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ADDENDUM MODIFICATION OF CYLINDRICAL SPUR GEARS WITH EQUALIZAED POWERS LOST BY FRICTION BETWEEN THE TEETH FLANKS AT THE POINTS WHERE THE MESHING STARS AND ENDS

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***Abstract**: The paper presents a computation method of the geometrical dimensions for the cylindrical spur gears based on the equalization of the powers lost by friction, between the teeth's flanks, during the meshing of the teeth. This method, compared to other, computes the addendum modifications considering the friction coefficient. If equalization of the power lost by friction succeeds, at the points where the meshing starts and ends, the specific addendum modification will depend also of the friction coefficient between teeth flanks.*

***Key words:** Power lost by friction, Friction coefficient, Specific addendum modification, Power equalization.*

1. INTRODUCTION

Cylindrical spur gears geometric dimensions, most often, are determined using specific addendum modification as these may be used to obtain a well defined distance between the axes, at the same time, ensuring the correct meshing of teeth flanks over a longer period. The technical literature [1] – [5] gives several recommendations to choose the x_1 and x_2 specific addendum modification. This paper describes a new method for choosing the specific addendum modifications x_1 and x_2 which takes into account the friction coefficient μ between the meshing teeth flanks. This dependency is achieved by equalizing the powers lost by friction at the points where the meshing starts and ends.

2. POWER LOSSES BY FRICTION BETWEEN THE TOOTH FLANKS

In paper [6] the following mathematical expression were deduced to compute the power lost by friction at point A, where the meshing begins and at point E, where the meshing ends:

$$P_{vzA} = \mu_A \, P_1 \frac{e_A (1 + i_{21})}{r_{b1} (1 - \mu_A \tan(\alpha_w)) + \mu_A \, e_A} . \tag{1}$$

and

$$P_{vzE} = \mu_E \, P_1 \frac{e_E (1 + i_{21})}{r_{b1} (1 + \mu_E \tan(\alpha_w)) + \mu_E \, e_E} . \tag{2}$$

where μ_A and μ_E are the friction coefficients between the teeth flanks at A and E points, $P_1 = T_1 \, \omega_1$ is the power on the driving wheel, α_w the meshing angle, $i_{21}=\omega_2/\omega_1$ the gear ratio, r_{b1} the base circle radius of the driving wheel and e_A with e_E are obtained from the following expressions:

$$y = \left(\frac{z_1 + z_2}{2}\right) \times \left(\frac{\cos(\alpha_0)}{\cos(\alpha_w)} - 1\right) . \tag{3}$$

$$k = x_1 + x_2 - y . \tag{4}$$

$$r_2 / r_{a2} = \frac{z_2}{z_2 + 2h_a + 2x_2 - 2k} . \tag{5}$$

$$r_1 / r_{a1} = \frac{z_1}{z_1 + 2h_a + 2x_1 - 2k} . \tag{6}$$

$$\cos(\alpha_A) = r_2 / r_{a2} \times \cos(\alpha_0)\,. \tag{7}$$

$$\cos(\alpha_E) = r_1 / r_{a1} \times \cos(\alpha_0)\,. \tag{8}$$

$$e_A = r_{b1} \frac{z_2}{z_1} \left(\tan(\alpha_A) - \tan(\alpha_w)\right). \tag{9}$$

$$e_E = r_{b1} \left(\tan(\alpha_E) - \tan(\alpha_w)\right). \tag{10}$$

where α_w is computed from the following equation:

$$x_1 + x_2 - \frac{\mathrm{inv}(\alpha_w) - \mathrm{inv}(\alpha_0)}{2\tan(\alpha_0)} \times (z_1 + z_2) = 0\,. \tag{11}$$

3. NUMERICAL REZULTS AND GRAPHICAL REPRESENTATION OF THE POWER LOSSES BY FRICTION BETWEEN THE TEETH FLANKS

Numerical results and graphical representations are given for (1) and (2) for the following input data: $\mu_A = \mu_E = \mu = 0.05$, $P_1 = 200$W, $r_{b1} = 20$mm. The x_1 and x_2 the values are in the range [-1, 1] while the z_1 and z_2 values are fixed.

Table 1

The P_{vzA} and P_{vzE} for $z_1=19$, $z_2=19$

x_1	x_2	P_{vzA}	P_{vzE}
-1.00	-1.00	24.11292	25.09856
-1.00	-0.50	21.50609	20.54361
-1.00	0.00	17.50523	13.70812
-1.00	0.50	8.92402	1.64889
-1.00	1.00	8.92494	0.00000
-0.50	-1.00	19.86342	22.23924
-0.50	-0.50	15.61204	15.97033
-0.50	0.00	6.93208	4.51052
-0.50	0.50	7.12249	2.74718
-0.50	1.00	7.45210	1.19680
0.00	-1.00	13.39881	17.90494
0.00	-0.50	4.62572	6.76043
0.00	0.00	5.13297	4.95176
0.00	0.50	5.61846	3.43860
0.00	1.00	6.06346	1.99303
0.50	-1.00	1.69131	8.70414
0.50	-0.50	2.84830	6.87226
0.50	0.00	3.58967	5.38318
0.50	0.50	4.18408	3.98662
0.50	1.00	4.69919	2.61844
1.00	-1.00	0.00000	8.61278
1.00	-0.50	1.24968	7.14141
1.00	0.00	2.09227	5.77860
1.00	0.50	2.76125	4.45728
1.00	1.00	3.33279	3.14791

Table 2

The P_{vzA} and P_{vzE} for $z_1=19$, $z_2=33$

x_1	x_2	P_{vzA}	P_{vzE}
-1.00	-1.00	42.37300	29.91934
-1.00	-0.50	36.32737	22.35129
-1.00	0.00	15.61283	2.57133
-1.00	0.50	13.71419	1.51018
-1.00	1.00	13.54741	0.00000
-0.50	-1.00	33.48587	25.03387
-0.50	-0.50	11.93589	7.41177
-0.50	0.00	10.62818	5.36555
-0.50	0.50	10.71462	3.75930
-0.50	1.00	10.96351	2.14670
0.00	-1.00	7.37031	10.81108
0.00	-0.50	7.14430	8.42066
0.00	0.00	7.60959	6.77609
0.00	0.50	8.08283	5.20254
0.00	1.00	8.53488	3.62578
0.50	-1.00	3.01743	11.06828
0.50	-0.50	4.11513	9.40414
0.50	0.00	4.92103	7.85584
0.50	0.50	5.58922	6.33247
0.50	1.00	6.17485	4.80162
1.00	-1.00	0.00000	11.78591
1.00	-0.50	1.35506	10.25619
1.00	0.00	2.34522	8.76967
1.00	0.50	3.14918	7.29091
1.00	1.00	3.83965	5.80191

Choosing the addendum modifications randomly, as shown by the numerical results from Table 1 and 2, might result in huge differences between the powers lost by friction that lead to nonuniform wearing of the teeth flanks. Rows in Table 1 and Table 2 with such huge differences are shown shaded. For example if we select x1=-1 and x2=1 for the addendum modification the highest differences between the P_{vzA} and P_{vzE} powers will be obtained. Wheels designed with such values will deteriorate faster and have a shorter lifetime. To inspect the equalization chances of these powers a graphical representation of the surfaces can be used. The intersection of the surfaces from Figures 1, 3 are given in Figures 2, 4 using Matlab as interpolated isolines on the second surface (2).

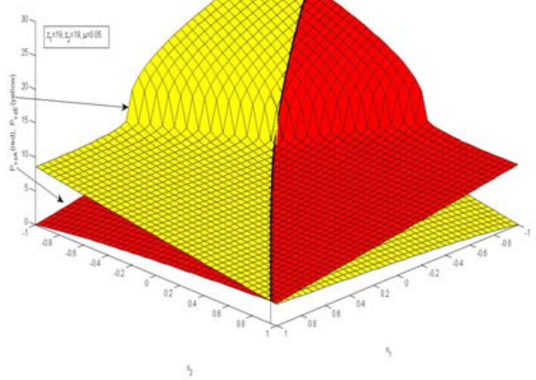

Fig. 1. Intersection of the (1) and (2) surfaces for z_1=19, z_2=19 and μ=0.05.

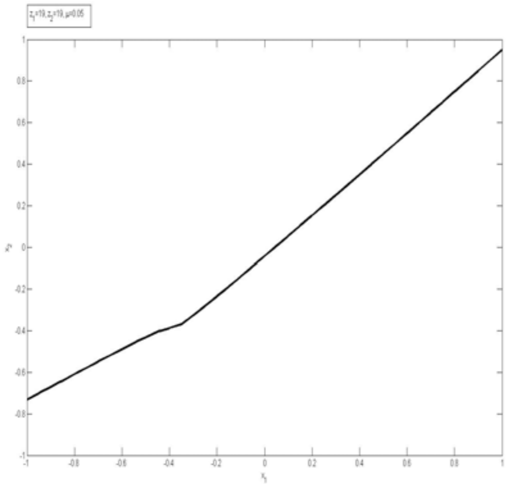

Fig. 2. The intersection curve of the two surfaces for z_1=19, z_2=19 and μ=0.05.

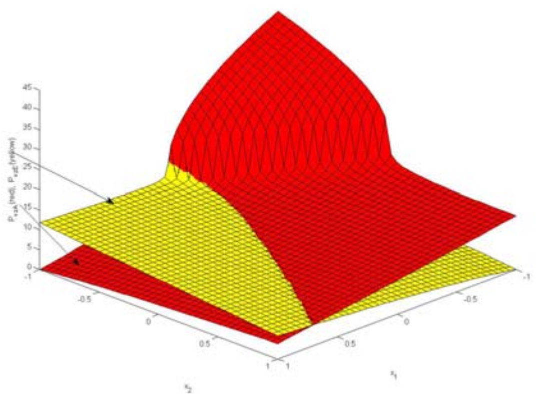

Fig. 3. Intersection of the (1) and (2) surfaces for z_1=19, z_2=33 and μ=0.05.

Although numerical and graphical results confirm the possibility of equalizing there are additional conditions that must be satisfied in order to use in real life the obtained solutions. These refer to the meshing angle and the limitations of the x_1 and x_2 values in order to avoid tooth undercut and sharpening.

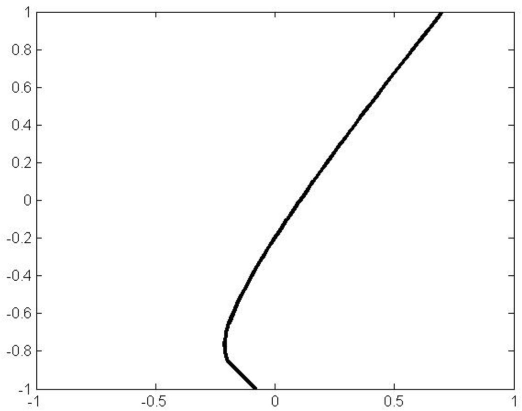

Fig. 4. The intersection curve of the two surfaces for z_1=19, z_2=33 and μ=0.05.

5. EQUALIZATION PLOTS AND NUMERICAL REZULTS

The following plots are showing the dependencies between the x_1 and x_2 addendum modification in the case of the equalized powers lost by friction at the A and E points where the meshing starts and ends.

In order to avoid the tooth undercut and sharpening the following limitations were applied to the obtained x_1, x_2 results [1], [3]:

$$x_{1\min} = \frac{17 - z_1}{17} \tag{12}$$

to avoid tooth undercut and

$$x_{1\max} = \frac{d_{v1}}{2m} - \frac{1}{12} - \frac{z_1}{2} - h_a + k \tag{13}$$

to avoid tooth sharpening, where (13) was obtained imposing the following limitation:

$$d_{a1} \le d_{v1} - \frac{m}{6} \tag{14}$$

with d_{a1} the outer circle diameter of wheel 1, d_{f1} the first (1) peak circle diameter of the sharpened tooth one where the two involute flanks of the tooth meet and m the module of the gear.

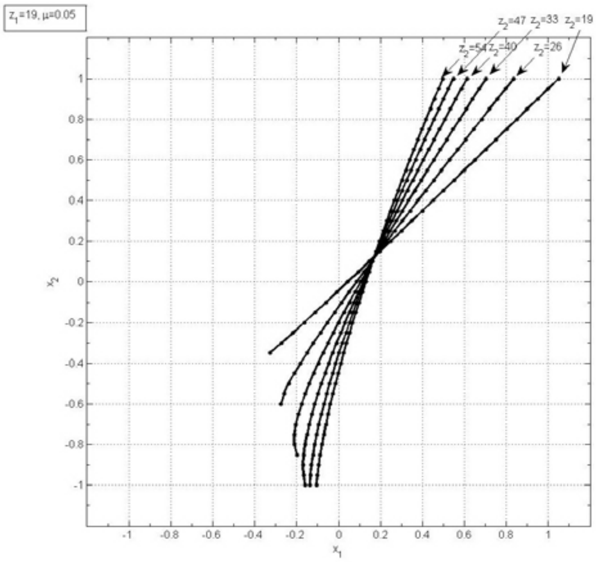

Fig. 5. The x_1 and x_2 plots for equalized powers lost by friction for z_1=19, z_2={19, 26, 33, 40, 47, 54}.

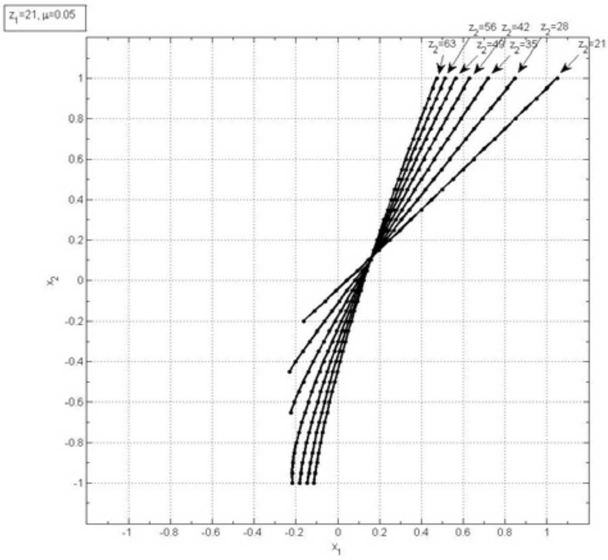

Fig. 8. The x_1 and x_2 plots with limitations for equalized powers lost by friction for z_1=21 z_2={21, 28, … 56, 63}.

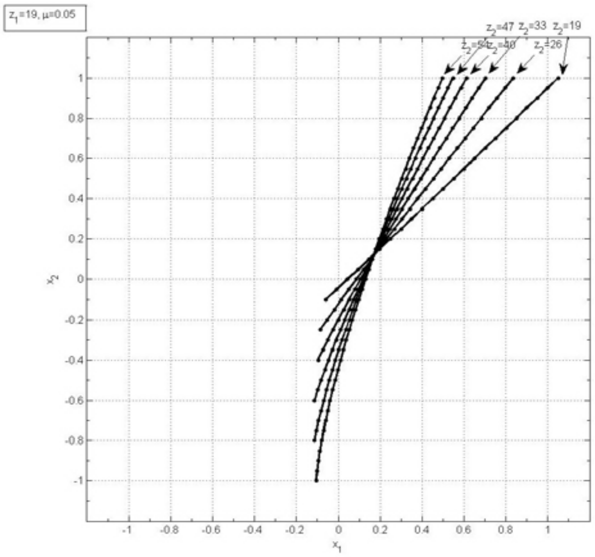

Fig. 6. The x_1 and x_2 plots with limitations for equalized powers lost by friction for z_1=19, z_2={19, …47, 54}.

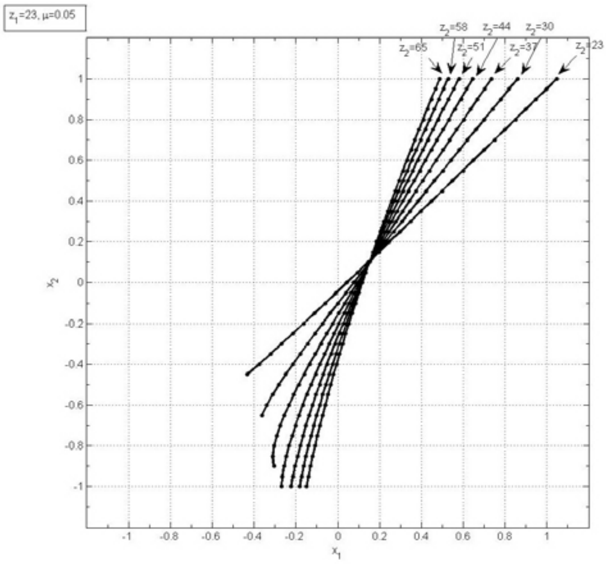

Fig. 9. The x_1 and x_2 plots for equalized powers lost by friction for z_1=23 z_2={23, 30, 37, 44, 51, 58, 65}.

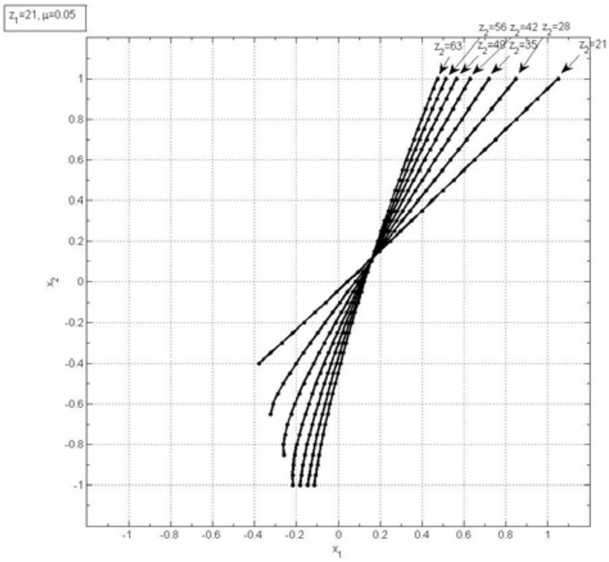

Fig. 7. The x_1 and x_2 plots for equalized powers lost by friction for z_1=21 z_2={21, 28, 35, 42, 49, 56, 63}.

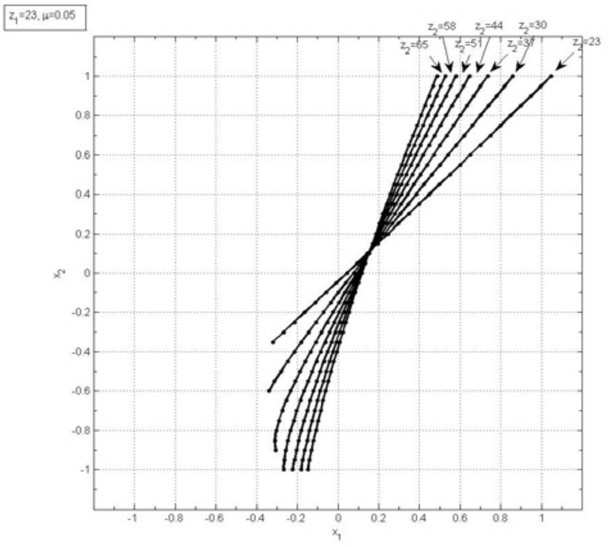

Fig. 10. The x_1 and x_2 plots with limitations for equalized powers lost by friction for z_1=23 z_2={23, 30, … 65}.

The equalization algorithm is based on the following Matlab code:

```
ha=1;a0=deg2rad(20);miua=0.05;
p1=200;rb1=20;
z1=23;
z2=65
x2in=-1.0;
x2fin=1.0;
n=41;
step=(x2fin-x2in)/(n-1);

plotcrt1x1=[];
plotcrt1x2=[];

for i=1:n
 x2=x2in+(i-1)*step;

% find astart for bisection method
% f_p = (1) - (2)

a_w=bisection(@f_p,deg2rad(astart),deg
2rad(astart+astarspas),x2,z1,z2,miua,h
a,a0,p1,rb1);
 if (bisection_converge)
   x1=(inv_f(a_w)-
inv_f(a0))*(z1+z2)/2./tan(a0)-x2;

pa=pvza1(x1,x2,z1,z2,a0,miua,p1,rb1,a_
w);

pe=pvze1(x1,x2,z1,z2,a0,miua,p1,rb1,a_
w);
  if (no_limitations)
   fprintf('%2d %9.5f %9.5f %9.5f
%9.5f %9.5f %9.5f %9.5f
\n',i,x1_min(z1),x1,
x1_max_v1(z1,z2,x1,x2), x2_min(z2),
x2, x2_max_v1(z1,z2,x1,x2),
rad2deg(a_w),pa,pe);
   plotcrt1x1=[plotcrt1x1 ; x1];
   plotcrt1x2=[plotcrt1x2 ; x2];
  end %if
 end %if
end %for

try
 plot(plotcrt1x1,
plotcrt1x2,'Parent',axes1,'Marker','o'
,'MarkerSize',3,'LineWidth',2,'Color',
'k');
end
catch err
end
```

Without the equalization condition the x_1 and x_2 values can be chosen independently, imposing the equalization condition, only one, of the two values, will remain independent. As seen in the algorithm, the x_2 remains independent (it is given), while the x_1 value results from the equalization condition. Table 3 shows the results and the limitations computed by the code:

Table 3

The equalized P_{vzA} and P_{vzE} for z_1=23, z_2=65 and their corresponding x_1, x_2 addendum modifications.

i	x_{1min}	x_1	x_{1max}	x_{2min}	x_2	x_{2max}	α_w (^{0})	$P_{vzA}=P_{vzE}$ (W)
1	-0.35294	-0.14819	0.56938	-2.82353	-1.00000	0.12813	14.38049	9.85731
2	-0.35294	-0.13928	0.55378	-2.82353	-0.95000	0.14827	14.79275	9.79109
3	-0.35294	-0.12969	0.54048	-2.82353	-0.90000	0.17005	15.18663	9.72594
4	-0.35294	-0.11949	0.52931	-2.82353	-0.85000	0.19335	15.56395	9.66154
5	-0.35294	-0.10872	0.52011	-2.82353	-0.80000	0.21805	15.92629	9.59763
6	-0.35294	-0.09741	0.51275	-2.82353	-0.75000	0.24405	16.27500	9.53401
7	-0.35294	-0.08562	0.50712	-2.82353	-0.70000	0.27126	16.61122	9.47054
8	-0.35294	-0.07336	0.50310	-2.82353	-0.65000	0.29961	16.93599	9.40709
9	-0.35294	-0.06067	0.50060	-2.82353	-0.60000	0.32902	17.25018	9.34357
10	-0.35294	-0.04758	0.49955	-2.82353	-0.55000	0.35943	17.55458	9.27990
11	-0.35294	-0.03410	0.49985	-2.82353	-0.50000	0.39078	17.84988	9.21603
12	-0.35294	-0.02026	0.50144	-2.82353	-0.45000	0.42302	18.13672	9.15191
13	-0.35294	-0.00607	0.50425	-2.82353	-0.40000	0.45609	18.41563	9.08750
14	-0.35294	0.00845	0.50823	-2.82353	-0.35000	0.48996	18.68714	9.02278
15	-0.35294	0.02328	0.51333	-2.82353	-0.30000	0.52458	18.95167	8.95771
16	-0.35294	0.03841	0.51948	-2.82353	-0.25000	0.55992	19.20966	8.89229
17	-0.35294	0.05382	0.52664	-2.82353	-0.20000	0.59594	19.46146	8.82650
18	-0.35294	0.06951	0.53478	-2.82353	-0.15000	0.63261	19.70742	8.76034
19	-0.35294	0.08546	0.54385	-2.82353	-0.10000	0.66991	19.94785	8.69378
20	-0.35294	0.10167	0.55381	-2.82353	-0.05000	0.70780	20.18303	8.62684
21	-0.35294	0.11812	0.56463	-2.82353	0.00000	0.74626	20.41323	8.55950
22	-0.35294	0.13480	0.57628	-2.82353	0.05000	0.78527	20.63869	8.49177

23	-0.35294	0.15171	0.58873	-2.82353	0.10000	0.82481	20.85964	8.42364
24	-0.35294	0.16884	0.60194	-2.82353	0.15000	0.86485	21.07627	8.35512
25	-0.35294	0.18618	0.61590	-2.82353	0.20000	0.90537	21.28879	8.28620
26	-0.35294	0.20372	0.63057	-2.82353	0.25000	0.94636	21.49738	8.21690
27	-0.35294	0.22146	0.64594	-2.82353	0.30000	0.98781	21.70219	8.14720
28	-0.35294	0.23940	0.66197	-2.82353	0.35000	1.02969	21.90340	8.07712
29	-0.35294	0.25752	0.67866	-2.82353	0.40000	1.07198	22.10113	8.00666
30	-0.35294	0.27582	0.69597	-2.82353	0.45000	1.11469	22.29554	7.93582
31	-0.35294	0.29429	0.71389	-2.82353	0.50000	1.15778	22.48675	7.86461
32	-0.35294	0.31294	0.73240	-2.82353	0.55000	1.20126	22.67488	7.79304
33	-0.35294	0.33176	0.75149	-2.82353	0.60000	1.24510	22.86004	7.72109
34	-0.35294	0.35073	0.77114	-2.82353	0.65000	1.28930	23.04234	7.64879
35	-0.35294	0.36987	0.79133	-2.82353	0.70000	1.33384	23.22188	7.57613
36	-0.35294	0.38916	0.81204	-2.82353	0.75000	1.37872	23.39876	7.50311
37	-0.35294	0.40860	0.83327	-2.82353	0.80000	1.42393	23.57307	7.42976
38	-0.35294	0.42818	0.85500	-2.82353	0.85000	1.46945	23.74489	7.35606
39	-0.35294	0.44791	0.87721	-2.82353	0.90000	1.51528	23.91431	7.28202
40	-0.35294	0.46778	0.89991	-2.82353	0.95000	1.56141	24.08140	7.20764
41	-0.35294	0.48779	0.92306	-2.82353	1.00000	1.60783	24.24623	7.13294

4. CONCLUSION

The values of the specific addendum modifications x_1 and x_2 determined by this equalization procedure provide uniform wearing of tooth flanks at the points where the meshing ends and begins during movement and load transmission.

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Deplasările specifice de profil ale angrenajelor cilindrice cu dinţi drepţi determinate în condiţiile egalizării puterilor pierdute prin frecare în punctele de început şi de sfârşit ale angrenării

Rezumat: Lucrarea îşi propune determinarea deplasărilor specifice de profil, în condiţiile egalizării expresiilor puterilor pierdute prin frecare, în punctele în care angrenarea începe (A) şi se termină (E). Rezultatele numerice din Matlab confirmă posibilitatea egalizării acestor expresii şi permite determinarea deplasărilor specifice de profil în condiţiile evitării fenomenelor de subtăiere, la baza dintelui şi de ascuţire, la capul dintelui.

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