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THE DYNAMIC STUDY OF TRTTRR1 MODULAR SERIAL ROBOT

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Abstract: *The dynamic equations of TRTTRR1 modular serial robot are established in this paper, using Lagrangian formulation. After the description of the robot's structural kinematic scheme and of the used geometric, kinematic and dynamic amounts, starting from the robot's base, the kinetic energies corresponding to the robot's modules and to the gripper are successively determined. The whole robot kinetic energy is obtained by summing up the kinetic energies of the modules from the robot's structure. Following the robot's structural kinematic scheme, the virtual elementary displacements from the joints are determined as well as the elementary virtual work and the generalized driving forces. Using Lagrange's equations of the second kind and the determined mechanical amounts (kinetic energy, generalized forces), the differential equations of TRTTRR1 robot are established. Three optimisation variants of the construction and operation of the robot are specified, based on solving the inverse problem of the dynamics of this robot.*

Key words: *serial robot, dynamics, Lagrange's equation.*

1. TRTTRR1 MODULAR SERIAL ROBOT – DYNAMIC STUDY

1.1 The Dynamic Equations of TRTTRR1 Robot Using Lagrangian Formulation

The dynamic modelling of robots is frequently achieved using Lagrangian formulation. Fig. 1 presents the structural kinematic scheme of TRTTRR1 modular serial industrial robot. Starting from the robot's base, the following parts can be mentioned:

- the module 1 (MTB) is the translation module from the robot's base;
- the module 2 (MRB) is the rotation module of the robot arm;
- the module 3 (MTV) is the vertical translation module;
- the module 4 (MT) is the translation module of the gripper (DP);
- the module 5 (MR) is the rotation module from the arm structure;
- the orientation module (MO) is the orientation module of the gripper.

In the robot's structural kinematic diagram, according to [1], the following notations are used:

Δ_i , $i = 1 \div 6$ – motion axes;

Δ_7 – axis parallel to robot's arm rotation axis Δ_2, passing through the mass centre C_7 of the gripper;

$k = 1 \div 6$ – degrees of freedom;

$q_k, \dot{q}_k, \ddot{q}_k$ – generalized coordinates, velocities and accelerations;

l_0, l_i, $i = 1 \div 7$ – constructive parameters of the robot; m_i, $i = 1 \div 7$ – masses;

O_i, $i = 1 \div 6$ – origins of the mobile frames coincident to the mass centres of the modules;

$\overline{G}_i$, $i = 1 \div 7$ – gravity forces of the robot's modules and of the gripper;

$\overline{F}_1$, $\overline{F}_3$, $\overline{F}_4$ – driving forces;

$\overline{M}_2$, $\overline{M}_5$, $\overline{M}_6$ – driving moments;

O_0 – measuring base (zero reference point);

$O_0x_0y_0z_0$ – fixed Cartesian reference frame;

$O_ix_iy_iz_i$, $i = 1 \div 6$ – mobile Cartesian reference frames, rigidly fixed to the mobile parts of the robot's modules;

$Cx_7y_7z_7$ – mobile reference frame, rigidly fixed to the gripper, having the origin C_7 into the mass centre of DP;

$J_{\Delta_2}^{(2)}$– mechanical moment of inertia of the mobile part of MRB module, determined with respect to Δ_2 rotation axis;

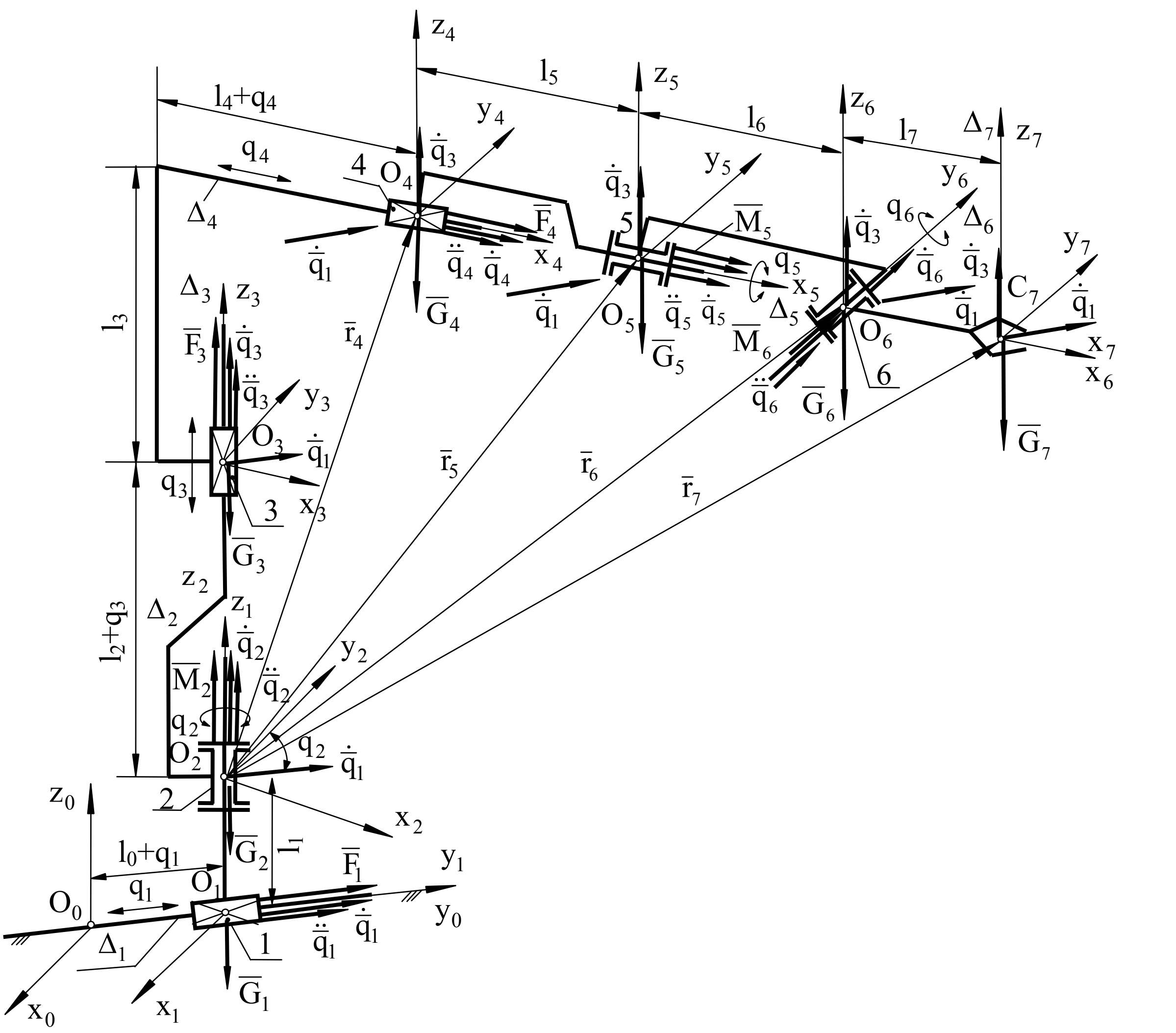

Fig.1. The structural kinematic scheme of TRTTRR1 modular serial industrial robot, used for the dynamic study

$J_{\Delta_2}^{(3)}$ – mechanical moment of inertia of MTV translation module, determined with respect to Δ_2 axis;

$J_{z_4}^{(4)}$ – mechanical moment of inertia of MT translation module, determined with respect to $O_4 z_4$ axis;

$J_{\Delta_5}^{(5)}$ – mechanical moment of inertia of the mobile part of MO orientation module, determined with respect to Δ_5 rotation axis;

$J_{z_5}^{(5)}$ – mechanical moment of inertia of MO module, determined with respect to O_5 z_5 axis;

$J_{z_6}^{(6)}$ – mechanical moment of inertia of MO module, determined with respect to $O_6 z_6$ axis;

$J_{x_6}^{(6)}$ – mechanical moment of inertia of MO module, determined with respect to $O_6 x_6$ axis;

$J_{\Delta_6}^{(6)}$ – mechanical moment of inertia of MO module, determined with respect to Δ_6 axis;

$J_{x_7}^{(7)}$ – mechanical moment of inertia of the gripper, determined with respect to $C_7 x_7$ axis;

$J_{y_7}^{(7)}$ – mechanical moment of inertia of the gripper, determined with respect to $C_7 y_7$ axis.

The motion differential equations of the robot can be determined according to [2] and [3], using 2nd kind Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_k}\right)-\frac{\partial Ec}{\partial q_k}=Q_k,\ k=1\div 6. \quad (1)$$

The following notations are involved in equation (1):

Ec – robot's kinetic energy;

$q_k, \dot{q}_k$ – generalized coordinates and velocities;

Q_k – generalized forces;

k – number of degrees of freedom.

According to [2], the kinetic energy of a module i from the robot's mechanical structure has the expression:

$$E_{c_i}=\frac{1}{2}M_i\left(v_x^2+v_y^2+v_z^2\right)_i+\frac{1}{2}\left(J_x\omega_x^2+J_y\omega_y^2+J_z\omega_z^2\right)_i, \quad i=1\div 7, \quad (2)$$

where $\omega_x, \omega_y, \omega_z$ are the Cartesian components of the angular velocity, v_x, v_y, v_z are the Cartesian components of the mass centre velocity of the element i, and J_x, J_y, J_z are the principal axial moments of inertia of the element i.

In order to express the kinetic energy of an element with the relation (2), the mass centre has to coincide to the origin of the reference frame $Oxyz$, such that $x_c = y_c = z_c = 0$ and the frame $Oxzy$ must be the principal frame of inertia, i.e. $J_{xy}=J_{yz}=J_{zx}=0$.

Starting from the robot's base and following fig.1 and the relation (2), the kinetic energies of the robot's modules and gripper can be successively determined as follows:

- for the modules MTB, MRB, MTV and MT, the expressions of the kinetic energy are determined with the relation (2) and they are:

$$\begin{aligned}
&E_{c_1}=\frac{1}{2}m_1\dot{q}_1^2,\\
&E_{c_2}=\frac{1}{2}m_2\dot{q}_1^2+\frac{1}{2}J_{\Delta_2}^{(2)}\dot{q}_2^2,\\
&E_{c_3}=\frac{1}{2}m_3\left(\dot{q}_1^2+\dot{q}_3^2\right)+\frac{1}{2}J_{\Delta_2}^{(3)}\dot{q}_2^2,\\
&E_{c_4}=\frac{1}{2}m_4\left(\dot{q}_1^2+\dot{q}_3^2+\dot{q}_4^2\right)+\\
&+\frac{1}{2}\left[J_{z_4}^{(4)}+m_4\left(l_4+q_4\right)^2\right]\dot{q}_2^2+\\
&+m_4\left[\left(l_4+q_4\right)cq_2\dot{q}_1\dot{q}_2+sq_2\dot{q}_1\dot{q}_4\right];
\end{aligned} \quad (3)$$

- for the rotation module MR, the kinematic parameters that describe the motion are:

$$\begin{aligned}
&\omega_{x_5}=\dot{q}_5,\ \omega_{y_5}=0,\ \omega_{z_5}=\dot{q}_2;\\
&\bar{v}_{O_5}=\dot{\bar{q}}_1+\dot{\bar{q}}_2\times\bar{r}_5+\dot{\bar{q}}_3+\dot{\bar{q}}_4,\\
&\bar{r}_5=\bar{l}_2+\bar{q}_3+\bar{l}_3+\bar{l}_4+\bar{q}_4+\bar{l}_5,\\
&\dot{\bar{q}}_2\times\bar{r}_5=\dot{\bar{q}}_2\times\left(\bar{l}_4+\bar{q}_4+\bar{l}_5\right),\\
&\bar{v}_{O_5}=\dot{\bar{q}}_1+\dot{\bar{q}}_2\times\left(\bar{l}_4+\bar{q}_4+\bar{l}_5\right)+\dot{\bar{q}}_3+\dot{\bar{q}}_4,\\
&v_{O_5}^2=\dot{q}_1^2+\left(l_4+l_5+q_4\right)^2\dot{q}_2^2+\dot{q}_3^2+\dot{q}_4^2+\\
&+2\left(l_4+l_5+q_4\right)cq_2\dot{q}_1\dot{q}_2+2sq_2\dot{q}_1\dot{q}_4,
\end{aligned} \quad (4)$$

the other terms of the expression being zero, because they are dot products of pairs of perpendicular vectors, and the principal mechanical moments of inertia are:

$$J_{x_5}=J_{\Delta_5}^{(5)},\quad J_{y_5}=J_{y_5}^{(5)},\quad J_{z_5}=J_{z_5}^{(5)}. \quad (5)$$

The kinetic energy of the rotation module, considering (2), (4) and (5), is expressed as:

$$\begin{aligned}
&E_{c_5}=\frac{1}{2}m_5\left(\dot{q}_1^2+\dot{q}_3^2+\dot{q}_4^2\right)+\\
&+\frac{1}{2}\left[J_{z_5}^{(5)}+m_5\left(l_4+l_5+q_4\right)^2\right]\dot{q}_2^2+\\
&+\frac{1}{2}J_{\Delta_5}^{(5)}\dot{q}_5^2+m_5\left(l_4+l_5+q_4\right)cq_2\dot{q}_1\dot{q}_2+\\
&+m_5sq_2\dot{q}_1\dot{q}_2.
\end{aligned} \quad (6)$$

- for the orientation module MO, the kinematic parameters that describe the motion are:

$$\omega_{x_6}=\dot{q}_5,\ \omega_{y_6}=\dot{q}_6,\ \omega_{z_6}=\dot{q}_2,$$

$$\bar{v}_{O_6}=\dot{\bar{q}}_1+\dot{\bar{q}}_2\times\bar{r}_6+\dot{\bar{q}}_3+\dot{\bar{q}}_4.$$

Following fig.1 and fig.2, the below relations can be written:

$$\bar{r}_6=\bar{l}_2+\bar{q}_3+\bar{l}_3+\bar{l}_4+\bar{q}_4+\bar{l}_5+\bar{l}_6,$$

$$\dot{\bar{q}}_2\times\bar{r}_6=\dot{\bar{q}}_2\times\left(\bar{l}_4+\bar{l}_5+\bar{l}_6+\bar{q}_4\right),$$

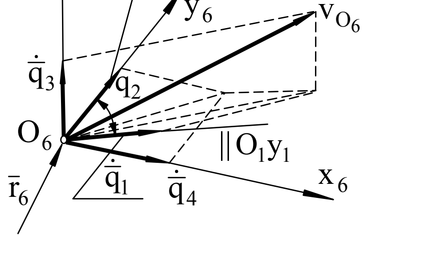

Fig.2. The velocity of the point O_6

$$\bar{v}_{O_6} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4\right) + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 , \qquad (7)$$

$$v_{O_6}^2 = \dot{q}_1^2 + (l_4 + l_5 + l_6 + q_4)^2 \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_4^2 + \\ + 2(l_4 + l_5 + l_6 + q_4) cq_2 \dot{q}_1 \dot{q}_2 + 2 sq_2 \dot{q}_1 \dot{q}_4 ,$$

because

$$\dot{\bar{q}}_3 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4\right)\right] = 0, \quad \dot{\bar{q}}_1 \cdot \dot{\bar{q}}_3 = 0,$$

$$\dot{\bar{q}}_3 \cdot \dot{\bar{q}}_4 = 0, \quad \dot{\bar{q}}_4 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4\right)\right] = 0,$$

the respective vectors being perpendicular (fig. 2).

The principal moments of inertia corresponding to this module are:

$$J_{x_6} = J_{x_6}^{(6)}, \quad J_{y_6} = J_{\Delta_6}^{(6)}, \quad J_{z_6} = J_{z_6}^{(6)} . \qquad (8)$$

The kinetic energy of the orientation module MO is obtained by replacing into the relation (2) the notations from the relations (7), (8). Therefore,

$$\begin{aligned} E_{c_6} &= \frac{1}{2} m_6 \left(\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2\right) + \\ &+ \frac{1}{2}\left[J_{z_6}^{(6)} + m_6 (l_4 + l_5 + l_6 + q_4)^2\right]\dot{q}_2^2 + \\ &+ \frac{1}{2} J_{x_6}^{(6)} \dot{q}_5^2 + \frac{1}{2} J_{\Delta_6}^{(6)} \dot{q}_6^2 + \\ &+ m_6 (l_4 + l_5 + l_6 + q_4) cq_2 \dot{q}_1 \dot{q}_2 + m_6 sq_2 \dot{q}_1 \dot{q}_4 \ ; \end{aligned} \qquad (9)$$

- for the gripper, the kinematic parameters that describe the motion are:

$$\begin{aligned} &\omega_{x_7} = \dot{q}_5 , \ \omega_{y_7} = \dot{q}_6 , \ \omega_{z_7} = \dot{q}_2 , \\ &\bar{v}_{C_7} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \bar{r}_7 + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 + \dot{\bar{q}}_6 \times \bar{l}_7 , \\ &\bar{r}_7 = \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 , \\ &\dot{\bar{q}}_2 \times \bar{r}_7 = \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 + \bar{q}_4\right), \\ &\bar{v}_{C_7} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 + \bar{q}_4\right) + \\ &\qquad + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 + \dot{\bar{q}}_6 \times \bar{l}_7 , \end{aligned} \qquad (10)$$

$$\begin{aligned} v_{c_7}^2 &= \dot{q}_1^2 + (l_4 + l_5 + l_6 + l_7 + q_4)^2 \dot{q}_2^2 + \dot{q}_3^2 + \\ &+ \dot{q}_4^2 + l_7^2 \dot{q}_6^2 + 2(l_4 + l_5 + l_6 + l_7 + q_4) \cdot \\ &\cdot \dot{q}_1 \dot{q}_2 cq_2 + 2 sq_2 \dot{q}_1 \dot{q}_4 - 2 l_7 \dot{q}_3 \dot{q}_6 , \end{aligned}$$

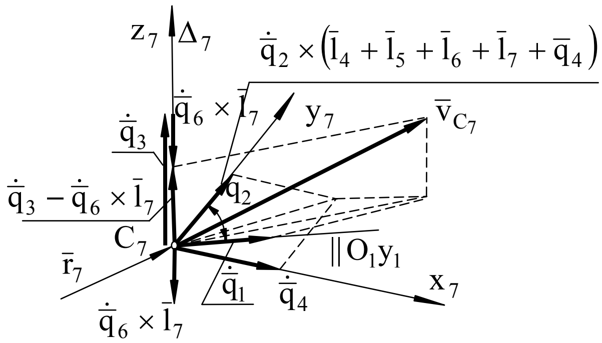

Fig.3. The velocity of the mass centre C_7

because the dot products

$$\dot{\bar{q}}_1 \cdot \dot{\bar{q}}_3 , \ \dot{\bar{q}}_1 \cdot \left(\dot{\bar{q}}_6 \times l_7\right), \dot{\bar{q}}_3 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 + \bar{q}_4\right)\right],$$

$$\dot{\bar{q}}_4 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 + \bar{q}_4\right)\right], \\ \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 + \bar{q}_4\right)\right] \cdot \left(\dot{\bar{q}}_6 \times l_7\right),$$

$$\dot{\bar{q}}_3 \cdot \dot{\bar{q}}_4 , \ \dot{\bar{q}}_4 \cdot \left(\dot{\bar{q}}_5 \times l_7\right)$$

are zero, the respective vectors being perpendicular (fig. 3).

The principal moments of inertia for the gripper are:

$$J_{x_7} = J_{x_7}^{(7)}, \ J_{y_7} = J_{y_7}^{(7)}, \ J_{z_7} = J_{\Delta_7}^{(7)} . \qquad (11)$$

Considering (2), (10) and (11), the kinetic energy of the gripper has the value:

$$\begin{aligned} E_{c_7} &= \frac{1}{2} m_7 \left(\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2\right) + \\ &+ \frac{1}{2}\left[J_{\Delta_7}^{(7)} + m_7 (l_4 + l_5 + l_6 + l_7 + q_4)^2\right]\dot{q}_2^2 + \\ &+ \frac{1}{2} J_{x_7}^{(7)} \dot{q}_5^2 + \frac{1}{2}\left[J_{y_7}^{(7)} + m_7 l_7^2\right]\dot{q}_6^2 + \\ &+ m_7 \left[(l_4 + l_5 + l_6 + l_7 + q_4) cq_2 \dot{q}_1 \dot{q}_2 + \right. \\ &\left. + sq_2 \dot{q}_1 \dot{q}_4 - l_7 \dot{q}_3 \dot{q}_3 \right]. \end{aligned} \qquad (12)$$

By summing up the kinetic energies of the component modules, the kinetic energy of TRTTRR1 robot is obtained. Considering (3), (6), (9), (12), the following expression is obtained:

$$E_c = \frac{1}{2}\left(\sum_{i=1}^{7} m_i\right)\dot{q}_1^2 + \frac{1}{2}\left[\sum_{i=2}^{3} J_{\Delta_2}^{(i)} + \sum_{i=4}^{6} J_{z_i}^{(i)} + J_{\Delta_7}^{(7)} + m_4(l_4+q_4)^2 + m_5(l_4+l_5+q_4)^2 + \right.$$
$$\left. + m_6(l_4+l_5+l_6+q_4)^2 + m_7(l_4+l_5+l_6+l_7+q_4)^2\right]\dot{q}_2^2 + \frac{1}{2}\left(\sum_{i=3}^{7} m_i\right)\dot{q}_3^2 +$$
$$+ \frac{1}{2}\left(\sum_{i=4}^{7} m_i\right)\dot{q}_4^2 + \frac{1}{2}\left[J_{\Delta_5}^{(5)} + \sum_{i=6}^{7} J_{x_i}^{(i)}\right]\dot{q}_5^2 + \frac{1}{2}\left[J_{\Delta_6}^{(6)} + J_{y_7}^{(7)} + m_7 l_7^2\right]\dot{q}_6^2 + \tag{13}$$
$$+ \left[m_4(l_4+q_4) + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4) + \right.$$
$$\left. + m_7(l_4+l_5+l_6+l_7+q_4)\right] cq_2 \dot{q}_1\dot{q}_2 + \left(\sum_{i=4}^{7} m_i\right) sq_2 \dot{q}_1\dot{q}_4 - m_7 l_7 \dot{q}_3\dot{q}_6 .$$

The following notations are used in (13):

$$\frac{1}{2}\left(\sum_{i=1}^{7} m_i\right) = c_1,$$
$$\frac{1}{2}\left[\sum_{i=2}^{3} J_{\Delta_2}^{(i)} + \sum_{i=4}^{6} J_{z_i}^{(i)} + J_{\Delta_7}^{(7)} + m_4(l_4+q_4)^2 + m_5(l_4+l_5+q_4)^2 + m_6(l_4+l_5+l_6+q_4)^2 + \right.$$
$$\left. + m_7(l_4+l_5+l_6+l_7+q_4)^2\right] = c_2,\ \frac{1}{2}\left(\sum_{i=3}^{7} m_i\right) = c_3,\ \frac{1}{2}\left(\sum_{i=4}^{7} m_i\right) = c_4, \tag{14}$$
$$\frac{1}{2}\left[J_{\Delta_5}^{(5)} + \sum_{i=6}^{7} J_{x_i}^{(i)}\right] = c_5,\ \frac{1}{2}\left[J_{\Delta_6}^{(6)} + J_{y_7}^{(7)} + m_7 l_7^2\right] = c_6,$$
$$\left[m_4(l_4+q_4) + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)\right] cq_2 = c_{12},$$
$$\left(\sum_{i=4}^{7} m_i\right) sq_2 = c_{14},\ -m_7 l_7 = c_{36}.$$

Using the above notations, the robot's kinetic energy becomes:

$$E_c = c_1\dot{q}_1^2 + c_2\dot{q}_2^2 + c_3\dot{q}_3^2 + c_4\dot{q}_4^2 + c_5\dot{q}_5^2 + c_6\dot{q}_6^2 + c_{12}\dot{q}_1\dot{q}_2 + c_{14}\dot{q}_1\dot{q}_4 + c_{36}\dot{q}_3\dot{q}_6 . \tag{15}$$

Observing the equations (1), (14), (15), the following specifications are achieved:

$$\frac{\partial E_c}{\partial q_1} = 0,\ \frac{\partial E_c}{\partial q_2} = \frac{\partial c_{12}}{\partial q_2}\dot{q}_1\dot{q}_2 + \frac{\partial c_{14}}{\partial q_2}\dot{q}_1\dot{q}_4 = -\left[m_4(l_4+q_4) + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4) + \right.$$
$$\left. + m_7(l_4+l_5+l_6+l_7+q_4)\right] sq_2\dot{q}_1\dot{q}_2 + \left(\sum_{i=4}^{7} m_i\right) cq_2\dot{q}_1\dot{q}_4,\ \frac{\partial E_c}{\partial q_3} = 0,\ \frac{\partial E_c}{\partial q_4} = \frac{\partial c_2}{\partial q_4}\dot{q}_2^2 + \frac{\partial c_{12}}{\partial q_4}\dot{q}_1\dot{q}_2 = \tag{16}$$
$$= \left[m_4(l_4+q_4) + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)\right]\dot{q}_2^2 +$$
$$+ \left(\sum_{i=4}^{7} m_i\right) cq_2\dot{q}_1\dot{q}_2,\ \frac{\partial E_c}{\partial q_5} = 0,\ \frac{\partial E_c}{\partial q_6} = 0 ;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_1}\right)=2c_1\ddot{q}_1+\frac{dc_{12}}{dt}\dot{q}_2+c_{12}\ddot{q}_2+\frac{dc_{14}}{dt}\dot{q}_4+c_{14}\ddot{q}_4=\left(\sum_{i=1}^{7}m_i\right)\ddot{q}_1+\left[m_4\left(l_4+q_4\right)+\right.$$
$$+m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right)+\left.m_7\left(l_4+l_5+l_6+l_7+q_4\right)\right]cq_2\ddot{q}_2+\left(\sum_{i=4}^{7}m_i\right)sq_2\ddot{q}_4-$$
$$-\left[m_4\left(l_4+q_4\right)+m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right)+m_7\left(l_4+l_5+l_6+l_7+q_4\right)\right]sq_2\dot{q}_2^2+$$
$$+2\left(\sum_{i=4}^{7}m_i\right)cq_2\dot{q}_2\dot{q}_4;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_2}\right)=\left[m_4\left(l_4+q_4\right)+m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right)+m_7\left(l_4+l_5+l_6+l_7+q_4\right)\right]\cdot$$
$$\cdot cq_2\ddot{q}_1+\left[\sum_{i=2}^{3}J_{\Delta_2}^{(i)}+\sum_{i=4}^{6}J_{z_i}^{(i)}+J_{\Delta_7}^{(7)}+m_4\left(l_4+q_4\right)^2+m_5\left(l_4+l_5+q_4\right)^2+m_6\left(l_4+l_5+l_6+q_4\right)^2+\right.$$
$$\left.+m_7\left(l_4+l_5+l_6+l_7+q_4\right)^2\right]\ddot{q}_2-\left[m_4\left(l_4+q_4\right)+m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right)+\right.$$
$$\left.+m_7\left(l_4+l_5+l_6+l_7+q_4\right)\right]sq_2\dot{q}_1\dot{q}_2+\left(\sum_{i=4}^{7}m_i\right)cq_2\dot{q}_1\dot{q}_4+2\left[m_4\left(l_4+q_4\right)+m_5\left(l_4+l_5+q_4\right)+\right.$$
$$\left.+m_6\left(l_4+l_5+l_6+q_4\right)+m_7\left(l_4+l_5+l_6+l_7+q_4\right)\right]\dot{q}_2\dot{q}_4;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_3}\right)=2c_3\ddot{q}_3+c_{36}\ddot{q}_6=\left(\sum_{i=3}^{7}m_i\right)\ddot{q}_3-m_7l_7\ddot{q}_6;\tag{17}$$
$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_4}\right)=c_{14}\ddot{q}_1+2c_4\ddot{q}_4+\frac{dc_{14}}{dt}\dot{q}_1=\left(\sum_{i=4}^{7}m_i\right)\left(sq_2\ddot{q}_1+\ddot{q}_4+cq_2\dot{q}_1\dot{q}_2\right);$$
$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_5}\right)=\left[J_{\Delta_5}^{(5)}+\sum_{i=6}^{7}J_{x_i}^{(i)}\right]\ddot{q}_5;\quad \frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_6}\right)=\left[J_{\Delta_6}^{(6)}+J_{y_7}^{(7)}+m_7l_7^2\right]\ddot{q}_6-m_7l_7\ddot{q}_3.$$

Virtual displacements are performed, such that the parameters q_k will modify with $\delta q_1, \delta q_2, \ldots, \delta q_5, \delta q_6$, respectively, one by one, in order to determine the generalized forces corresponding to the six robot's modules, after that the virtual elementary work is determined.

According to fig. 1, the virtual work corresponding to the external forces and moments and to the elementary virtual displacement compatible to the robot's connections has the following expression:

$$\begin{aligned}\delta L=&F_1\delta q_1+M_2\delta q_2+\left(F_3-G_3\right)\delta q_3+\\&+\left(\overline{F}_4+\overline{G}_4\right)\cdot\delta\overline{O_0O_4}+\overline{G}_5\cdot\delta\overline{O_0O_5}+\\&+M_5\delta q_5+\overline{G}_6\cdot\delta\overline{O_0O_6}+\\&+M_6\delta q_6+\overline{G}_7\cdot\delta\overline{O_0C_7}.\end{aligned}\tag{18}$$

Considering that:

$$\overline{O_0O_4}=\bar{l}_0+\bar{q}_1+\bar{l}_1+\bar{l}_2+\bar{q}_3+\bar{l}_3+\bar{l}_4+\bar{q}_4,$$
$$\delta\overline{O_0O_4}=\delta\bar{q}_1+\delta\bar{q}_3+\delta\bar{q}_4$$
$$\overline{O_0O_5}=\bar{l}_0+\bar{q}_1+\bar{l}_1+\bar{l}_2+\bar{q}_3+\bar{l}_3+\bar{l}_4+\bar{q}_4+\bar{l}_5,$$
$$\delta\overline{O_0O_5}=\delta\bar{q}_1+\delta\bar{q}_3+\delta\bar{q}_4$$
$$\begin{aligned}\overline{O_0O_6}=&\bar{l}_0+\bar{q}_1+\bar{l}_1+\bar{l}_2+\bar{q}_3+\bar{l}_3+\bar{l}_4+\\&+\bar{q}_4+\bar{l}_5+\bar{l}_6,\ \delta\overline{O_0O_6}=\delta\bar{q}_1+\delta\bar{q}_3+\delta\bar{q}_4\end{aligned}\tag{19}$$

$$\overline{O_0C_7} = \bar{l}_0 + \bar{q}_1 + \bar{l}_1 + \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \\ + \bar{q}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7, \ \delta\overline{O_0C_7} = \delta\bar{q}_1 + \delta\bar{q}_3 + \delta\bar{q}_4,$$

the expression (18) of the virtual work reaches the form:

$$\delta L = (F_1 + F_4 s q_2)\delta q_1 + M_2 \delta q_2 + \\ + \left(F_3 - \sum_{i=3}^{7} G_i\right)\delta q_3 + F_4 \delta q_4 + \\ + M_5 \delta q_5 + M_6 \delta q_6. \qquad (20)$$

The generalized forces, according to (2), are obtained from the relations:

$$\mathrm{Q_k} = \frac{\delta \mathrm{L}}{\delta \mathrm{q_k}}, \quad \mathrm{k} = 1 \div 6. \qquad (21)$$

If the robot has holonomous connections, these connections do not explicitly depend on the generalized velocities and, consequently, they do not depend on displacements also. For this reason, the elementary displacements are independent, therefore they can be considered zero, with one exception. Thus, if $\delta q_1 \neq 0$, then $\delta q_2 = \delta q_3 = \ldots = \delta q_5 = \delta q_6 = 0$, if $\delta q_1 = \delta q_3 = \ldots$ $\ldots = \delta q_5 = \delta q_6 = 0$, then $\delta q_2 \neq 0$, and so on.

Considering this approach and the relations (20 and (21), the generalized forces are obtained and their expressions are:

$$Q_1 = F_1 + F_4 s q_2, \quad Q_2 = M_2, \\ Q_3 = F_3 - \sum_{i=3}^{7} G_i, \quad Q_4 = F_4, \\ Q_5 = M_5, \quad Q_6 = M_6. \qquad (22)$$

The robot's differential equations are obtained from (1), after substituting the relations (16), (17), (22). Therefore,

$$\left(\sum_{i=1}^{7} m_i\right)\ddot{q}_1 + [m_4(l_4 + q_4) + m_5(l_4 + l_5 + q_4) + m_6(l_4 + l_5 + l_6 + q_4) + \\ + m_7(l_4 + l_5 + l_6 + l_7 + q_4)]cq_2\ddot{q}_2 + \left(\sum_{i=4}^{7} m_i\right)sq_2\ddot{q}_4 - [m_4(l_4 + q_4) + m_5(l_4 + l_5 + q_4) + \\ + m_6(l_4 + l_5 + l_6 + q_4) + m_7(l_4 + l_5 + l_6 + l_7 + q_4)]sq_2\dot{q}_2^2 + 2\left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_2\dot{q}_4 = F_1 + F_4 sq_2$$

$$[m_4(l_4 + q_4) + m_5(l_4 + l_5 + q_4) + m_6(l_4 + l_5 + l_6 + q_4) + m_7(l_4 + l_5 + l_6 + l_7 + q_4)]cq_2\ddot{q}_1 + \\ + \left[\sum_{i=2}^{3} J_{\Delta_2}^{(i)} + \sum_{i=4}^{6} J_{z_i}^{(i)} + m_4(l_4 + q_4)^2 + m_5(l_4 + l_5 + q_4)^2 + m_6(l_4 + l_5 + l_6 + q_4)^2 + \right. \\ \left. + m_7(l_4 + l_5 + l_6 + l_7 + q_4)^2\right]\ddot{q}_2 + 2[m_4(l_4 + q_4) + m_5(l_4 + l_5 + q_4) + m_6(l_4 + l_5 + l_6 + q_4) + \\ + m_7(l_4 + l_5 + l_6 + l_7 + q_4)]\dot{q}_2\dot{q}_4 = M_2$$

$$\left(\sum_{i=3}^{7} m_i\right)\ddot{q}_3 - m_7 l_7 \ddot{q}_6 = F_3 - \sum_{i=3}^{7} G_i \qquad (23)$$

$$\left(\sum_{i=4}^{7} m_i\right)(sq_1\ddot{q}_1 + \ddot{q}_4 + cq_2\dot{q}_1\dot{q}_2) - [m_4(l_4 + q_4) + m_5(l_4 + l_5 + q_4) + m_6(l_4 + l_5 + l_6 + q_4) + \\ + m_7(l_4 + l_5 + l_6 + l_7 + q_4)]\dot{q}_2^2 - \left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_1\dot{q}_2 = F_4$$

$$\left[J_{\Delta_5}^{(5)} + \sum_{i=6}^{7} J_{x_i}^{(i)}\right]\ddot{q}_5 = M_5$$

$$-m_7 l_7 \ddot{q}_3 + \left[J_{\Delta_6}^{(6)} + J_{y_7}^{(7)} + m_7 l_7^2\right]\ddot{q}_6 = M_6 .$$

The differential equation system (23) of TRTTRR1 robot was determined with the assumption that all the movements take place simultaneously.

3. CONCLUSION

By solving the inverse problem of the robot's dynamics, it results:

- an optimal way to arrange the modules from the robot structure, in order to have minimal energy consumption;
- an optimal way to choose the motion laws in each robot joint, such that the energy consumption to be minimal;
- a way of choosing the modules actuation engines, considering their composition and the whole robot dynamics.

4. REFERENCES

1. Gui, Ramona-Maria, "*Modelarea geometrică, cinematică şi dinamică a roboților industriali propuşi a fi implementați în fabricarea robotizată a reperelor de tip flanşă* ", Raport de Cercetare II, Cluj-Napoca, 2010.
2. Ispas, V., „*Manipulatoare şi roboți industriali*", Editura Didactică şi Pedagogică, Bucureşti, 2004, ISBN 973-30-1349-8.
3. Ripianu, A., *„Mecanică. Cinematică şi Dinamică."*, Lito IPCN, Cluj-Napoca, 1977.
4. Deteşan, O.A., *"Cercetări privind modelarea, simularea şi construcția miniroboților"*, Teză de doctorat, Universitatea Tehnică din Cluj-Napoca, 2007.
5. Siciliano, B., Khatib, O., "*The Handbook of Robotics*", Springer, Berlin, Heidelberg, Germany, 2008.
6. Paul, R.P., „*Robot Manipulators: Mathematics, Programming and Control*", MIT Press, Cambridge, Mass., 1981.

STUDIUL DINAMIC AL ROBOTULUI SERIAL MODULAR TRTTRR1

Rezumat: În această lucrare se stabilesc ecuațiile dinamice pentru robotul serial modular TRTTRR1, utilizând formalismul lui Lagrange. După prezentarea schemei cinematice structurale a robotului şi precizarea mărimilor geometrice, cinematice şi dinamice utilizate, se determină succesiv, pornind de la baza robotului, energiile cinetice corespunzătoare modulelor robotului şi dispozitivului de prehensiune. Energia cinetică a întregului robot se obține însumând energiile cinetice ale modulelor componente. Urmărind schema cinematică structurală a robotului, se determină deplasările elementare virtuale din cuple, lucrul mecanic virtual elementar şi forțele generalizate motoare. Utilizând ecuațiile lui Lagrange de speța a II-a şi mărimile mecanice (energie cinetică, forțe generalizate) determinate, se stabilesc ecuațiile diferențiale ale robotului TRTTRR1. În final sunt enunțate trei variante de optimizare a construcției şi funcționării robotului, rezultate din rezolvarea problemei inverse a dinamicii acestui robot.

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