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INFLUENCE OF ROTATION ANGLE ON BEARING ROLLING BODIES LOAD DISTRIBUTION. PART 1: MATHEMATICAL MODEL

Cristina TUDOSE, Florina RUSU, Lucian TUDOSE

Abstract: *The theoretical analysis of a single-row radial bearing with radial clearance under constant external radial load is presented in this paper. This analysis is focused on finding the rolling element deflection that allows determining the number of active rolling elements that participate in the load transfer. Taking into account the bearing internal geometry, a mathematical model to calculate the rolling elements deflections during the bearing rotation has been derived.*
Key words: *rolling bodies load distribution, radial ring shift*

1. INTRODUCTION

The main objective of a radial bearing is to reduce rotational friction and support radial and/or axial loads. When a ball or roller bearing is subjected to a radial load, the load is transferred unequally to the rolling elements making up the bearing assembly. Usually, less than half of the rolling elements are loaded at any given time [7]. The actual load on each rolling element depends on its position in the bearing because only rolling elements that have contact with both bearing rings participate in the load transfer. The load distribution on the rolling elements is one of the most important operating characteristics of a bearing, due to the influence on its operating ability and the accuracy of its operation [12].

An analysis of the studies regarding load distribution and methods used to determine the number of active rolling elements of a rolling radial bearing with or without internal clearance that was loaded with a constant external radial load will be presented.

This problem of determining the number of active rolling elements and the load distribution has been researched by a large number of authors. In 1907, Stribeck [11] conducted a research on the variation of loads on the elements of a radially loaded ball bearing. He found that the load on the most loaded ball is 4.37 times the average ball load for zero internal clearance.

Later, Jones published a model describing the behavior of an ideal ball bearing, without deviations in dimensions, shape or position of the operating surface of the coupled parts [3]. Using Hertz‘s theory of contact stresses he obtained a system of non-linear equations for determining the parameters for the load distribution on the rolling elements. This model was later developed by Harris [2] in his own work. He developed a two-dimensional elastic-deformational model of rolling bearings using the concept of Sjovall's integral. The problem of the contact zone size is defined by Harris with the help of a load distribution factor and, using the trial and error method, the load distribution is solved for a given bearing with a known clearance under a given load. After a value for the load distribution factor is assumed, the bearing radial deflection is calculated. This value, together with the value of Sjovall's integral for the load distribution factor, is inserted into the equilibrium equation. If the equilibrium equation is not satisfied, then the process is repeated. The method developed by Harris to calculate the distribution of load among the balls or rollers of rolling bearings can be used in bearing applications where rotational speeds are slow to moderate, because under these speed conditions, effects of rolling element centrifugal forces and gyroscopic moments are significantly reduced. A disadvantage of Harris' model is that approximates a finite sum with an integral causing an imprecise solution for the problem of the internal load distribution.

Chana [1] conducted a preliminary study of rolling element bearing behavior under radial loading. He considered a roller bearing without internal clearance and concluded that, when rotating, a roller bearing under a constant radial load shows periodic variation of the reaction forces in the bearing. The stiffness characteristics of the roller bearing are affected by this variation that is dependent on the number of rolling elements and on the speed of rotation of the supported shaft.

Mitrović studied the influence of internal radial clearance on the static load rating of the rolling bearing. He concluded that the static load rating of the rolling bearing depends on the size of the internal radial clearance and that the standard expressions for the static load rating of the bearings do not take into account the size of the internal radial clearance, but only its presence. He also showed that, in the case of zero clearance and a mean clearance, the actual static load rating and the static load rating determined under ISO are equal, while deviations are evident in other cases [6].

In the early 2000s, Lazovic [4] analyzed the effect of the internal geometry of rolling bearing parameters on the load distribution on rolling elements based on analytically described values and their mutual relations. A few years later, taking into consideration the influence of the internal radial clearance and the shape of the rolling bearing races, he developed a mathematical model of the load distribution on the rolling element of a bearing [5].

In paper [8], Ricci created a new procedure for getting numerically, accurately, and quickly the static load distribution of a single-row, angular-contact ball bearing, subjected to a known thrust load which is applied to a variable distance from the geometric bearing center line. In another paper [9], he used the same procedure to obtain the static load distribution of a ball bearing under axial and radial loads, taking into account the influence of fits and thermal gradients. The innovation of his method is given by the choice of the set of the nonlinear equations, which must be solved simultaneously [10].

Tomović [12] presented a mathematical model for necessary boundary bearing-deflection level to support the inner bearing ring on q rolling elements. His model takes into account two boundary positions of the inner ring: supported on an even or an odd number of rolling elements. The size of the boundary bearing deflection was determined by the size of the internal radial clearance and the total number of rolling elements in a bearing. In [13], using the mathematical model he developed, Tomović presented the procedure for determining the number of the active rolling elements. He also made a comparison of the developed model with some models for the calculation of a load distribution that have been most frequently used so far. In [14], he conducted an investigation on the effect of rolling bearing construction on internal load distribution and the number of active rolling elements. He showed that higher values of external load and smaller values of internal radial clearance create a more favorable load distribution and ensure that higher number of rolling elements participate in the load transfer. As far as the total number of rolling elements in a bearing is concerned, the analysis showed that a more favorable load distribution is easier to achieve in bearings with a higher total number of rolling elements.

This analysis of the above papers shows that there is only one study in the open access literature that addresses the problem of the bearing behavior under radial loading while rotating. That study that does not approach the problem of calculating the rolling elements deflections during bearing rotation. In this work, a mathematical model is presented for solving this problem.

2. STARTING ASSUMPTIONS

Due to the complexity of a rolling bearing the following assumptions were introduced in this study:

- The analyzed bearing is a single-row radial bearing with balls or rollers as rolling elements.
- The external load on rolling bearing is radial and constant.
- The rolling elements and the inner ring have translational movement only in the bearing radial plane, while the outer bearing ring is considered to be absolutely fixed.
- Elastic deformations occur along the contact surfaces of the bearing rolling elements while inner and outer ring are rigid.
- The bearing contact parts (rolling elements and races) are ideally shaped.
- The cages of the rolling bearing keep the rolling elements evenly distributed around the complete circumference preventing direct contact between neighboring rolling elements, therefore, the mutual influence between them is excluded. They also ensure a constant separation angle between them in order to provide even load distribution and quiet and uniform running. The separation angle between the rolling elements is given by the following equation:

$$\varphi_Z = \frac{2\cdot\pi}{Z} \quad (1)$$

  where Z is the total number of rolling elements.
- The support and the elements in contact with the analyzed bearing (shaft and housing) are rigid and ideal in terms of dimensions, shape and position of the surfaces.
- The bearing operates under isothermal conditions.
- As a result of external load, relative motions of the bearing rings occur. Their influence on the angle between the rolling elements is neglected.

3. MATHEMATICAL MODEL OF RADIAL BEARING ROLLING ELEMENT DEFLECTION

3.1 Rolling bodies deflections at initial apex position

Fig. 1 is a sketch representation of a cross-section through a radial bearing with internal clearance *s*. The outer circle represents the outer raceway with the cross-sectional groove radius R_o. The inner circle represents the ball/roller envelope with the cross-sectional radius R_{re}. Further on, the direction of the load that is applied perpendicular to the shaft axis, passes through the center of the bearing and is included in the radial plane will be referred to as *radial direction*.

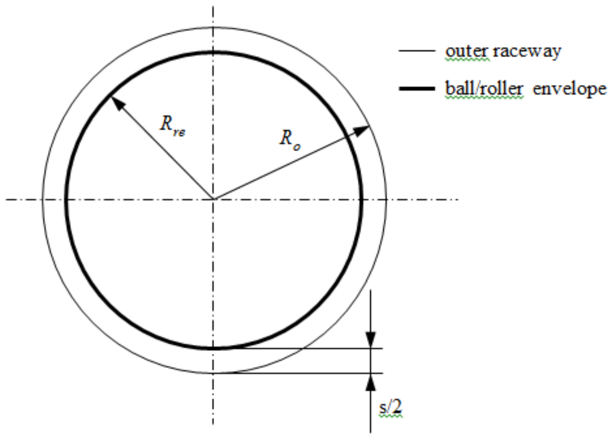

Fig. 1. Scheme of a radial bearing with internal clearance.

Loading the bearing in the radial direction, the lower half of the bearing supports the load applied, causing the center of the ball/ roller envelope to translate with the radial ring shift δ_r in the direction of the radial external load as shown in Fig. 2.

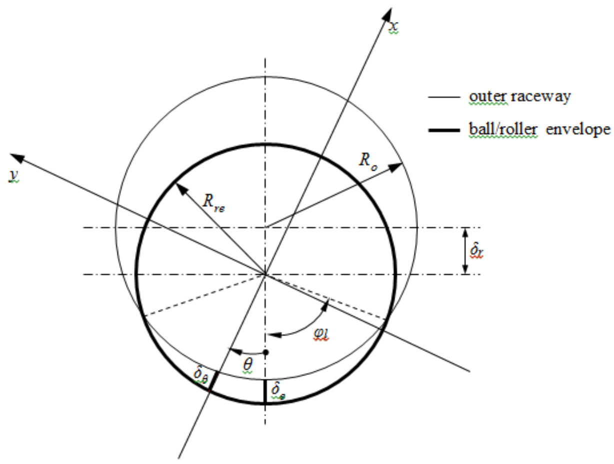

Fig. 2. Approaching of the bearing rings under radial load.

The equation of the circle that represents the outer raceway is:

$$x^2 + y^2 = R_{re}^2 \quad (2)$$

and for the circle that represents the ball/roller envelope is:

$$(x - \delta_r \cos\theta)^2 + (y - \delta_r \sin\theta)^2 = R_e^2 \quad (3)$$

Using these equations, the abscissas of the circles intersection points with the negative x-axis are given by the equations:

$$x_{re} = -R_{re} \quad (4)$$

and

$$x_e = -\sqrt{R_e^2 - \delta_r^2 \sin^2\theta} + \delta_r \cos\theta \approx -R_e + \delta_r \cos\theta \quad (5)$$

Based on these, an expression for the deflection of a body situated at an angle θ with the radial direction can be calculated:

$$\delta_\theta = x_e - x_{re} = \delta_r \cos\theta - \frac{s}{2} \quad (6)$$

For θ=0, the deflection on the radial direction is obtained:

$$\delta_e = \delta_r - \frac{s}{2} \quad (7)$$

and, therefore:

$$\delta_\theta = \delta_e \cos\theta - \frac{s}{2}(1 - \cos\theta) \quad (8)$$

On the basis of Hertz's contact theory, the load of a rolling element at an angle θ with the radial direction is given by the following non-linear equation:

$$Q_\theta = K_n \delta_\theta^n \quad (9)$$

where:

$$n = \begin{cases} \frac{3}{2} & \text{for ball bearings} \\ \frac{10}{9} & \text{for roller bearings} \end{cases} \quad ()$$

and K_n is a constant value, that depends on the bearing internal geometry.

3.2 Rolling bodies deflections during shaft rotation

The same problem of determining the rolling element deflection and the load distribution for a bearing with internal clearance can be addressed during the shaft rotation of angle φ.

Fig. 3 is a sketch representation of a cross-section through a radial bearing that shows the deflections of the rolling elements that occur at a certain angle with the radial direction. The bearing has internal clearance and is radially loaded with a force denoted by F_r. The direction of this force passes through the center of the rings.

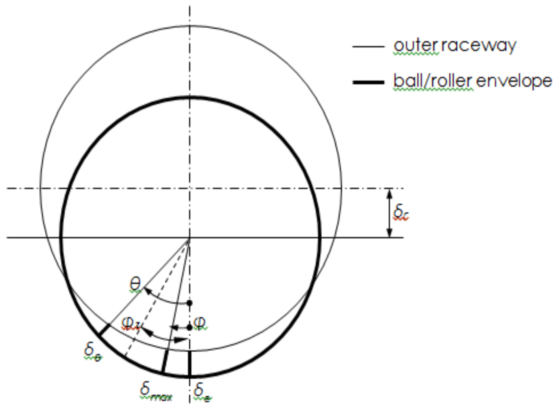

Fig. 3. Deflections of the rolling elements as they rotate from initial apex position to the next successive apex position.

In the initial apex position the rolling elements that participate to the transfer of the external radial load are symmetrically distributed in relation to the direction of load and the most loaded rolling element is situated on the radial direction. This is not true during rotation, when the most loaded rolling element is situated at an angle φ with the radial direction, where $0 \leq \varphi \leq \varphi_z/2$. Using equation (8), the maximum deflection is given by:

$$\delta_{max} = \delta_e \cos\varphi - \frac{s}{2}(1 - \cos\varphi) \quad (10)$$

Expressing δ_e from (11):

$$\delta_e = \frac{\delta_{max} + \frac{s}{2}(1 - \cos\varphi)}{\cos\varphi} \quad (11)$$

the deflection at an angle θ with the radial direction equals:

$$\delta_\theta = \delta_{max} \cdot \frac{\cos\theta}{\cos\varphi} - \frac{s}{2} \cdot \left(1 - \frac{\cos\theta}{\cos\varphi}\right) \quad (12)$$

and the corresponding load equals:

$$Q_\theta = K_n \cdot \left(\frac{s}{2}\right)^n \left[\left(\frac{2\delta_{max}}{s} + 1\right) \cdot \frac{\cos\theta}{\cos\varphi} - 1\right]^n \quad (13)$$

Using the notation:

$$\Delta_{max} = \left(\frac{2\delta_{max}}{s} + 1\right) \cdot \frac{1}{\cos\varphi} \quad (14)$$

equation (14) can be written:

$$Q_\theta = K_n \left(\frac{s}{2}\right)^n (\Delta_{max} \cos\theta - 1)^n \quad (15)$$

Defining

$$\langle x \rangle = \max(x, 0) = \begin{cases} x, \text{if } x > 0 \\ 0, \text{otherwise} \end{cases} \quad (16)$$

and

$$\varphi_k = (k-1) \cdot \varphi_Z + \varphi;\ k = 1, 2, \dots, Z \quad (17)$$

the equation of the load of the *k*-th rolling body is:

$$Q_k = K_n \left(\frac{s}{2}\right)^n \langle \Delta_{max} \cos\varphi_k - 1 \rangle^n;\ k = 1, 2, \dots, Z \quad (18)$$

The sum of the radial components of these loads must be equal to the applied load F_r:

$$F_r = \sum_{k=1}^{Z} Q_k \cos\varphi_k =$$

$$= K_n \left(\frac{s}{2}\right)^n \cdot \sum_{k=1}^{Z} \left[\langle \Delta_{max} \cos\varphi_k - 1 \rangle^n \cos\varphi_k\right] \quad (19)$$

Equation (20) can also be written as:

$$\sum_{k=1}^{Z} \left[\langle \Delta_{max} \cos\varphi_k - 1 \rangle^n \cos\varphi_k\right] - \frac{F_r}{K_n} \cdot \left(\frac{2}{s}\right)^n = 0 \quad (20)$$

Using a numerical method, Δ_{max} can be found.

Therefore, the deflection of the *k*-th rolling element is:

$$\delta_k = \left[\max(\Delta_{max} \cos\varphi_k - 1, 0)\right]^n;\ k = 1, 2, \dots, Z \quad (21)$$

and its load is:

$$Q_k = K_n\, \delta_k^n;\ k = 1, 2, \dots, Z \quad (22)$$

4. CONCLUSIONS

Based on the above study described in this paper one can draw the following conclusions:

- The mathematical model of calculating rolling elements deflections and load distribution during the bearing rotation has a great practical importance for the development of the elastic-dynamic model and the model of dynamic behavior of the rolling bearing, based on the analysis of each rolling element load.
- The number of the active rolling elements has a decisive influence on the basic static and dynamic load rating of a rolling bearing, on his dynamic behavior, on a level of noise and vibrations generated by the bearings, working ability, working accuracy and working life of a rolling bearing. This mathematical model provides the foundation for the calculation of the number of active rolling elements participating in the external radial load transfer and for a future research to find out if the number of active rolling elements remains constant or not during bearing rotation.
- The analysis presented in this paper, was created only taking into account the basic internal geometry of the bearing, without requiring a discussion about an even or odd number of the rolling elements. Some of the previous mathematical models of a rolling bearing considered one boundary position of the support (on an odd number of the rolling elements of a bearing) or two boundary positions of the inner ring support (on even and odd number of the rolling elements).
- The developed mathematical model of calculating the load distribution and rolling elements deflections is related to an ideal radial ball or roller bearing. The model can fairly well serve as a starting point of the model development of a real bearing, with errors in macro and micro geometry − that in the practical applications cannot be avoided − and for the development of the models for other bearing types.
- Further investigation can be undertaken based on this mathematical model in order to analyze if the sum of transversal reactions to the constant external radial load applied equals zero:

$$\sum_{k=1}^{Z} Q_k \sin\varphi_k = 0 \quad (24)$$

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INFLUENȚA UNGHIULUI DE ROTAȚIE ASUPRA DISTRIBUŢIEI FORŢELOR PE CORPURILE DE RULARE ALE RULMENŢILOR. PARTEA 1: MODEL MATEMATIC

Rezumat: În această lucrare este prezentată analiza teoretică a unui rulment cu un rând de bile sau role cu joc intern atunci când acesta este încărcat radial cu o forță externă constantă. Analiza este centrată pe calcularea deformației elementelor de rulare, care permite determinarea numărului de corpuri de rulare încărcate ce participă la transferul sarcinii. Luând în considerare geometria internă a rulmentului, s-a dezvoltat un model matematic pentru calcularea deformației elementelor de rulare ale rulmentului în timpul rotației acestuia.

Cristina TUDOSE, Assist. Prof. Dr. Eng., Technical University of Cluj-Napoca, Mechanical System Engineering Department, Cristina.Stanescu@omt.utcluj.ro, AC&O Dept. RKB Bearing Industries, Switzerland

Florina RUSU, PhD student, Technical University of Cluj-Napoca, Mechanical System Engineering Department, Florina.Rusu@omt.utcluj.ro

Lucian TUDOSE, Prof. Dr. Eng., Technical University of Cluj-Napoca, Mechanical System Engineering Department, Lucian.Tudose@omt.utcluj.ro, Head of AC&O Dept. RKB Bearing Industries, Switzerland