TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue IV, 2012

KINEMATIC ANALYSIS OF THE CRANK-ROCKER MECHANISM QUADRILATERAL

Adriana Ioana PRECUB, Maria Felicia SUCALĂ

Abstract: *In this paper based on the transmission functions, it has been determined the variation of the angular velocity and the angular acceleration for different speeds of the drive motor and for different lengths of crank. Therefore, this mechanism represent a part of the stand for experimental tests of the overrunning clutch, and it has been made a program allowing a graphical display of these.*
Key words: *analysis, kinematic, mechanism, crank, rocker.*

1. INTRODUCTION

The mechanism with adjustable oscillations used in the stand construction, and for which it was determined the mathematical model (theoretical) is a crank-rocker mechanism quadrilateral .

Kinematic analysis consists in the studying of elements movement in terms of geometry without taking into account the forces determining the movement in the kinematic analysis we aim to solve the following problems [1], [2]:

- determination of the elements positions and the trajectories which describe the points elements;
- determination of the angular velocity of the elements;
- determination of the angular acceleration of the elements.

In order to determine the kinematic parameters we'll be using the transmission functions of order zero, one and two [1].

2. RELATION CALCULATION FOR THE KINEMATIC ANALYSIS OF THE MECHANISM

In the first figure it's represented the quadrilateral mechanism which is a part of the experimental test stand for overrunning clutch, and for which we aim to determing on the analytical way, with transmission functions, the angular velocity and the angular acceleration .

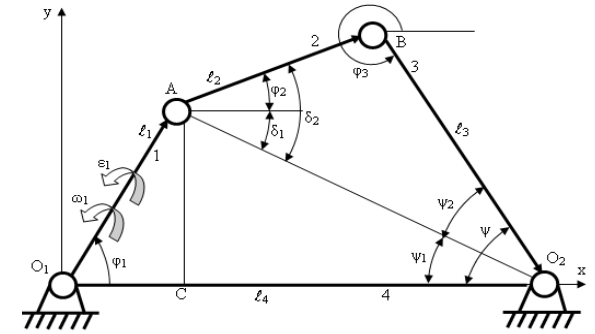


Fig. 1. Mechanism quadrilateral crank-rocker.

Transmission functions for the quadrilateral mechanism demonstrated in [1] and [2] are:

$$R_2'(\varphi_1) = \frac{\ell_1}{\ell_2} \cdot \frac{\ell_2 \cdot \sin(\varphi_1 - \varphi_2) - \ell_4 \cdot \sin\varphi_1}{\ell_1 \cdot \sin(\varphi_1 - \varphi_2) + \ell_4 \cdot \sin\varphi_2} \quad (1)$$

$$R_3'(\varphi_1) = \frac{\ell_1}{\ell_3} \cdot \frac{\ell_3 \cdot \sin(\varphi_1 - \varphi_3) - \ell_4 \cdot \sin\varphi_1}{\ell_1 \cdot \sin(\varphi_1 - \varphi_3) + \ell_4 \cdot \sin\varphi_3} \quad (2)$$

$$R_2''(\varphi_1) = \frac{\ell_1 \cdot [\ell_2 \cdot \cos(\varphi_1 - \varphi_2) - \ell_4 \cdot \cos\varphi_1] +}{\ell_2 \cdot [\ell_1 \cdot \sin(\varphi_1 - \varphi_2) + \ell_4 \cdot \sin\varphi_2]}$$
$$\frac{+\ell_2 \cdot \left[R_2'(\varphi_1)\right]^2 \cdot [\ell_1 \cdot \cos(\varphi_1 - \varphi_2) - \ell_4 \cdot \cos\varphi_2]}{\ell_2 \cdot [\ell_1 \cdot \sin(\varphi_1 - \varphi_2) + \ell_4 \cdot \sin\varphi_2]} \quad (3)$$

$$R_3''(\varphi_1) = \frac{\ell_1 \cdot [\ell_3 \cdot \cos(\varphi_1 - \varphi_3) - \ell_4 \cdot \cos\varphi_1] +}{\ell_3 \cdot [\ell_1 \cdot \sin(\varphi_1 - \varphi_3) + \ell_4 \cdot \sin\varphi_3]}$$
$$\frac{+\ell_3 \cdot \left[R_3'(\varphi_1)\right]^2 \cdot [\ell_1 \cdot \cos(\varphi_1 - \varphi_3) - \ell_4 \cdot \cos\varphi_3]}{\ell_3 \cdot [\ell_1 \cdot \sin(\varphi_1 - \varphi_3) + \ell_4 \cdot \sin\varphi_3]} \quad (4)$$

where the angles φ_2 and φ_3 are determined with relations: [1], [2]:

$$\varphi_2 = \arccos\frac{\ell_1^2+\ell_2^2-\ell_3^2+l_4^2-2\ell_1\ell_4\cdot\cos\varphi_1}{2l_2\cdot\sqrt{l_1^2+l_4^2-2l_1l_4\cdot\cos\varphi_1}} - \arctan\frac{\ell_1\cdot\sin\varphi_1}{\ell_4-\ell_1\cdot\cos\varphi_1} \quad (5)$$

$$\varphi_3 = 360^o - \arccos\frac{\ell_1^2-\ell_2^2+\ell_3^2+\ell_4^2-2\ell_1\ell_4\cdot\cos\varphi_1}{2\ell_3\cdot\sqrt{\ell_1^2+\ell_4^2-2\ell_1\ell_4\cdot\cos\varphi_1}} - \arctan\frac{\ell_1\cdot\sin\varphi_1}{\ell_4-\ell_1\cdot\cos\varphi_1} \quad (6)$$

Relations (1), (2), (3), (4), (5) and (6) will be used to determine the angular velocities and angular accelerations for elements 2, with equation (7) and 3, with equation (8) :

$$\omega_2 = \omega_1\cdot R_2'(\varphi_1)$$
$$\varepsilon_2 = \omega_1^2\cdot R_2''(\varphi_1)+\varepsilon_1\cdot R_2'(\varphi_1) \quad (7)$$

$$\omega_3 = \omega_1\cdot R_3'(\varphi_1)$$
$$\varepsilon_3 = \omega_1^2\cdot R_3''(\varphi_1)+\varepsilon_1\cdot R_3'(\varphi_1) \quad (8)$$

3. KINEMATIC ANALYSIS OF THE CRANK-ROCKER MECHANISM QUADRILATERAL

Based on the relations (1) - (8) it has been developed a calculation program that allows a graphical display of angular velocity and angular acceleration of the mechanism. At this mechanism (Fig. 1), is considered known the angular velocity and angular acceleration of the element driving 1, and are also known the lengths of the elements:

- $\ell_1 = 10,20,30[mm]$ - variable length;
- $\ell_2 = 70[mm]$, $\ell_3 = 40[mm]$, $\ell_4 = 75[mm]$;
- the angle φ_1 takes values from 0^o to 360^o, $\varphi_1 = 0 \div 360^o$.

Further we'll present the graphs with the angular velocities and the angular accelerations realized by the quadrilateral mechanism. They were made at different engine speeds of the motor drive.

Further more in figure 2 it's presented the interface of the calculation program with the following meanings:

1- includes input data and calculated values based on the equations presented above;

2- includes the graphic display of angular velocities ω_2 and ω_3;

3- includes the graphic display of angular accelerations ε_2 and ε_3;

4- the movement of the mechanism for a rotation of 360^o.

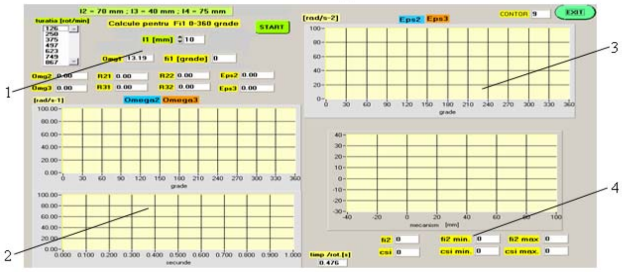
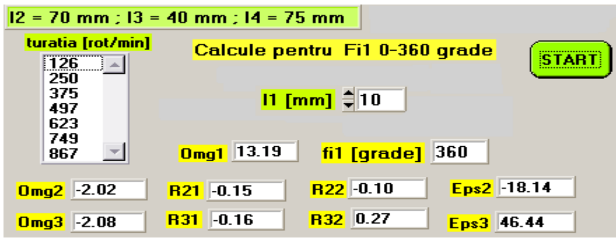
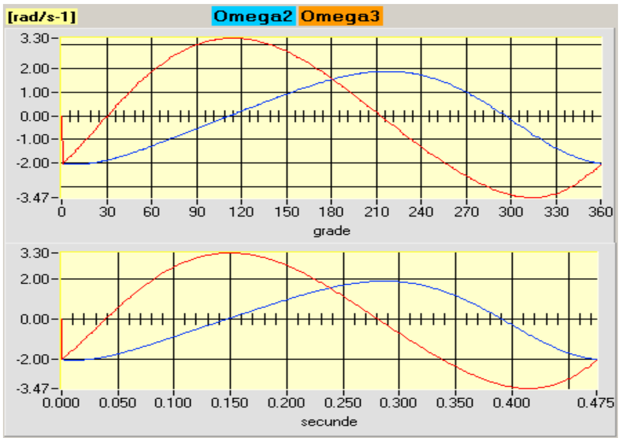


Fig. 2. The program interface

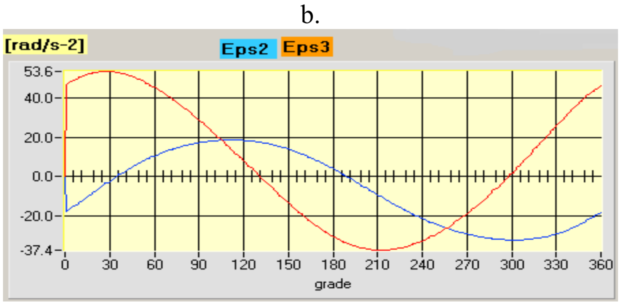
Figure 3 presents the graphs for variation of the angular velocity and acceleration for speed: $n = 126 rot/\min$, $\ell_1 = 10mm$, $\ell_2 = 70mm$, $\ell_3 = 40mm$ and $\ell_4 = 75mm$.



a.



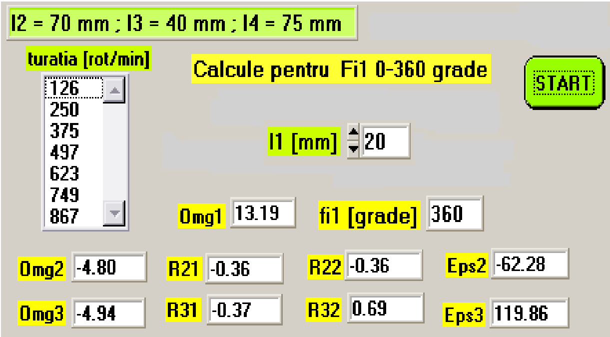
b.



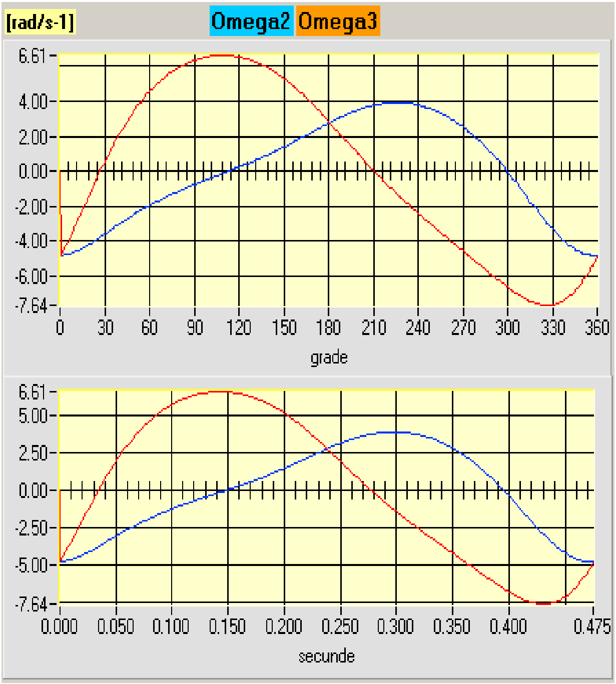
c.

Fig. 3. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 126 rot/\min$, $\ell_1 = 10mm$.

In Figure 4 are represented the graphs for variation of the angular velocity and acceleration for speed: $n = 126 rot/\min$, $\ell_1 = 20mm$, $\ell_2 = 70mm$, $\ell_3 = 40mm$ and $\ell_4 = 75mm$.



a.

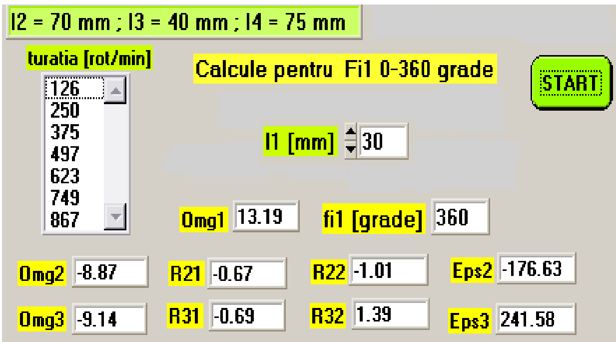


b.

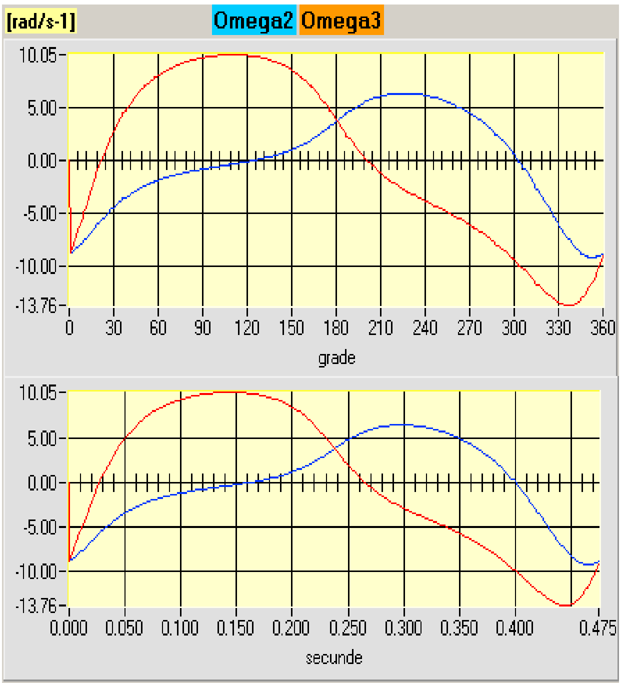
c.

Fig. 4. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 126 rot/\min$, $\ell_1 = 20mm$.

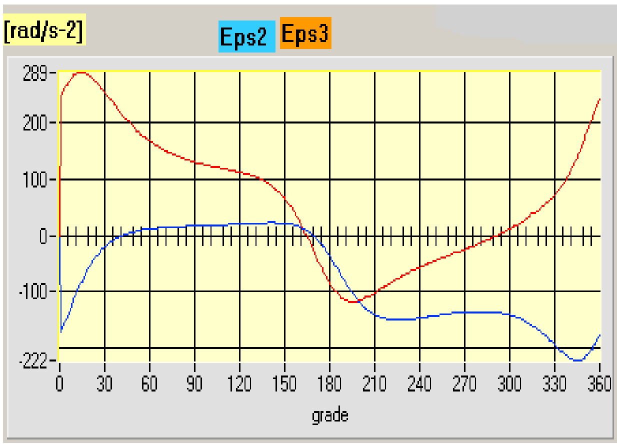
In Figure 5 are represented the graphs for variation of the angular velocity and acceleration for speed: $n = 126 rot/\min$, $\ell_1 = 30mm$, $\ell_2 = 70mm$, $\ell_3 = 40mm$ and $\ell_4 = 75mm$.



a.



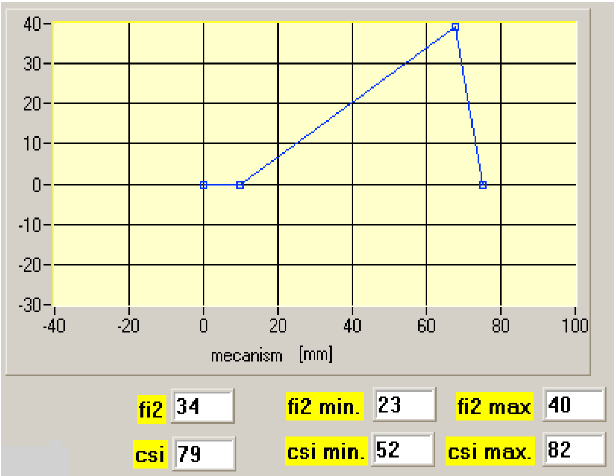
b.



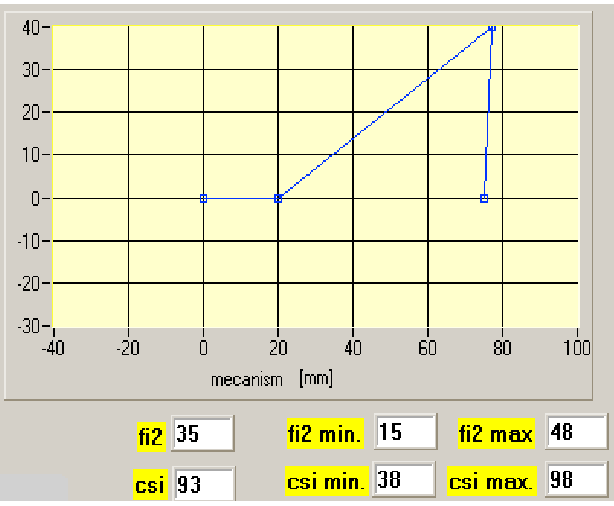
c.

Fig. 5. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 126 rot/\min$, $\ell_1 = 30mm$.

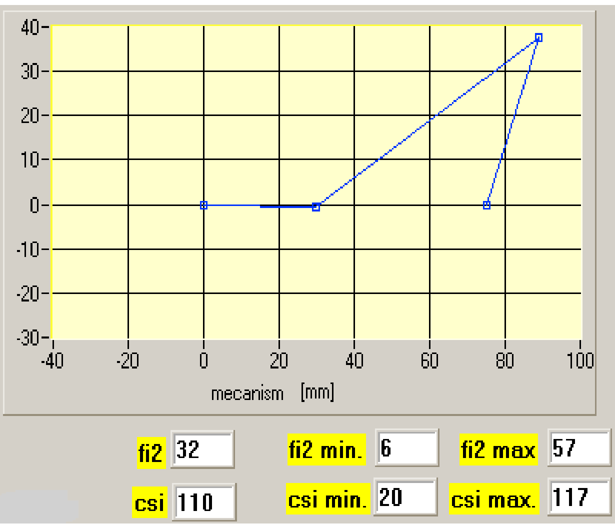
In Figure 6 it's presented the mechanism together with the angle values made by the mechanism during functioning at speed: $n = 126 rot/\min$ and $\ell_1 = 10, 20, 30 mm$.



a.



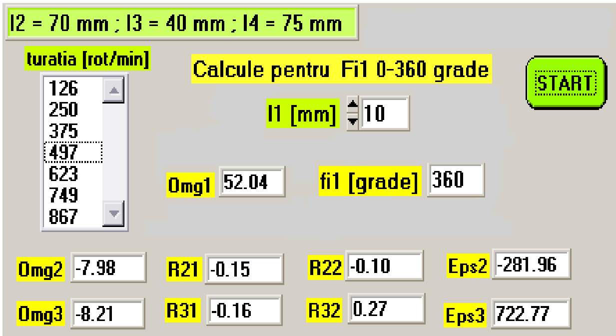
b.



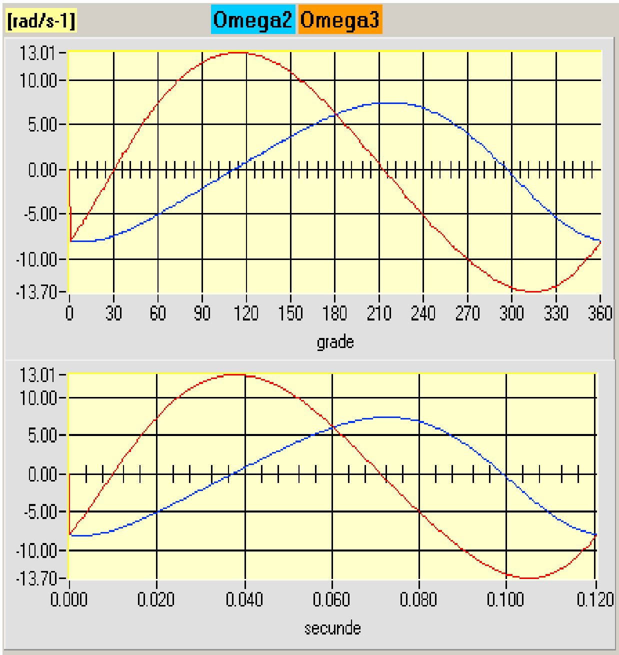
c.

Fig. 6. Simulation of the quadrilateral mechanism for: a) $\ell_1 = 10$ [mm], b) $\ell_1 = 20$ [mm], c) $\ell_1 = 30$ [mm].

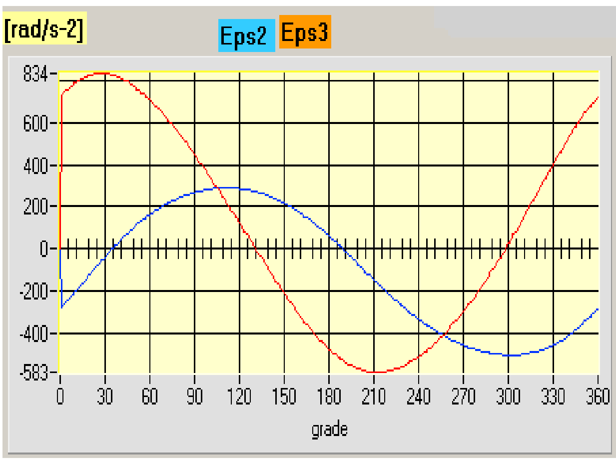
Figure 7 presents the graphs for variation of the angular velocity and acceleration for speed: $n = 497 rot/\min$, $\ell_1 = 10 mm$, $\ell_2 = 70 mm$, $\ell_3 = 40 mm$ and $\ell_4 = 75 mm$.



a.



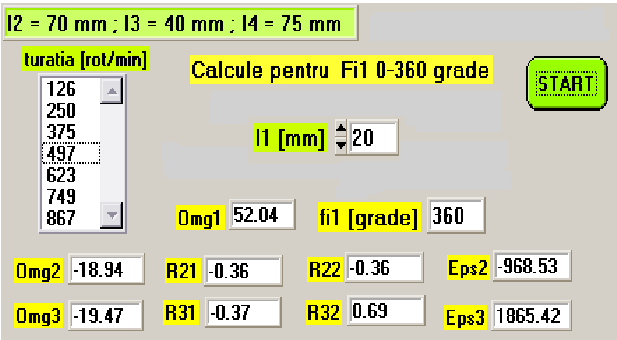
b.



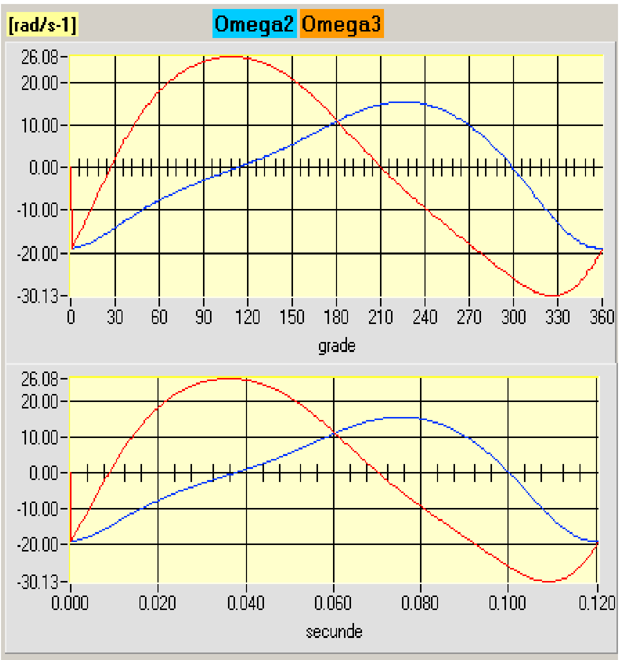
c.

Fig. 7. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 497 rot/\min$, $\ell_1 = 10 mm$.

Figure 8 presents the graphs for variation of the angular velocity and acceleration for speed: $n = 497 rot / \min$, $\ell_1 = 20mm$, $\ell_2 = 70mm$, $\ell_3 = 40mm$ and $\ell_4 = 75mm$.



a.

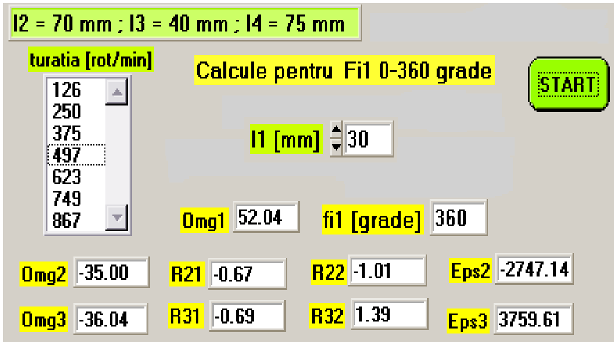


b.

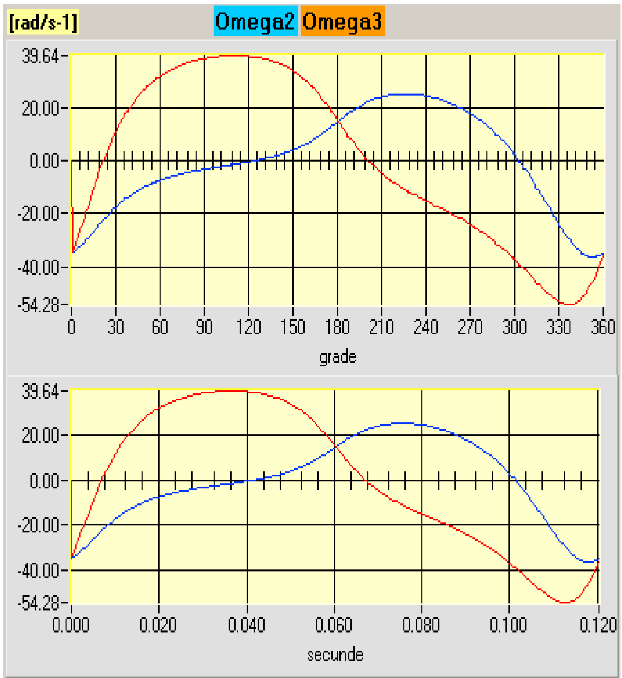
c.

Fig. 8. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 497 rot / \min$, $\ell_1 = 20mm$.

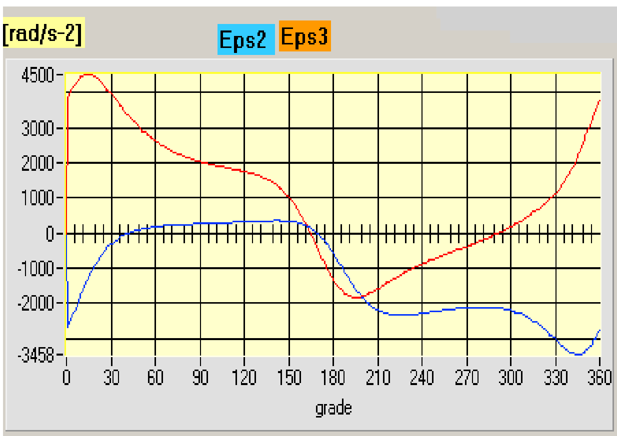
In figure 9 are represented the graphs for variation of the angular velocity and acceleration for speed: $n = 497 rot / \min$, $\ell_1 = 30mm$, $\ell_2 = 70mm$, $\ell_3 = 40mm$ and $\ell_4 = 75mm$.



a.



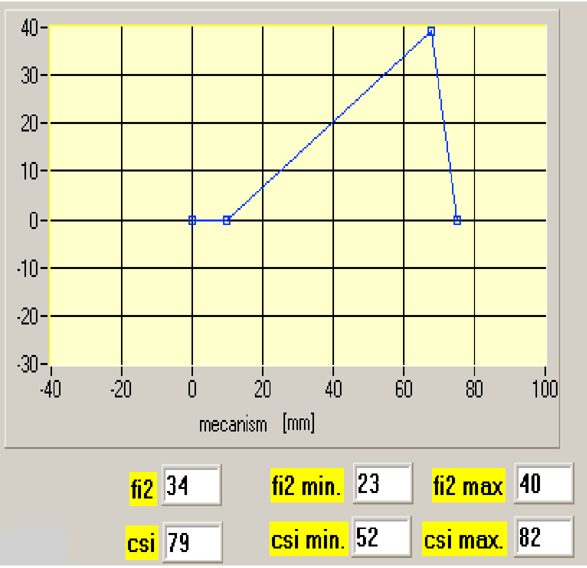
b.



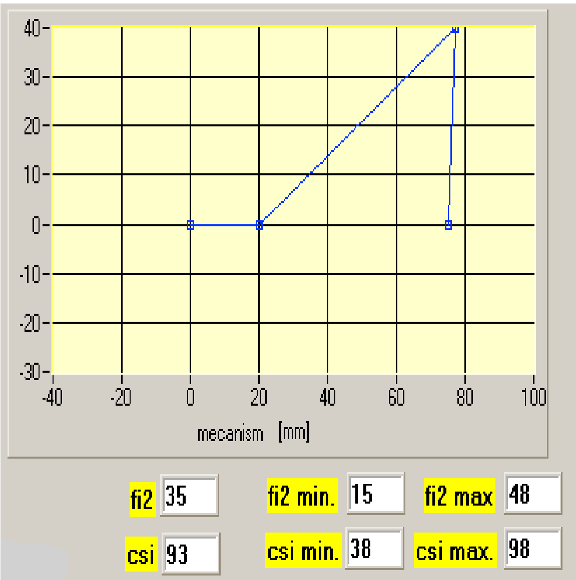
c.

Fig. 9. Variation of the angular velocity (a) and acceleration (b) for speed: $n = 497 rot / \min$, $\ell_1 = 30mm$.

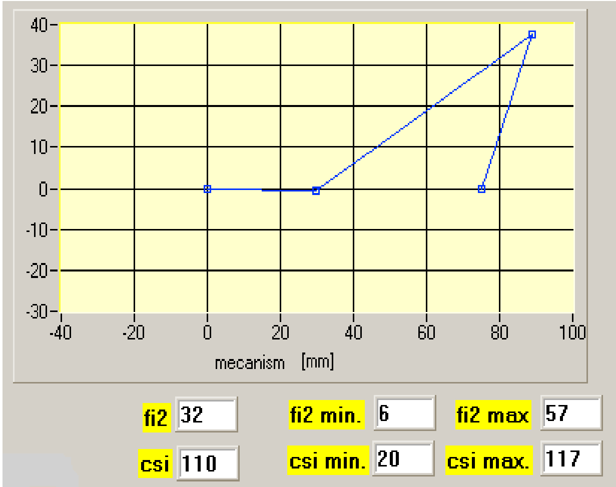
In Figure 10 it's presented the mechanism together with the angle values made by the mechanism during functioning at speed: $n = 497 rot/\min$ and $\ell_1 = 10,20,30mm$.



a.



b.



c.

Fig. 10. Simulation of the quadrilateral mechanism for: a) $\ell_1 = 10$ [mm], b) $\ell_1 = 20$ [mm], c) $\ell_1 = 30$ [mm].

4. CONCLUSION

After analyzing the mechanism quadrilateral crank-rocker it can be seen that the angular velocity and angular acceleration increases along with changing crank arm length and speed of the driving motor.

The time in which the mechanism makes a complete rotation decreases along with changing motot speed, so for a speed of 126 rot / min the time is 0745 s, and for a speed of 497 rot / min the time is 0.120 s.

5. REFERENCES

[1] Handra-Luca, V., *Mecanisme*, Atelierul de multiplicare al Institutului Politehnic, Cluj-Napoca, 1980.

[2] Sucală, F., ş.a., *Organe de maşini, mecanisme şi tribologie*, Studii de caz, Editura Todesco, Cluj-Napoca, 2008.

ANALIZA CINEMATICĂ A MECANISMULUI PATRULATER MANIVELĂ-BALANSIER

Abstract: În cadrul acestei lucrări s-au determinat pe baza funcţiilor de transmitere variaţiile vitezelor şi acceleraţiilor unghiulare pentru diferite turaţii ale motorului de antrenare şi pentru diferite lungimii ale manivelei mecanismului, mecanismul făcnd parte din componenţa standului de încercat cuplaje unisens. Pentru aceasta s-a realizat un program care să permită afişarea grafică a acestora.

Adriana Ioana PRECUB, PhD student, Technical University of Cluj-Napoca, Mechanical System Engineering Department, E-mail: ioana_precub@yahoo.com.

Maria Felicia SUCALĂ, Prof. Dr. ing., Technical University of Cluj-Napoca, Mechanical System Engineering Department, Felicia.Sucala@omt,utcluj.ro.