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DETERMINING THE MAXIMAL RELATIVE MOTION SPEED CONSIDERING THE INFLUENCE OF TWO PARAMETERS, USING A NEW OPTIMIZATION METHOD

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Abstract: *The paper presents the problem of optimizing particles speed in the relative motion on a vibratory conveyor. Developing a C program the relative motion characteristics were modeled. Using a new optimization method the maximal relative motion speed was determined considering the influence of two parameters.*
Key words: *differential equation, vibratory conveyor, optimization, relative motion, C programming.*

1. INTRODUCTION

A problem of theoretical interest is the study of the relative movements of different materials on a vibratory conveyor trough.

The study of the relative motion of the material on the vibrating surface is a starting point in the design and construction of the vibratory conveyors, and other types of equipment so that they fulfill certain technological imposed requirements.

Vibratory conveyors are generally used for displacement of the various materials between two working points.

These equipments are used in different domains, such as chemical industry, construction, food industry and others [1], [5], [6], [7], [8].

A vibrating conveyor consists generally of: one or more moving tables 1, one or more vibration generators 2, elastic elements of different types, their role is to support, or as links between the masses in motion or between mass and vibrations generator 3 and the frame 4 (Fig. 1).

Fig.1. Vibratory Conveyor

Optimization is a process in which an objective function is minimized or maximized.

The optimization problem of the parameter values that determine the vibratory movement has the purpose to maximize the relative motion of the materials on the vibrating surface.

The dynamics of different equipments are frequently studied in the literature, using various optimization methods. The vibratory conveyors were optimized with respect to the transmission of the dynamic force considering various design parameters.

The positions at which the elastic elements are attached to the plate were considered as design variables. The optimization of these parameters is performed using the mathematical modeling of the system dynamic behavior [10], [14].

Authors have developed a new optimization method proper only for two variable parameters based on geometrical considerations and has some similarities with the method proposed by Rosenbrock [11], [13].

2. MATHEMATICAL MODELING OF THE VIBRATORY MOVEMENT

Considering a vibratory plate that belongs to a vibratory conveyor that performs a harmonic motion throughout the direction Δ according the law $S = A \cdot \cos(\omega t)$ the mass m will move on this plate (figure 2).

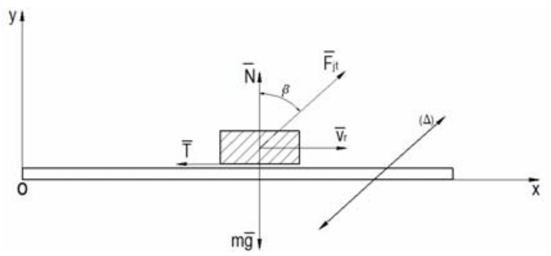

Fig.2. Schematic representation of a vobro-conveyor

Referring to the Oxy Cartesian frame the differential equations of the mass m relative motion is established.

The system (1) represents the differential simultaneous equations that model the relative motion of the object on the vibrating plate [1], [8]:

$$\begin{cases} m\ddot{x} = mg + F_{jt}\cos\beta - T \\ m\ddot{y} = -mg + F_{jt}\sin\beta + N \end{cases} \quad (1)$$

were

$$F_{jt} = m \cdot a_t = m\omega^2 A\cos\omega t$$

$$\overline{T} = -\mu N \frac{\overline{v}_r}{|\overline{v}_r|}$$

μ being the friction coefficient.

If the mass m will not leave the plate belonging to the vibratory conveyor, results the condition: $y = 0$ and $N \geq 0$.

Because of this condition the normal force N is as follows:

$$N = mg - F_{jt}\sin\beta \geq 0 \quad (2)$$

Using a C program the displacement and speed of the relative motion were determined.

The system of equation was solved using fourth order Runge-Kutta method, performing n iteration (figure 3).

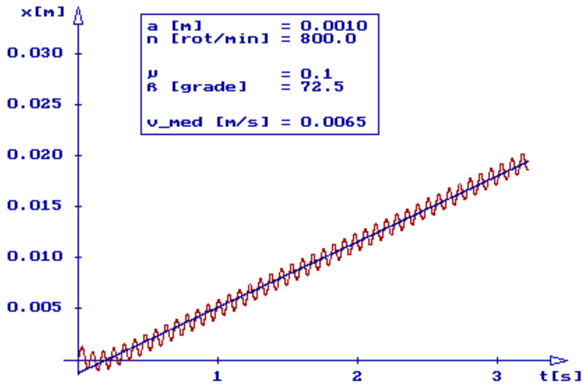

Fig.3. Displacement diagram

The obtained displacement diagram was replaced with a line according to the least square method, $x(t)=c_1 \cdot t+c_2$.

The coefficient c1 and c2 are determined solving the system (3).

$$\begin{cases} \left(\sum_{i=1}^{n} t_i^2\right) c_1 + \left(\sum_{i=1}^{n} t_i\right) c_2 = \sum_{i=1}^{n} x_i \cdot t_i \\ \left(\sum_{i=1}^{n} t_i\right) c_1 + n \cdot c_2 = \sum_{i=1}^{n} x_i \end{cases} \quad (3)$$

The slope c_1 of the line represents the relative motion.

There are different possibilities to determine the relative motion.

In first case is considered the situation in which the amplitude, frequency, coefficient of friction μ and angle β are considered variable parameters.

In the second case is considered the situation when the angle β and coefficient of friction μ are variable parameters and the amplitude and frequency are constant parameters.

In the third case is considered the situation when the amplitude and frequency are variable parameters and the coefficient of friction μ and the angle β are constant parameters.

Also, between these parameters are other possible combinations in the study of the relative motion speed.

Some numerical results of the studied cases are presented in this paper and are presented in figure 3.

3. A NEW OPTIMIZATION METHOD BASED ON GEOMETRICAL CONSIDERATION

The optimization method presented in this paper can be used only when there are two variable parameters.

As variable parameters were considered the coefficient of friction μ and the angle β.

It is considered a surface z=f(x, y). The analytical expression of this surface is unknown and the quote z results based on calculations that involve the coordinates x and y (figure 4).

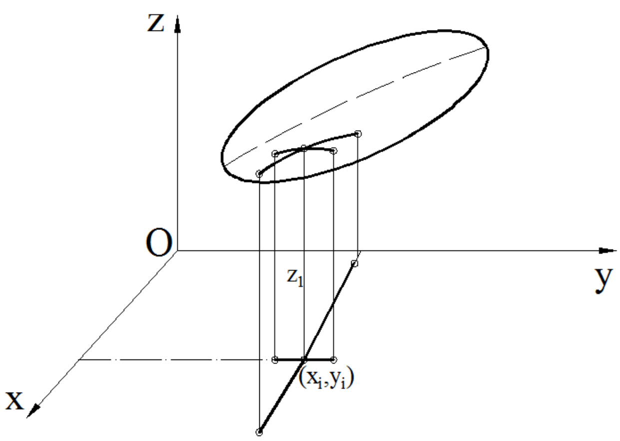

Fig.4. The unknown surface

In the frame xOy an initial point (x_i, y_i) is considered. The purpose is to determine which quote z_i is the highest from the neighborhood of this initial point.

Similar with the Rosenbrock method, the coordinates of four points z_2, z_3, z_4 and z_5 were determined and correspond to the following points from the plan: (x_i+h, y_i), (x_i-h,y_i), (x_i,y_i+h) şi (x_i, y_i-h) (figure 5).

The main problem is to determine the equations of the plane P that is tangent to a unknown surface at the point of coordinates (x_i, y_i, z_1) in such a way that interpolation parabolas which pass through the points 3, 1, 2 and 4, 1, 5 to be drawn trough the middle points defining this plane.

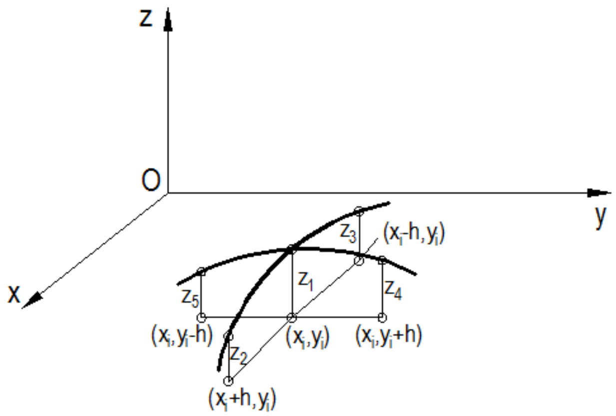

Fig.5. The coordinates of four points z_2, z_3, z_4 and z_5

Considering figure 6, the parabola of interpolation passing through the points 3, 1, 2 will have the equation $z=ax^2+bx+c$.

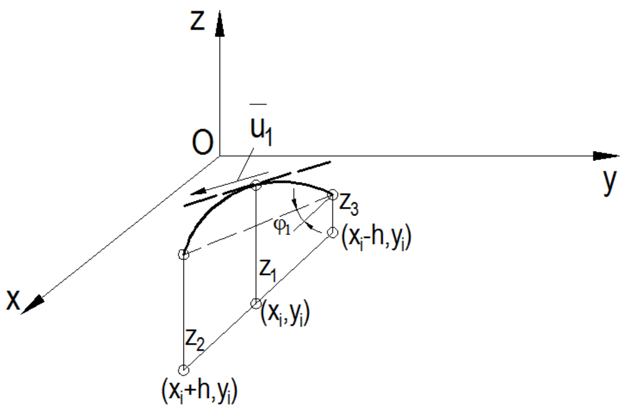

Fig.6. The parabola of interpolation

It is known that in the case of a parabola of interpolation that passes through three equidistant points the tangent drawn in the center point is parallel with the chord that cross through the start and the end points [2], [4], [9] [12].

The tangent slope is written in the middle points and the angle φ_1 with the horizontal plane is calculated.

Similarly, considering the parabola of interpolation passing through the points 4, 1 and 5 (figure 7) the coefficients of the equation $z=dy^2+ey+f$ can be determined and then it was calculated the tangent slope to this parable in the center point, resulting the angle φ_2 between the tangent and plan Oxy.

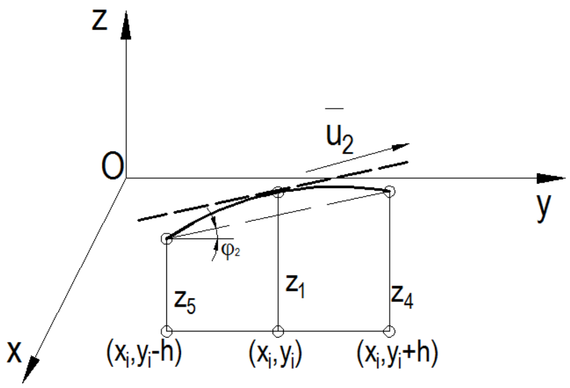

Fig.7. The parabol of interpolation

Knowing the angles φ_1 and φ_2 were determined the unit vectors $\bar{u}_1$ and $\bar{u}_2$ on the two lines tangent to the parabolas.

$$\begin{aligned}\bar{u}_1 &= \cos\varphi_1 \bar{i} + 0\cdot\bar{j} + \sin\varphi_2 \bar{k}\\ \bar{u}_2 &= 0\cdot\bar{i} + \cos\varphi_2\bar{j} + \sin\varphi_2\bar{k}.\end{aligned} \tag{4}$$

These two unit vectors are included in the plane therefore we can determine the expressions of the vector which is perpendicular to the tangential plane (figure 8).

$$\bar{n} = \bar{u}_1 \times \bar{u}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \cos\varphi_1 & 0 & \sin\varphi_1 \\ 0 & \cos\varphi_2 & \sin\varphi_2 \end{vmatrix} \tag{5}$$

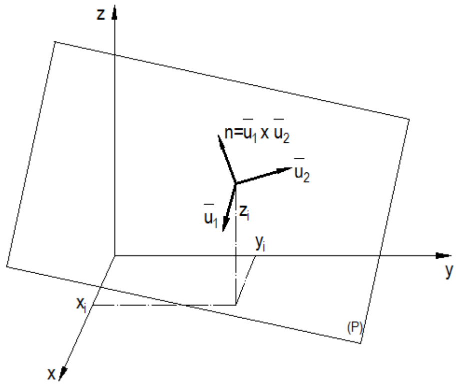

Fig.8.Tangential plane

The analytical equation of the plane P is obtained equalizing with zero the dot product between a vector belonging to the plane, having the equation $\bar{u} = (x - x_i)\bar{i} + (y - y_i)\bar{j} + (z - z_1)\bar{k}$ and the normal unit vector $\bar{n}$ to the plane.

$$\left[(x - x_i)\bar{i} + (y - y_i)\bar{j} + (z - z_1)\bar{k}\right]\cdot \cdot\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \cos\varphi_1 & 0 & \sin\varphi_1 \\ 0 & \cos\varphi_2 & \sin\varphi_2 \end{vmatrix} = 0$$

The previous equation may be written in the following form:

$$\left[(x - x_i)\bar{i} + (y - y_i)\bar{j} + (z - z_1)\bar{k}\right]\cdot \left(-\sin\varphi_1\cos\varphi_2\bar{i} - \cos\varphi_1\sin\varphi_2\bar{j} + \cos\varphi_1\cos\varphi_2\bar{k}\right) = 0$$

resulting the analytical equation of the plane

$$\begin{aligned}&-\sin\varphi_1\cos\varphi_2(x - x_i) - \cos\varphi_1\sin\varphi_2(y - y_i) + \\ &\qquad + \cos\varphi_1\cos\varphi_2(z - z_1) = 0\end{aligned} \tag{6}$$

If the plane P is parallel with the frame xOy plane the problem of determining the maximal relative motion speed is completed and is at the maximal point, obtained at the end of the calculations, belonging to the considered surface

Starting with the initial point (x_i, y_i, z_1) a movement on the highest slope of the line is performed.

In order to determine the line that intersect the plane P and the frame xOy, which has the equation z = 0, from (6) results:

$$\begin{aligned}&-x\cdot\sin\varphi_1\cdot\cos\varphi_2 - y\cdot\cos\varphi_1\cdot\sin\varphi_2 + \\ &+ x_i\cdot\sin\varphi_1\cdot\cos\varphi_2 + y_i\cdot\cos\varphi_1\cdot\sin\varphi_2 - \\ &- z_1\cdot\cos\varphi_1\cdot\cos\varphi_2 = 0\end{aligned}$$

Highlighting the expression of the angular coefficient belonging to the determined line with respect of the Ox axis, this equation is written in the following form

$$\begin{aligned}y &= \frac{\sin\varphi_1\cdot\cos\varphi_2}{\cos\varphi_1\cdot\sin\varphi_2}x + \\ &\quad + \frac{x_i\cdot\sin\varphi_1\cdot\cos\varphi_2 + y_i\cdot\cos\varphi_1\cdot\sin\varphi_2}{\cos\varphi_1\cdot\sin\varphi_2}\end{aligned} \tag{7}$$

According the equation (7) the slope of the line is

$$m_d = \frac{\sin\varphi_1 \cdot \cos\varphi_2}{\cos\varphi_1 \cdot \sin\varphi_2} = tg\varphi \qquad (8)$$

and the angular coefficient of the normal on the line is

$$m_n = -\frac{\cos\varphi_1 \cdot \sin\varphi_2}{\sin\varphi_1 \cdot \cos\varphi_2} \qquad (9)$$

4. NUMERICAL EXPERIMENT

There were considered two points, 6 and 7, belonging to the frame Oxy and we determine in which point is obtained the maximal quote of the surface. At the next iteration a new start point will be considered $x_i = x_6$, $y_i = y_6$ or $x_i = x_7$, $y_i = y_7$.

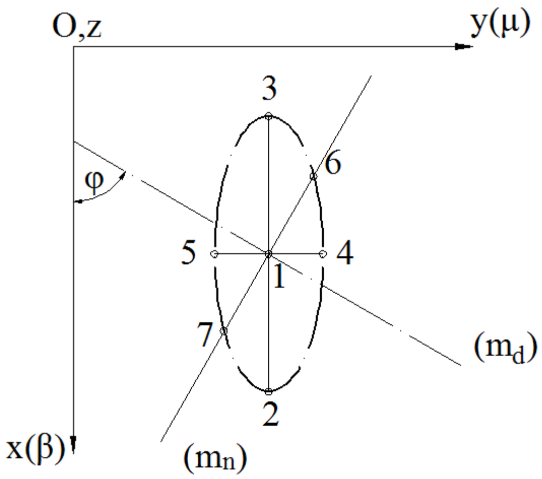

Fig.9. The direction of the β angle and the μ coefficient of friction

Obvious is the fact that if the line of intersection between the plane P, and the horizontal plane xOy is parallel to the one of the axis Ox or Oy, then new coordinates can be calculated directly, much simpler. Points 6 and 7 coincide with points 2 and 3 or 4 and 5.

In the figure 9 points 3 and 2 represent the direction on which the angle β change its value with a step h_β, and the points 5 and 4 represent the direction on which the coefficient of friction μ change its value with a step h_μ.

There is also the situation in which the optimal value is found on a direction which is different from the obtained points 3 and 2 or 5 and 4.

In this situation we assume that the considered points are located on an arc belonging to an ellipse that passes through the four points 2, 5, 3, and 4.

It can be seen from figure 9 that the line with slope m_n cut an ellipse in two points 6 and 7.

By solving the system of equations (10) the coordinates x_6, y_6 and x_7 and y_7 of the points 6 and 7 will be determined,

$$\begin{cases} \dfrac{(x-x_1)^2}{h_\beta^2} + \dfrac{(y-y_1)^2}{h_\mu^2} = 1 \\ y - y_1 = m \cdot (x - x_1) \end{cases} \qquad (10)$$

By substituting $y\text{-}y_1$ from the second equation in the first one on obtain:

$$(x-x_1)^2\left[\frac{1}{h_\beta^2} + \frac{m^2}{h_\mu^2}\right] = 1 \Rightarrow$$

$$(x-x_1)^2\left[h_\mu^2 + m^2 \cdot h_\beta^2\right] = h_\beta^2 \cdot h_\mu^2$$

resulting the point coordinates x_6, x_7:

$$x_{6,7} = x_1 \pm \frac{h_\beta \cdot h_\mu}{\sqrt{h_\mu^2 + m^2 \cdot h_\beta^2}} \qquad (11)$$

The values for y_6 and y_7 are determined using the following formula:

$$y_6 = m_n \cdot (x_6 - x_1) + y_1 \qquad (12)$$
$$y_7 = m_n \cdot (x_7 - x_1) + y_1$$

In figure 10 is presented the graphical representation of the directions in which the obtained points are moving.

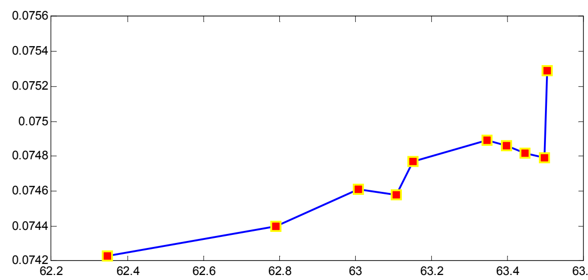

Fig.10. The graphical representation of the points directions

5. CONCLUSION

The relative motions of the parts moving along the trough were obtained using a C program and the system of equation was solved with fourth order Runge-Kutta method.

Using the least square method we determine the line that approximates the diagram of the parts displacement with respect of time.

Using the propose optimization method, after ten slops the expected result was obtained.

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DETERMINAREA VITEZEI MAXIME DE DEPLASARE CONSIDERÂND INFLUENȚA A DOI PARAMETRII, FOLOSIND O METODĂ NOUĂ DE OPTIMIZARE

Rezumat: În cadrul acestei lucrări este prezentată problema optimizării vitezei medii de transport. În urma rulării unui program realizat în limbajul de programare C s-a obținut viteza maximă de transport considerând influența a doi parametrii variabili, folosind o nouă metodă de optimizare dezvoltată de către autori.

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