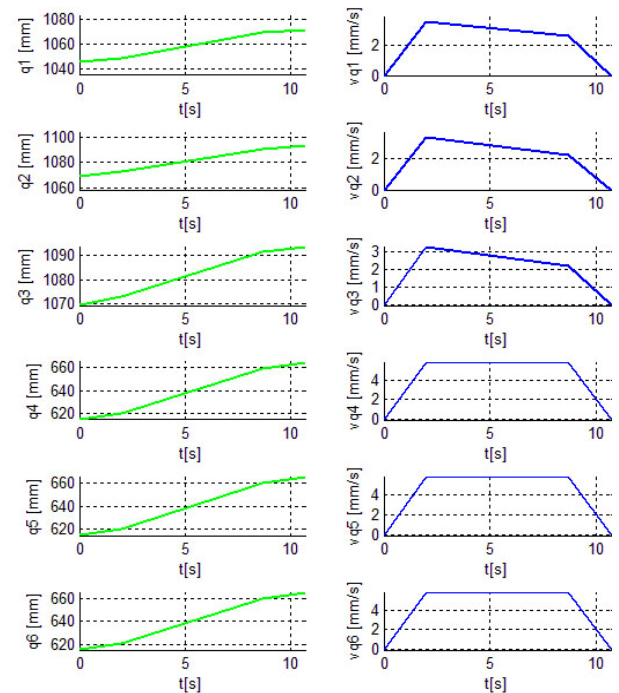


Fig. 4a. Task space dynamics – simulation results

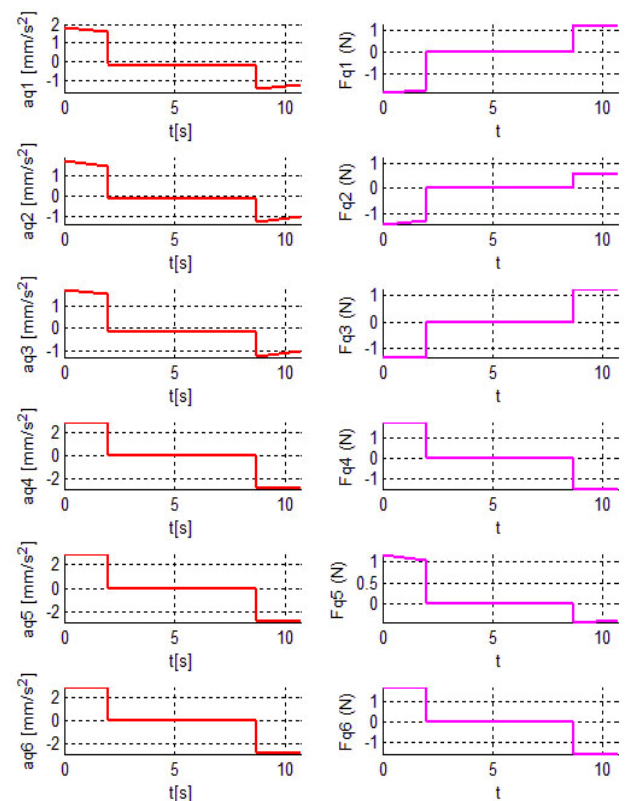


Fig. 4b. Task space dynamics – simulation results

## 6. ACKNOWLEDGEMENTS

This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC" contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

## 7. REFERENCES

1. Plitea, N., Hesselbach, J., Pislă, D., Simnofske, M., Prodan, B., Burish, A., Vidrean, A., Modelling of Parallel Microrobots With Two To Six Degrees Of Freedom, Series: Maschines Construction. Materials, 50, 2007
2. Mândru Dan, Curs de Acționări în Mecanică Fină - 2004
3. Gogu, G., „Isogliden TaRb: a Family of up to Five Axes Reconfigurable and Maximally Regular Parallel Kinematic Machines”, International Conference on Smart Machining Systems, vol. 36, pp. 419-426, 2007
4. Negri, S., et. al., Improvement and optimization of a reconfigurable parallel kinematic machine, 2002
5. Krefft, M., Last, P., Hesselbach, J., New concepts to adapt the PKM performace to
6. application Requirements, Parallel kinematic machines in research and practice, The 5<sup>th</sup> Chemnitz Parallel Kinematics Seminar, pp. 115-133, Chemnitz, 2006.
7. Borras, J., Thoms, F., Ottaviano, E., Ceccarelli, M., A Reconfigurable 5-Dof 5- SPU Parallel Platform, Proceedings of the 2009 ASME/IFTOMM International Conferece of Reconfigurable Mechanisms and Robots, pp. 617-623, 2009.
8. Plitea, N., Pislă, D., Vidrean, A., Vaida, C., Gyurka, B., Workspace and Singularity Analysis for a Reconfigurable Parallel Robot, Proceedings of SYROM 2009, the 10th IFTOMM International Symposium on Science of Mechanisms and Machines, 2009.
9. Plitea N., Pislă D., Vaida C., Gherman B., Pislă A., Dynamic modelling of a parallel robot used in minimally invasive surgery, Eucomes 2008, Cassino-Italy, 2008.
10. N Codourey A., and Burdet E.: A body oriented method for finding a linear form of the dynamic equatiojns of fully parallel robot, IEEE Conf. on Robotics and Automation, pp.1612-1619, Albuquerque, New Mexico, U.S., 1997.
11. Li, Y., Xu, Q.: Kinematics and inverse dynamics analysis for a general 3 -PRS spatial parallel mechanism. Robotica 23, 219–229, 2005.
12. Tsai L-W.: Solving the inverse dynamics of a Stewart-Gough manipulator by the principle of virtual work, J. of Mechanical design 122, 3-9, 2000.
13. Plitea, N., Pislă, D., Vaida, C., Lese, D-B., Konya, B., Dadarlat, R., Scurtu, I., Sabou, C., Familie de roboti paraleli cu sase grade de mobilitate, Patent nr. A/10013/2011B.
14. Konya B., Vaida C., Dadarlat R., Plitea N. : A new reconfigurable parallel robot with six degrees of freedom, International Conference on Quality and Innovation in Engineering and Management, (2011).
15. Plitea N., Konya B., Variants of a new reconfigurable parallel robot with six, five, four, three and two degrees of freedom, Acta Tehnica Napocensis, Applied Mathematics and Mechanics, 2011.
16. Konya B., Dadarlat R., Vaida C., Plitea N., Pislă, D., The kinematics of a new reconfigurable parallel robot with six degrees of freedom of a new reconfigurable parallel robot with six, five, four, three and two degrees of freedom, 4th European Conference on Mechanism Science, Santander Spain, 2012.
17. Dizioglu, D., Getriebelehre, Band 3, Dynamik. Fried Vieweg und Sohn, Braunschweig, 1966.
18. Khalil W. and Guegan S.: Inverse and Direct Dynamic Modeling of Gough –Stewart Robots, IEEE Transactions on Robotics and Automation, 20(4), 754-762, 2004.

### DINAMICA UNEI NOI CONFIGURARI DE ROBOT PARALEL CU ȘASE GRADE DE LIBERTATE

Lucrarea prezintă un nou robot paralel reconfigurabil cu șase grade de mobilitate și trei lanțuri cinematice de ghidare a platformei. Un robot reconfigurabil își poate schimba structura în scopul de a se adapta rapid la capacitatea și funcționalitate producției. Folosind metoda lucrului mecanic virtual, această lucrare prezintă dinamica robotului paralel reconfigurabil. Unele rezultate ale simulării și concluziile sunt prezentate la sfârșitul lucrării.

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