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THE DYNAMIC MODEL OF TRTTR1 MODULAR SERIAL ROBOT

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Abstract: *A dynamic study of TRTTR1 modular serial robot with five degrees of freedom is performed in this paper. The authors use Newton-Euler formulation. Some general considerations on the Newton-Euler iterative method are presented first, and then they are applied for determining the dynamic equations of TRTTR1 robot. The dynamic modelling is performed using the symbolic modelling program, called Robot_Symbolic, the module Robot_Dynamics. The program assumes that the mass distribution parameters and the geometric and kinematic parameters from the robot geometric and kinematic modelling are known. The robot mechanical structure is passed by outwards iterations from the beginning, obtaining the torsor of the external forces. Then, the mechanical structure is passed by inwards iterations, in order to determine the torsor of contact forces and the generalized driving forces from robot's joints. The expressions of these generalized forces are the system of motion differential equations of the robot.*

Key words: *serial robot, dynamics, Newton-Euler formulation.*

1. THEORETICAL APPROACH

1.1 Newton-Euler Dynamic Equations

The general motion of a rigid body can be considered at a given time to have in its composition two simultaneous motions: a translation with the velocity $\bar{v}_c$ and the acceleration $\bar{a}_c$ of its mass centre and a spherical motion about the mass centre with the angular velocity $\bar{\omega}$ and angular acceleration $\bar{\varepsilon}$.

The mass centre motion theorem is applied for the dynamic study of a rigid body in a general motion. For this purpose, Newton equation and the kinetic moment theorem applied with respect to the mass centre (Euler dynamic equation) are to be used [1]. According to this method, for the dynamic equations determination, d'Alembert principle has to be applied for each element from the robot's mechanical structure.

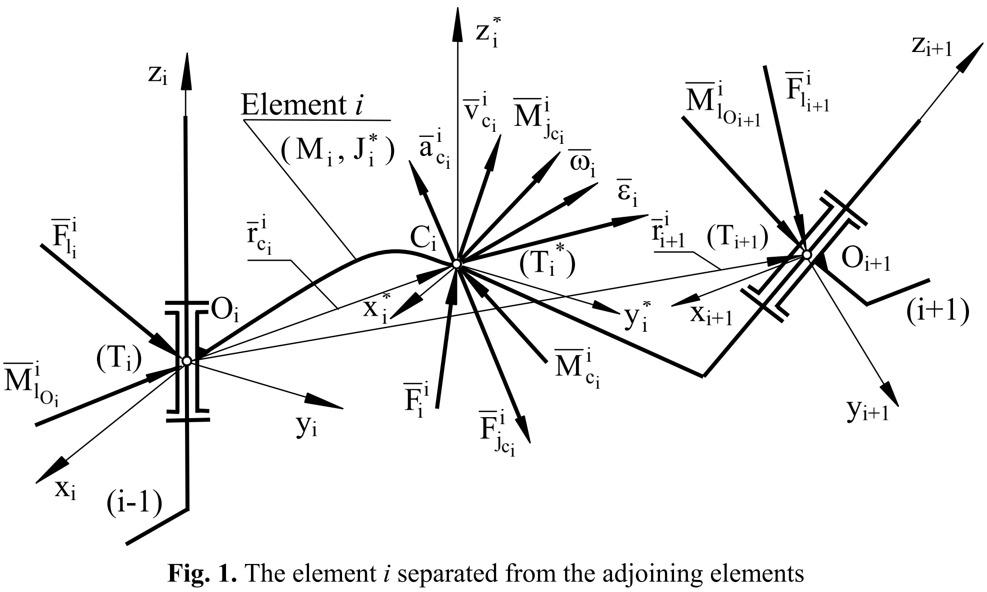

Fig. 1. The element *i* separated from the adjoining elements

From the dynamic equations written for the element *i* (fig. 1), considering that:

$$\overline{F}^i_{j_{c_i}} = -M_i \overline{a}^i_{c_i}$$

and

$$\overline{M}^i_{j_{c_i}} = -\dot{\overline{K}}^i_{c_i} = -J_i^* \overline{\varepsilon}_i - \overline{\omega}_i \times J_i^* \overline{\omega}_i ,$$

the vector expressions of $\overline{F}^i_{l_i}, \overline{M}^i_{l_{O_i}}$ can be obtained, yielding:

$$\begin{aligned} &\overline{F}^i_i + \overline{F}^i_{l_i} + \overline{F}^i_{l_{i+1}} + \overline{F}^i_{j_{c_i}} = 0 \\ &\overline{M}^i_{c_i} + \overline{M}^i_{l_{O_i}} + \overline{M}^i_{l_{O_{i+1}}} + \overline{r}^i_{c_i} \times \overline{F}^i_i + \\ &+ \overline{r}^i_{i+1} \times \overline{F}^i_{l_{i+1}} + \overline{M}^i_{j_{c_i}} + \overline{r}^i_{c_i} \times \overline{F}^i_{j_{c_i}} = 0. \end{aligned} \qquad (1)$$

The generalized driving forces $Q^i_m, i = 1 \div n$, from the joint *i* are obtained projecting the vectors $\overline{F}^i_{l_i}, \overline{M}^i_{l_{O_i}}$ on the joint axis, of versor $\overline{k}^i_i$. Therefore,

$$Q^i_m = \begin{cases} \overline{F}^i_{l_i} \cdot \overline{k}^i_i, \text{ for translation joints,} \\ \overline{M}^i_{l_{O_i}} \cdot \overline{k}^i_i, \text{ for rotation joints.} \end{cases} \qquad (2)$$

1.2 The Iterative Method

Using the iterative method presented below, the dynamic equations of a robot are determined, which emphasize the generalized variables, the generalized driving forces and the contact forces appearing between the robot parts. The computation algorithm is based on Luh-Walker-Paul method [2] and it has two stages:

1. Iterations outwards the end of the robot structure. Using Newton-Euler dynamic equations, the velocities and accelerations, linear and angular and the external forces and moments are determined for each element *i*, (*i*=1÷*n*), from the robot's structure.

2. Iterations inwards the mechanical structure of the robot. According to this stage, the torsor of the contact forces between the elements *i*, *i*+1, i.e. the generalized driving forces from the kinematic axes are determined for each element *i*, (*i*=1÷*n*), of the robot.

According to the kinematic structure of a robot with *n* degrees of freedom (fig. 2), [2], [4], the dynamic equations of a robot can be established. The kinematic structure of the robot is previously modelled from the geometric point of view.

Passing those iterations, outwards and inwards the robot's mechanical structure and considering the relation (2), the generalized driving forces Q^i_m are determined, representing the dynamic model of the robot:

$$Q^i_m = \Delta_i \left[M^i_{l_{O_i}} \right]^T \cdot \overline{k}^i_i + (1 - \Delta_i) \left[\overline{F}^i_{l_i} \right]^T \cdot \overline{k}^i_i + Q^i_f, \qquad (3)$$

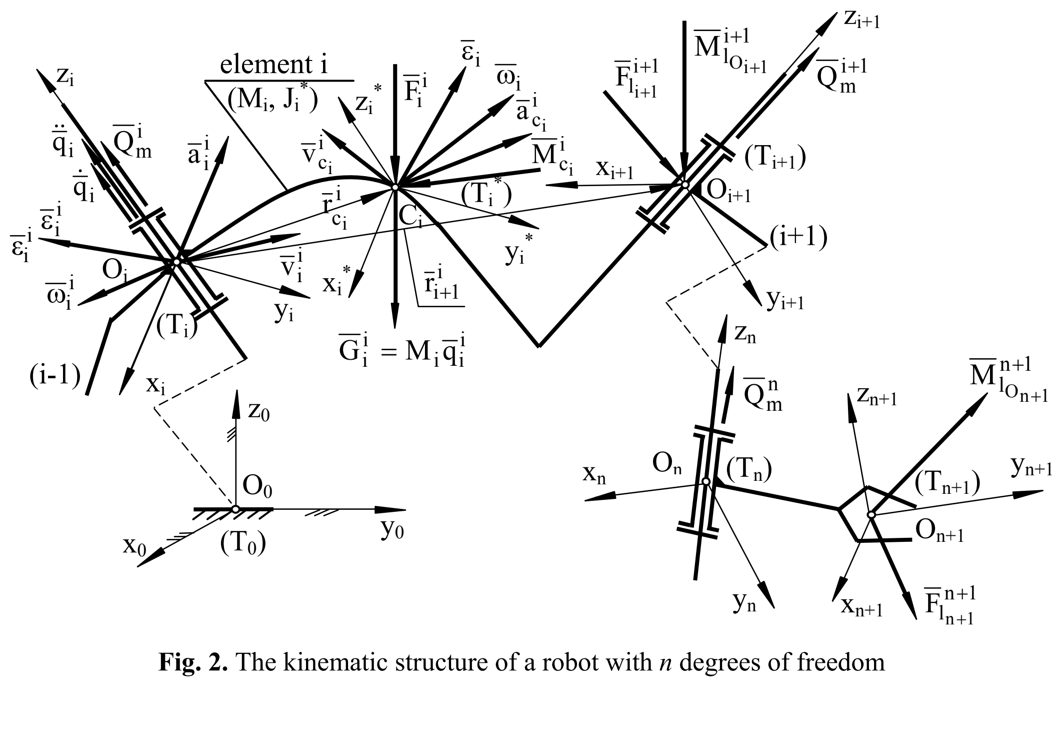

Fig. 2. The kinematic structure of a robot with *n* degrees of freedom

where Q_f^i is, according to [3], the generalized force due to frictions and it has the expression:

$$Q_f^i = b_i \dot{q}_i + Q_{f_c}^i . \tag{4}$$

The parameters b_i and $Q_{f_c}^i$ from (4) are:

b_i – the viscous friction coefficient;

$Q_{f_c}^i$ – the generalized force due to dry frictions (Coulomb frictions) and it has the expression:

$$Q_{f_c}^i = \Delta_i c_i \frac{d_i}{2} \left| \bar{k}_i^i \times \bar{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i + (1 - \Delta_i) c_i \left| \bar{k}_i^i \times \bar{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i . \tag{5}$$

In the above relations, c_i represents the dry friction coefficient, and d_i is the spindle diameter from the rotation joint.

The system (3) of dynamic equations can be written as:

$$\bar{Q}_m(t) = \left[Q_m^i(t) = f^{-1}\left(q_j(t),\ j = 1 \div n\right),\ i = 1 \div n \right]^T \tag{6}$$

and it represents the dynamic model of the robot having n degrees of freedom.

The column vector of the generalized driving forces within the direct problem of the robot dynamics is known. The following functions are determined:

$$\bar{q}(t) = f\left\{\bar{Q}_m(t)\right\} = \left[q_i(t),\ i = 1 \div n\right]^T , \tag{7}$$

representing the motion law of the robot in the configurative space of states.

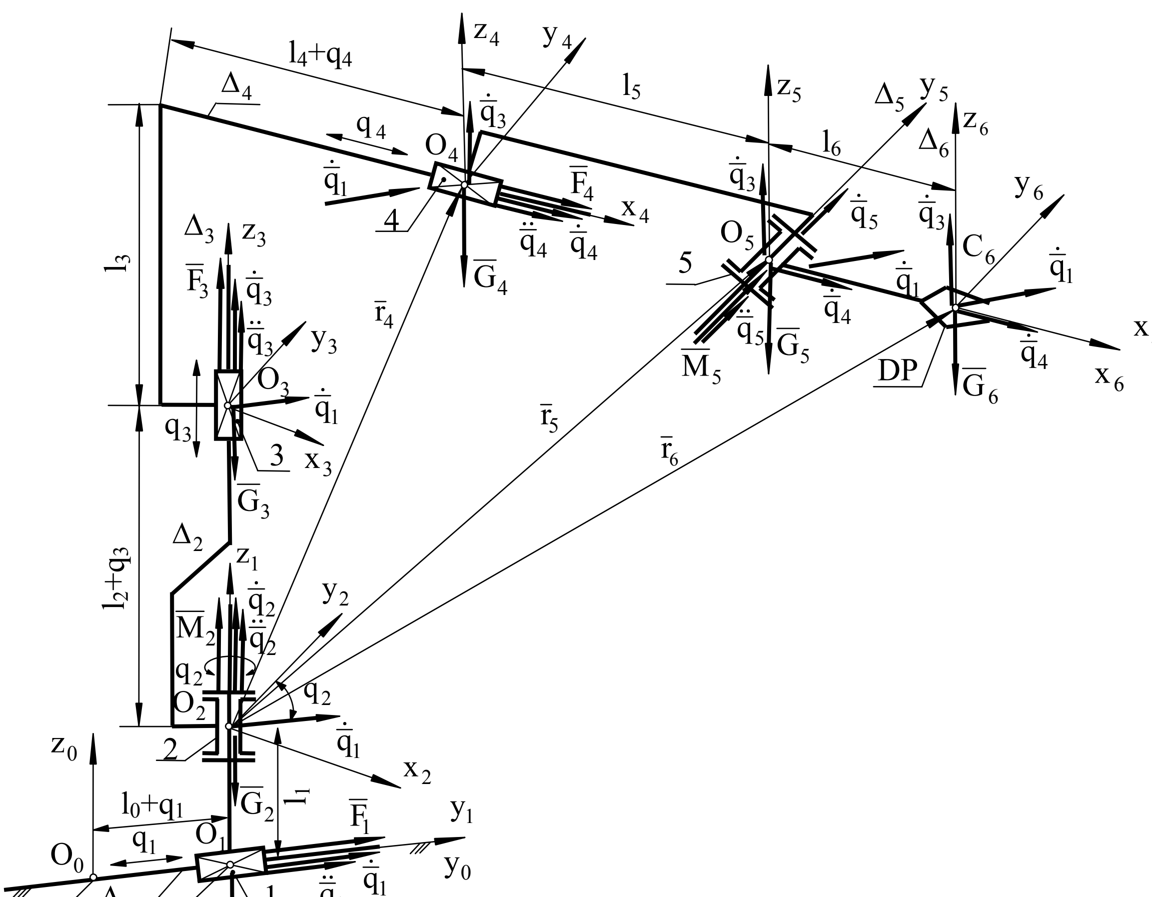

Fig. 3. The kinematic structure of TRTTR1 modular serial industrial robot

Within the inverse problem of the robot dynamics, called the inverse dynamic model, the functions $\overline{q}(t)$ are known and using the relation (3), the generalized driving forces $\overline{Q}_m(t)$ are determined.

Using Newton-Euler iterative method, the elements of torsor of contact forces between the robot elements can be determined.

2. THE DYNAMIC EQUATIONS OF THE TRTTR1 ROBOT USING NEWTON-EULER FORMULATION

The dynamic model of the TRTTR1 robot, whose kinematic scheme is presented in fig. 3, will be determined according to [5], [6], [7] and [8], using the symbolic modelling program *Robot_Symbolic*, the module *Robot_Dynamics*.

In order to apply the formulation, the geometric and kinematic model have to be completed, the mass distribution parameters have to be well established and the following simplifying hypotheses are enrolled:

- the mass centres C_i are chosen into the origins O_i of the frames $O_ix_iy_iz_i$, $i = 1 \div 5$, the mass centres position vectors being therefore cancelled;
- choosing the mobile frames such that their axes coincide with the principal inertia directions, corresponding to these frames origins, resulting in the cancelled centrifugal mechanical inertia moments.

According to [9], in the table 1, the mass distribution parameters are presented: the element i, the mass of the element i, the mass centres and the inertia tensors.

In the table 1, J_x^{*i}, J_y^{*i}, J_z^{*i}, $i = 1, 2, 3, 4, 5$, are the axial mechanical moments of inertia, with respect to the frame i, with the origin in the mass centre C_i and having the same orientation as the frame attached to each robot element.

The acceleration corresponding to the mass centres are determined with the following relations:

Table 1

Mass distribution parameters of TRTTR1 robot

Element i	Mass M_i	Mass centre, $\overline{r}_{c_i}^i$	Inertial tensor, J_i^{*i}
1	M_1	$[0\ 0\ 0]^T$	$\begin{bmatrix} J_x^{*1} & 0 & 0 \\ 0 & J_y^{*1} & 0 \\ 0 & 0 & J_z^{*1} \end{bmatrix}$
2	M_2	$[0\ 0\ 0]^T$	$\begin{bmatrix} J_x^{*2} & 0 & 0 \\ 0 & J_y^{*2} & 0 \\ 0 & 0 & J_z^{*2} \end{bmatrix}$
3	M_3	$[0\ 0\ 0]^T$	$\begin{bmatrix} J_x^{*3} & 0 & 0 \\ 0 & J_y^{*3} & 0 \\ 0 & 0 & J_z^{*3} \end{bmatrix}$
4	M_4	$[0\ 0\ 0]^T$	$\begin{bmatrix} J_x^{*4} & 0 & 0 \\ 0 & J_y^{*4} & 0 \\ 0 & 0 & J_z^{*4} \end{bmatrix}$
5	M_5	$[0\ 0\ 0]^T$	$\begin{bmatrix} J_x^{*5} & 0 & 0 \\ 0 & J_y^{*5} & 0 \\ 0 & 0 & J_z^{*5} \end{bmatrix}$

$$\overline{a}_{c_1}^1 = \overline{a}_1^1 + \overline{\varepsilon}_1^1 \times \overline{r}_{c_1}^1 + \overline{\omega}_1^1 \times \left(\overline{\omega}_1^1 \times \overline{r}_{c_1}^1\right) \tag{8}$$

$$[\overline{a}_c]_1^1 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ g \end{bmatrix}$$

$$\overline{a}_{c_2}^2 = \overline{a}_2^2 + \overline{\varepsilon}_2^2 \times \overline{r}_{c_2}^2 + \overline{\omega}_2^2 \times \left(\overline{\omega}_2^2 \times \overline{r}_{c_2}^2\right); \tag{9}$$

$$[\overline{a}_c]_2^2 = \begin{bmatrix} sq_2\ddot{q}_1 \\ cq_2\ddot{q}_1 \\ g \end{bmatrix};$$

$$\overline{a}_{c_3}^3 = \overline{a}_3^3 + \overline{\varepsilon}_3^3 \times \overline{r}_{c_3}^3 + \overline{\omega}_3^3 \times \left(\overline{\omega}_3^3 \times \overline{r}_{c_3}^3\right); \tag{10}$$

$$[\overline{a}_c]_3^3 = \begin{bmatrix} sq_2\ddot{q}_1 \\ cq_2\ddot{q}_1 \\ g + \ddot{q}_3 \end{bmatrix};$$

$$\overline{a}_{c_4}^4 = \overline{a}_4^4 + \overline{\varepsilon}_4^4 \times \overline{r}_{c_4}^4 + \overline{\omega}_4^4 \times \left(\overline{\omega}_4^4 \times \overline{r}_{c_4}^4\right); \tag{11}$$

$$\left[\overline{a}_c\right]_4^4 = \begin{bmatrix} sq_2\ddot{q}_1 - \dot{q}_2^2 q_4 - \dot{q}_2^2 l_4 + \ddot{q}_4 \\ cq_4\ddot{q}_1 + \ddot{q}_2 q_4 + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2 \\ g + \ddot{q}_3 \end{bmatrix};$$

$$\overline{a}_{c_5}^5 = \overline{a}_5^5 + \overline{\varepsilon}_5^5 \times \overline{r}_{c_5}^5 + \overline{\omega}_5^5 \times \left(\overline{\omega}_5^5 \times \overline{r}_{c_5}^5\right); \tag{12}$$

$$\left[\overline{a}_c\right]_5^5 = \begin{bmatrix} cq_5 sq_2\ddot{q}_1 - cq_5\dot{q}_2^2 q_4 - \\ - cq_5\dot{q}_2^2 l_4 + cq_5\ddot{q}_4 - \\ - cq_2\dot{q}_2^2 l_5 - sq_5 g - sq_5\ddot{q}_3 \\ \hdashline cq_2\ddot{q}_1 + \ddot{q}_2 q_4 + \\ + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2 + \ddot{q}_2 l_5 \\ \hdashline sq_5 sq_2\ddot{q}_1 - sq_5\dot{q}_2^2 q_4 - \\ - sq_5\dot{q}_2^2 l_4 + sq_5\ddot{q}_4 - \\ - sq_2\dot{q}_2^2 l_5 + cq_5 g + cq_5\ddot{q}_3 \end{bmatrix} \tag{13}$$

According to Newton-Euler formulation, for the beginning, the mechanical structure is passed by outwards iterations, resulting in the system of external forces:

$$\left[\overline{F}\right]_1^1 = M_1\left[\overline{a}_c\right]_1^1, \quad \overline{F}_1^1 = \begin{bmatrix} 0 \\ M_1\ddot{q}_1 \\ M_1 g \end{bmatrix}; \tag{14}$$

$$\overline{F}_2^2 = M_2\left[\overline{a}_c\right]_2^2, \quad \left[\overline{F}\right]_2^2 = \begin{bmatrix} M_2 sq_2\ddot{q}_1 \\ M_2 cq_2\ddot{q}_1 \\ M_2 g \end{bmatrix}; \tag{15}$$

$$\left[\overline{F}\right]_3^3 = M_3\left[\overline{a}_c\right]_3^3, \quad \left[\overline{F}\right]_3^3 = \begin{bmatrix} M_3 sq_2\ddot{q}_1 \\ M_3 cq_2\ddot{q}_1 \\ M_3(g + \ddot{q}_3) \end{bmatrix}; \tag{16}$$

$$\left[\overline{F}\right]_4^4 = M_4\left[\overline{a}_c\right]_4^4, \quad \left[\overline{F}\right]_4^4 = \begin{bmatrix} -M_4\left(-sq_2\ddot{q}_1 + \dot{q}_2^2 q_4 + \dot{q}_2^2 l_4 - \ddot{q}_4\right) \\ M_4\left(cq_2\ddot{q}_1 + \ddot{q}_2 q_4 + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2\right) \\ M_4\left(g + \ddot{q}_3\right) \end{bmatrix}; \tag{17}$$

$$\left[\overline{F}\right]_5^5 = M_5\left[\overline{a}_c\right]_5^5 \tag{18}$$

$$\left[\overline{F}\right]_5^5 = \begin{bmatrix} -M_5\left(-cq_5 sq_2\ddot{q}_1 + cq_5\dot{q}_2^2 q_4 + cq_5\dot{q}_2^2 l_4 - cq_5\ddot{q}_4 + cq_5\dot{q}_2^2 l_5 + sq_5 g + sq_5\ddot{q}_3\right) \\ M_5\left(cq_2\ddot{q}_1 + \ddot{q}_2 q_4 + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2 + \ddot{q}_2 l_5\right) \\ -M_5\left(-sq_5 sq_2\ddot{q}_1 + sq_5\dot{q}_2^2 q_4 + sq_5\dot{q}_2^2 l_4 - sq_5\ddot{q}_4 + sq_5\dot{q}_2^2 l_5 - cq_5 g - cq_5\ddot{q}_3\right) \end{bmatrix} \tag{19}$$

In the same way, the moments of the external forces are obtained:

$$\overline{M}_{c_1}^1 = J_1^{*1}\overline{\varepsilon}_1^1 + \overline{\omega}_1^1 \times J_1^{*1}\overline{\omega}_1^1, \quad \left[\overline{M}_c\right]_1^1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T; \tag{20}$$

$$\overline{M}_{c_2}^2 = J_2^{*2}\overline{\varepsilon}_2^2 + \overline{\omega}_2^2 \times J_2^{*2}\overline{\omega}_2^2, \quad \left[\overline{M}_c\right]_2^2 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*2}\ddot{q}_2 \end{bmatrix}; \tag{21}$$

$$\overline{M}_{c_3}^3 = J_3^{*3}\overline{\varepsilon}_3^3 + \overline{\omega}_3^3 \times J_3^{*3}\overline{\omega}_3^3, \quad \left[\overline{M}_c\right]_3^3 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*3}\ddot{q}_2 \end{bmatrix}; \tag{22}$$

$$\overline{M}_{c_4}^4 = J_4^{*4}\overline{\varepsilon}_4^4 + \overline{\omega}_4^4 \times J_4^{*4}\overline{\omega}_4^4, \quad \left[\overline{M}_c\right]_4^4 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*4}\ddot{q}_2 \end{bmatrix}; \tag{23}$$

$$\overline{M}_{c_5}^5 = J_5^{*5}\overline{\varepsilon}_5^5 + \overline{\omega}_5^5 \times J_5^{*5}\overline{\omega}_5^5, \quad \left[\overline{M}_c\right]_5^5 = \begin{bmatrix} -J_x^{*5}sq_5\ddot{q}_2 - J_x^{*5}\dot{q}_5 cq_5\dot{q}_2 - cq_5\dot{q}_2 J_y^{*5}\dot{q}_5 + \dot{q}_5 J_z^{*5} cq_5\dot{q}_2 \\ J_y^{*5}\ddot{q}_5 - cq_5\dot{q}_2^2 J_x^{*5} sq_5 + sq_5\dot{q}_2^2 J_z^{*5} cq_5 \\ J_z^{*5} cq_5\ddot{q}_2 - J_z^{*5}\dot{q}_5 sq_5\dot{q}_2 + \dot{q}_5 J_x^{*5} sq_5\dot{q}_2 - sq_5\dot{q}_2 J_y^{*5}\dot{q}_5 \end{bmatrix}; \tag{24}$$

In the second part of Newton-Euler method, the mechanical structure is passed by iterations inwards the robot's mechanical structure. The torsor of the contact forces and moments is determined, namely the generalized driving forces from the robot's joints [4].

The elements of the payload torsor are expressed by the relations:

$$F_6^6 = \begin{bmatrix} F_x^6 \\ F_y^6 \\ F_z^6 \end{bmatrix}; \quad M_{O_6}^6 = \begin{bmatrix} M_x^6 \\ M_y^6 \\ M_z^6 \end{bmatrix}. \tag{25}$$

The contact forces have these expressions:

$$\overline{F}_{l_5}^5 = [R]_6^5 \cdot \overline{F}_{l_6}^6 + \overline{F}_5^5; \tag{26}$$

$$\overline{F}_{l_4}^4 = [R]_5^4 \cdot \overline{F}_{l_5}^5 + \overline{F}_4^4; \tag{27}$$

$$\overline{F}_{l_3}^3 = [R]_4^3 \cdot \overline{F}_{l_4}^4 + \overline{F}_3^3; \tag{28}$$

$$\overline{F}_{l_2}^2 = [R]_3^2 \cdot \overline{F}_{l_3}^3 + \overline{F}_2^2; \tag{29}$$

$$\overline{F}_{l_1}^1 = [R]_2^1 \cdot \overline{F}_{l_2}^2 + \overline{F}_1^1; \tag{30}$$

$$\left[\overline{F}_l\right]_5^5 = \begin{bmatrix} F_{l_x}^6 + M_5 cq_5 sq_2\ddot{q}_1 - M_5 cq_5\dot{q}_2^2 q_4 - M_5 cq_5\dot{q}_2^2 l_4 + M_5 cq_5\ddot{q}_4 - M_5 cq_5\dot{q}_2^2 l_5 - \\ - M_5 sq_5 g - M_5 sq_5\ddot{q}_3 \\ \hline F_{l_y}^6 + M_5 cq_2\ddot{q}_1 + M_5\ddot{q}_2 q_4 + M_5\ddot{q}_2 l_4 + 2M_5\dot{q}_4\dot{q}_2 + M_5\ddot{q}_2 l_5 \\ \hline F_{l_z}^6 + M_5 sq_5 sq_2\ddot{q}_1 - M_5 sq_5\dot{q}_2^2 q_4 - M_5 sq_5\dot{q}_2^2 l_4 + M_5 sq_5\ddot{q}_4 - M_5 sq_5\dot{q}_2^2 l_5 + \\ + M_5 cq_5 g + M_5 cq_5\ddot{q}_3 \end{bmatrix}; \tag{31}$$

$$\left[\overline{F}_l\right]_4^4 = \begin{bmatrix} cq_5 F_{l_x}^6 + sq_5 F_{l_z}^6 + M_5 sq_2\ddot{q}_1 - M_5\dot{q}_2^2 q_4 - M_5\dot{q}_2^2 l_4 + M_5\ddot{q}_4 - M_5\dot{q}_2^2 l_5 + \\ + M_4 sq_2\ddot{q}_1 - M_4\dot{q}_2^2 q_4 - M_4\dot{q}_2^2 l_4 + M_4\ddot{q}_4 \\ \hline F_{l_y}^6 + M_5 cq_2\ddot{q}_1 + M_5\ddot{q}_2 q_4 + M_5\ddot{q}_2 l_4 + 2M_5\dot{q}_4\dot{q}_2 + M_5\ddot{q}_2 l_5 + \\ + M_4 cq_2\ddot{q}_1 + M_4\ddot{q}_2 q_4 + M_4\ddot{q}_2 l_4 + 2M_4\dot{q}_4\dot{q}_2 \\ \hline - sq_5 F_{l_x}^6 + M_5 g + M_5\ddot{q}_3 + cq_5 F_{l_z}^6 + M_4 g + M_4\ddot{q}_3 \end{bmatrix}; \tag{32}$$

$$\left[\overline{F}_l\right]_3^3 = \begin{bmatrix} cq_5 F_{l_x}^6 + sq_5 F_{l_z}^6 + M_5 sq_2\ddot{q}_1 - M_5\dot{q}_2^2 q_4 - M_5\dot{q}_2^2 l_4 + M_5\ddot{q}_4 - M_5\dot{q}_2^2 l_5 + \\ + M_4 sq_2\ddot{q}_1 - M_4\dot{q}_2^2 q_4 - M_4\dot{q}_2^2 l_4 + M_4\ddot{q}_4 + M_3 sq_2\ddot{q}_1 \\ \hline F_{l_y}^6 + M_5 cq_2\ddot{q}_1 + M_5\ddot{q}_2 q_4 + M_5\ddot{q}_2 l_4 + 2M_5\dot{q}_4\dot{q}_2 + M_5\ddot{q}_2 l_5 + \\ + M_4 cq_2\ddot{q}_1 + M_4\ddot{q}_2 q_4 + M_4\ddot{q}_2 l_4 + 2M_4\dot{q}_4\dot{q}_2 + M_3 cq_2\ddot{q}_1 \\ \hline - sq_5 F_{l_x}^6 + M_5 g + M_5\ddot{q}_3 + cq_5 F_{l_z}^6 + M_4 g + M_4\ddot{q}_3 + M_3 g + M_3\ddot{q}_3 \end{bmatrix}; \tag{33}$$

$$[\overline{F}_l]^2_2 = \begin{bmatrix} \begin{array}{l} cq_5 F^6_{l_x} + sq_5 F^6_{l_z} + M_5 sq_2 \ddot{q}_1 - M_5 \dot{q}_2^2 q_4 - M_5 \dot{q}_2^2 l_4 + M_5 \ddot{q}_4 - M_5 \dot{q}_2^2 l_5 + \\ \quad + M_4 sq_2 \ddot{q}_1 - M_4 \dot{q}_2^2 q_4 - M_4 \dot{q}_2^2 l_4 + M_4 \ddot{q}_4 + M_3 sq_2 \ddot{q}_1 + M_2 sq_2 \ddot{q}_1 \end{array} \\ \hline \begin{array}{l} F^6_{l_y} + M_5 cq_2 \ddot{q}_1 + M_5 \ddot{q}_2 q_4 + M_5 \ddot{q}_2 l_4 + 2M_5 \dot{q}_4 \dot{q}_2 + M_5 \ddot{q}_2 l_5 + \\ \quad + M_4 cq_2 \ddot{q}_1 + M_4 \ddot{q}_2 q_4 + M_4 \ddot{q}_2 l_4 + 2M_4 \dot{q}_4 \dot{q}_2 + M_3 cq_2 \ddot{q}_1 + M_2 cq_2 \ddot{q}_1 \end{array} \\ \hline -sq_5 F^6_{l_x} + M_5 g + M_5 \ddot{q}_3 + cq_5 F^6_{l_z} + M_4 g + M_4 \ddot{q}_3 + M_3 g + M_3 \ddot{q}_3 + M_2 g \end{bmatrix}; \tag{34}$$

$$[\overline{F}_l]^1_1 = \begin{bmatrix} \begin{array}{l} -cq_2 M_5 \dot{q}_2^2 l_4 + cq_2 M_5 \ddot{q}_4 - cq_2 M_5 \dot{q}_2^2 q_4 + cq_2 cq_5 F^6_{l_x} + cq_2 sq_5 F^6_{l_z} + cq_2 M_4 \ddot{q}_4 - \\ \quad - sq_2 F^6_{l_y} - cq_2 M_5 \dot{q}_2^2 l_5 - cq_2 M_4 \dot{q}_2^2 q_4 - cq_2 M_4 \dot{q}_2^2 l_4 - sq_2 M_5 \ddot{q}_2 q_4 - \\ \quad - sq_2 M_5 \ddot{q}_2 l_4 - 2sq_2 M_5 \dot{q}_4 \dot{q}_2 - sq_2 M_5 \ddot{q}_2 l_5 - sq_2 M_4 \ddot{q}_2 q_4 - \\ \quad - sq_2 M_4 \ddot{q}_2 l_4 - 2sq_2 M_4 \dot{q}_4 \dot{q}_2 \end{array} \\ \hline \begin{array}{l} M_5 \ddot{q}_1 + M_4 \ddot{q}_1 + M_3 \ddot{q}_1 + sq_2 M_5 \ddot{q}_4 + sq_2 sq_5 F^6_{l_z} + sq_2 cq_5 F^6_{l_x} + M_1 \ddot{q}_1 - sq_2 M_5 \dot{q}_2^2 q_4 - \\ \quad - sq_2 M_5 \dot{q}_2^2 l_4 - sq_2 M_5 \dot{q}_2^2 l_5 + cq_2 F^6_{l_y} + M_2 \ddot{q}_1 + cq_2 M_5 \ddot{q}_2 l_5 + cq_2 M_4 \ddot{q}_2 q_4 + \\ \quad + cq_2 M_4 \ddot{q}_2 l_4 + 2cq_2 M_4 \dot{q}_4 \dot{q}_2 + 2cq_2 M_5 \dot{q}_4 \dot{q}_2 - sq_2 M_4 \dot{q}_2^2 q_4 - sq_2 M_4 \dot{q}_2^2 l_4 + \\ \quad + cq_2 M_5 \ddot{q}_2 q_4 + cq_2 M_5 \ddot{q}_2 l_4 + sq_2 M_4 \ddot{q}_4 \end{array} \\ \hline -sq_5 F^6_{l_x} + M_5 g + M_5 \ddot{q}_3 + cq_5 F^6_{l_z} + M_4 g + M_4 \ddot{q}_3 + M_3 g + M_3 \ddot{q}_3 + M_2 g + M_1 g \end{bmatrix}. \tag{35}$$

According to [5] and [2], the moments of the contact forces have the following expressions:

$$\overline{M}^5_{l_{O_5}} = [R]^5_6 \cdot \overline{M}^6_{l_{O_6}} + \overline{r}^5_{c_5} \times \overline{F}^5_5 + \overline{r}^5_6 \times [R]^5_6 \cdot \overline{F}^6_{l_6} + \overline{M}^5_{c_5}; \tag{36}$$

$$[\overline{M}_l]^5_{O_5} = \begin{bmatrix} M^6_{l_x} - J^{*5}_x sq_5 \ddot{q}_2 - J^{*5}_x \dot{q}_5 cq_5 \dot{q}_2 - cq_5 \dot{q}_2 J^{*5}_y \dot{q}_5 + \dot{q}_5 J^{*5}_z cq_5 \dot{q}_2 \\ M^6_{l_y} - l_6 F^5_{l_y} + J^{*5}_y \ddot{q}_5 - cq_5 \dot{q}_2^2 J^{*5}_x sq_5 + sq_5 \dot{q}_2^2 J^{*5}_z cq_5 \\ M^6_{l_z} + l_6 F^6_{l_x} + J^{*5}_z cq_5 \ddot{q}_2 - J^{*5}_z \dot{q}_5 sq_5 \dot{q}_2 + \dot{q}_5 J^{*5}_x sq_5 \dot{q}_2 - sq_5 \dot{q}_2 J^{*5}_y \dot{q}_5 \end{bmatrix}; \tag{37}$$

$$\overline{M}^4_{l_{O_4}} = [R]^4_5 \cdot \overline{M}^5_{l_{O_5}} + \overline{r}^4_{c_4} \times \overline{F}^4_4 + \overline{r}^4_5 \times [R]^4_5 \cdot \overline{F}^5_{l_5} + \overline{M}^4_{c_4}; \tag{38}$$

$$[\overline{M}_l]^4_{O_4} = \begin{bmatrix} \begin{array}{l} cq_5 M^6_{l_x} - cq_5 J^{*5}_x sq_5 \ddot{q}_2 - 2J^{*5}_x \dot{q}_5 c^2 q_5 \dot{q}_2 + 2\dot{q}_5 J^{*5}_z c^2 q_5 \dot{q}_2 + sq_5 M^6_{l_z} + sq_5 l_6 F^6_{l_y} + \\ \quad + sq_5 J^{*5}_z cq_5 \ddot{q}_2 - J^{*5}_z \dot{q}_5 \dot{q}_2 + \dot{q}_5 J^{*5}_x \dot{q}_2 - \dot{q}_2 J^{*5}_y \dot{q}_5 \end{array} \\ \hline \begin{array}{l} M^6_{l_y} - l_6 F^6_{l_z} + J^{*5}_y \ddot{q}_5 - cq_5 \dot{q}_2^2 J^{*5}_x sq_5 + sq_5 \dot{q}_2^2 J^{*5}_z cq_5 + l_5 sq_5 F^6_{l_x} - l_5 M_5 g - \\ \quad - l_5 M_5 \ddot{q}_3 - l_5 cq_5 F^6_{l_z} \end{array} \\ \hline \begin{array}{l} -sq_5 M^6_{l_x} + J^{*5}_x \ddot{q}_2 - J^{*5}_x \ddot{q}_2 c^2 q_5 + 2sq_5 J^{*5}_x \dot{q}_5 cq_5 \dot{q}_2 - 2sq_5 \dot{q}_5 J^{*5}_z cq_5 \dot{q}_2 + cq_5 M^6_{l_z} + \\ \quad + cq_5 l_6 F^6_{l_z} + J^{*5}_z c^2 q_5 \ddot{q}_2 + l_5 F^6_{l_y} + l_5 M_5 cq_2 \ddot{q}_1 + l_5 M_5 \ddot{q}_2 q_4 + l_5 M_5 \ddot{q}_2 l_4 + \\ \quad + 2l_5 M_5 \dot{q}_4 \dot{q}_2 + M_5 \ddot{q}_2 l_5^2 + J^{*4}_z \dot{q}_2 \end{array} \end{bmatrix}; \tag{39}$$

Given the high complexity of the vectors: $\overline{M}^{3}_{l_{O_3}}$, $\overline{M}^{2}_{l_{O_2}}$ and $\overline{M}^{1}_{l_{O_1}}$, they will be expressed by their Cartesian components:

$$\overline{M}^{3}_{l_{O_3}} = \left[R\right]^{3}_{4} \cdot \overline{M}^{4}_{l_{O_4}} + \overline{r}^{\,3}_{c_3} \times \overline{F}^{3}_{3} + \overline{r}^{\,3}_{4} \times \left[R\right]^{3}_{4} \cdot \overline{F}^{4}_{l_4} + \overline{M}^{3}_{c_3}; \tag{40}$$

$$\begin{aligned} M^{3}_{l_x} = {} & cq_5 M^{6}_{l_x} - cq_5 J^{*5}_{x} sq_5 \ddot{q}_2 - 2J^{*5}_{x} \dot{q}_5 c^2 q_5 \dot{q}_2 + 2\dot{q}_5 J^{*5}_{z} c^2 q_5 \dot{q}_2 + sq_5 M^{6}_{l_z} + sq_5 l_6 F^{6}_{l_y} + \\ & + sq_5 J^{*5}_{z} cq_5 \ddot{q}_2 - J^{*5}_{z} \dot{q}_5 \dot{q}_2 + \dot{q}_5 J^{*5}_{x} \ddot{q}_2 - \dot{q}_2 J^{*5}_{y} \dot{q}_5 - l_3 F^{6}_{l_y} - l_3 M_5 cq_2 \ddot{q}_1 - \\ & - l_3 M_5 \ddot{q}_2 q_4 - l_3 M_5 \ddot{q}_2 l_4 - 2l_3 M_5 \dot{q}_4 \dot{q}_2 - l_3 M_5 \ddot{q}_2 l_5 - l_3 M_4 cq_2 \ddot{q}_1 - \\ & - l_3 M_4 \ddot{q}_2 q_4 - l_3 M_4 \ddot{q}_2 l_4 - 2l_3 M_4 \dot{q}_4 \dot{q}_2; \end{aligned} \tag{41}$$

$$\begin{aligned} M^{3}_{l_y} = {} & l_3 M_5 \ddot{q}_4 + l_3 M_4 \ddot{q}_4 - q_4 M_5 g - q_4 M_5 \ddot{q}_3 - q_4 M_4 g - q_4 M_4 \ddot{q}_3 - l_4 M_5 g - l_4 M_5 \ddot{q}_3 - \\ & - l_4 M_4 g - l_4 M_4 \ddot{q}_3 + l_3 cq_5 F^{6}_{l_x} + l_3 sq_5 F^{6}_{l_z} + q_4 sq_5 F^{6}_{l_x} - q_4 cq_5 F^{6}_{l_z} + \\ & + l_4 sq_5 F^{6}_{l_x} - l_4 cq_5 F^{6}_{l_z} - cq_5 \dot{q}_2^2 J^{*5}_{x} sq_5 + sq_5 \dot{q}_2^2 J^{*5}_{z} cq_5 + M^{6}_{l_y} - l_5 M_5 \ddot{q}_3 + \\ & + l_5 sq_5 F^{6}_{l_x} - l_5 cq_5 F^{6}_{l_z} + J^{*5}_{y} \ddot{q}_5 - l_6 F^{6}_{l_z} - l_5 M_5 g - l_3 M_5 \dot{q}_2^2 q_4 - \\ & - l_3 M_5 \dot{q}_2^2 l_4 - l_3 M_5 \dot{q}_2^2 l_5 - l_3 M_4 \dot{q}_2^2 q_4 - l_3 M_4 \dot{q}_2^2 l_4 + l_3 M_5 sq_2 \ddot{q}_1 + l_3 M_4 sq_2 \ddot{q}_1; \end{aligned} \tag{42}$$

$$\begin{aligned} M^{3}_{l_z} = {} & J^{*5}_{x} \ddot{q}_2 - J^{*5}_{x} \ddot{q}_2 c^2 q_5 + 2l_5 M_5 \ddot{q}_2 q_4 + 2l_5 M_5 \ddot{q}_2 l_4 + 2l_5 M_5 \dot{q}_4 \dot{q}_2 + l_5 M_5 cq_2 \ddot{q}_1 + \\ & + J^{*3}_{z} \ddot{q}_2 + 2sq_5 J^{*5}_{x} \dot{q}_5 cq_5 \dot{q}_2 - 2sq_5 \dot{q}_5 J^{*5}_{z} cq_5 \dot{q}_2 + l_5 F^{6}_{l_y} - sq_5 M^{6}_{l_x} + cq_5 M^{6}_{l_z} + \\ & + cq_5 l_6 F^{6}_{l_y} + J^{*5}_{z} c^2 q_5 \ddot{q}_2 + M_5 \ddot{q}_2 l_5^2 + 2q_4 M_5 \ddot{q}_2 l_4 + 2q_4 M_5 \dot{q}_4 \dot{q}_2 + 2q_4 M_4 \ddot{q}_2 l_4 + \\ & + 2q_4 M_4 \dot{q}_4 \dot{q}_2 + 2l_4 M_5 \dot{q}_4 \dot{q}_2 + 2l_4 M_4 \dot{q}_4 \dot{q}_2 + q_4 M_5 cq_2 \ddot{q}_1 + q_4 M_4 cq_2 \ddot{q}_1 + \\ & + l_4 M_5 cq_2 \ddot{q}_1 + l_4 M_4 cq_2 \ddot{q}_1 + M_5 \ddot{q}_2 q_4^2 + M_4 \ddot{q}_2 q_4^2 + M_5 \ddot{q}_2 l_4^2 + M_4 \ddot{q}_2 l_4^2 + \\ & + J^{*4}_{z} \ddot{q}_2 + q_4 F^{6}_{l_y} + l_4 F^{6}_{l_y}. \end{aligned} \tag{43}$$

$$\overline{M}^{2}_{l_{O_2}} = \left[R\right]^{2}_{3} \cdot \overline{M}^{3}_{l_{O_3}} + \overline{r}^{\,2}_{c_2} \times \overline{F}^{2}_{2} + \overline{r}^{\,2}_{3} \times \left[R\right]^{2}_{3} \cdot \overline{F}^{3}_{l_3} + \overline{M}^{2}_{c_2}; \tag{44}$$

$$\begin{aligned} M^{2}_{l_x} = {} & sq_5 l_6 F^{6}_{l_y} - \dot{q}_2^2 J^{*5}_{y} \dot{q}_5 + \dot{q}_5 J^{*5}_{x} \dot{q}_2 - J^{*5}_{z} \dot{q}_5 \dot{q}_2 - l_2 M_3 cq_2 \ddot{q}_1 - q_3 M_3 cq_2 \ddot{q}_1 - l_2 M_5 cq_2 \ddot{q}_1 - \\ & - l_2 M_4 cq_2 \ddot{q}_1 - q_3 M_5 cq_2 \ddot{q}_1 - q_3 M_4 cq_2 \ddot{q}_1 - l_3 F^{6}_{l_y} + sq_5 M^{6}_{l_z} + cq_5 M^{6}_{l_x} - \\ & - l_2 F^{6}_{l_y} - q_3 M_5 \ddot{q}_2 q_4 - q_3 M_4 \ddot{q}_2 q_4 - q_3 M_4 \ddot{q}_2 l_4 - q_3 M_5 \ddot{q}_2 l_4 - 2q_3 M_5 \dot{q}_4 \dot{q}_2 - \\ & - q_3 M_5 \ddot{q}_2 l_5 - 2q_3 M_4 \dot{q}_4 \dot{q}_2 - l_2 M_5 \ddot{q}_2 l_5 - l_2 M_4 \ddot{q}_2 q_4 - l_2 M_5 \ddot{q}_2 q_4 - l_2 M_5 \ddot{q}_2 l_4 - \\ & - 2l_2 M_5 \dot{q}_4 \dot{q}_2 - 2J^{*5}_{x} \dot{q}_5 c^2 q_5 \dot{q}_2 - cq_5 J^{*5}_{x} sq_5 \ddot{q}_2 - l_3 M_5 cq_2 \ddot{q}_1 + sq_5 J^{*5}_{z} cq_5 \ddot{q}_2 + \\ & + 2\dot{q}_5 J^{*5}_{z} c^2 q_5 \dot{q}_2 - 2l_3 M_5 \dot{q}_4 \dot{q}_2 - l_3 M_5 \ddot{q}_2 l_4 - l_3 M_5 \ddot{q}_2 q_4 - l_2 M_4 \ddot{q}_2 l_4 - \\ & - 2l_2 M_4 \dot{q}_4 \dot{q}_2 - l_3 M_5 \ddot{q}_2 l_5 - q_3 F^{6}_{l_y} - l_3 M_4 \ddot{q}_2 q_4 - l_3 M_4 cq_2 \ddot{q}_1 - 2l_3 M_4 \dot{q}_4 \dot{q}_2 - \\ & - l_3 M_4 \ddot{q}_2 l_4; \end{aligned} \tag{45}$$

$$\begin{aligned}
M^2_{l_y} = &-l_5 M_5 g + l_3 M_5 \ddot{q}_4 + l_3 M_4 \ddot{q}_4 - q_4 M_5 g - q_4 M_4 g - q_4 M_5 \ddot{q}_3 - q_4 M_4 \ddot{q}_3 - l_4 M_5 g - \\
&- l_4 M_5 \ddot{q}_3 - l_4 M_4 g - l_4 M_4 \ddot{q}_3 + l_3 cq_5 F^6_{l_x} + l_3 sq_5 F^6_{l_z} + q_4 sq_5 F^6_{l_x} - q_4 cq_5 F^6_{l_z} + \\
&+ l_4 sq_5 F^6_{l_x} - l_4 cq_5 F^6_{l_z} - l_5 M_5 \ddot{q}_3 + l_5 sq_5 F^6_{l_x} - l_5 cq_5 F^6_{l_z} + J^{*5}_y \ddot{q}_5 - l_5 F^6_{l_z} + \\
&+ q_3 M_5 sq_2 \ddot{q}_1 + q_3 M_4 sq_2 \ddot{q}_1 + q_3 M_3 sq_2 \ddot{q}_1 + l_2 M_5 sq_2 \ddot{q}_1 + l_2 M_4 sq_2 \ddot{q}_1 + \\
&+ l_2 M_3 sq_2 \ddot{q}_1 + M^6_{l_y} - cq_5 \dot{q}_2^2 J^{*5}_x sq_5 + sq_5 \dot{q}_2^2 J^{*5}_z cq_5 - l_3 M_4 \dot{q}_2^2 q_4 - l_3 M_4 \dot{q}_2^2 l_4 - \\
&- l_3 M_5 \dot{q}_2^2 l_5 - l_3 M_5 \dot{q}_2^2 l_4 + l_3 M_4 sq_2 \ddot{q}_1 - l_3 M_5 \dot{q}_2^2 q_4 + l_3 M_5 sq_2 \ddot{q}_1 + q_3 M_5 \ddot{q}_4 + \\
&+ q_3 M_4 \ddot{q}_4 + l_2 M_5 \ddot{q}_4 + l_2 M_4 \ddot{q}_4 + q_3 cq_5 F^6_{l_x} + q_3 sq_5 F^6_{l_z} + l_2 cq_5 F^6_{l_x} + l_2 sq_5 F^6_{l_z} - \\
&- q_3 M_5 \dot{q}_2^2 q_4 - q_3 M_5 \dot{q}_2^2 l_4 - q_3 M_5 \dot{q}_2^2 l_5 - q_3 M_4 \dot{q}_2^2 q_4 - q_3 M_4 \dot{q}_2^2 l_4 - l_2 M_5 \dot{q}_2^2 q_4 - \\
&- l_2 M_5 \dot{q}_2^2 l_4 - l_2 M_5 \dot{q}_2^2 l_5 - l_2 M_4 \dot{q}_2^2 q_4 - l_2 M_4 \dot{q}_2^2 l_4 \ ;
\end{aligned} \tag{46}$$

$$\begin{aligned}
M^2_{l_z} = &-2 sq_5 \dot{q}_5 J^{*5}_z cq_5 \dot{q}_2 + 2 sq_5 J^{*5}_x \dot{q}_5 cq_5 \dot{q}_2 - J^{*5}_x \ddot{q}_2 c^2 q_5 + cq_5 l_6 F^6_{l_y} + J^{*5}_z c^2 q_5 \ddot{q}_2 + M_5 \ddot{q}_2 l_5^2 + \\
&+ M_5 \ddot{q}_2 q_4^2 + M_4 \ddot{q}_2 q_4^2 + M_5 \ddot{q}_2 l_4^2 + M_4 \ddot{q}_2 l_4^2 + 2 q_4 M_4 \ddot{q}_2 l_4 + J^{*5}_x \ddot{q}_2 + \\
&+ J^{*3}_z \ddot{q}_2 + l_5 F^6_{l_y} - sq_5 M^6_{l_x} + cq_5 M^6_{l_z} + q_4 F^6_{l_y} + l_4 F^6_{l_y} + 2 q_4 M_5 \dot{q}_4 \dot{q}_2 + \\
&+ 2 q_4 M_5 \ddot{q}_2 l_4 + 2 l_5 M_5 \dot{q}_4 \dot{q}_2 + J^{*2}_z \ddot{q}_2 + 2 l_5 M_5 \ddot{q}_2 q_4 + 2 l_5 M_5 \ddot{q}_2 l_4 + \\
&+ q_4 M_5 cq_2 \ddot{q}_1 + l_5 M_5 cq_2 \ddot{q}_1 + q_4 M_4 cq_2 \ddot{q}_1 + l_4 M_5 cq_2 \ddot{q}_1 + l_4 M_4 cq_2 \ddot{q}_1 + \\
&+ J^{*4}_z \ddot{q}_2 + 2 l_4 M_4 \dot{q}_4 \dot{q}_2 + 2 l_4 M_5 \dot{q}_4 \dot{q}_2 + 2 q_4 M_4 \dot{q}_4 \dot{q}_2 \ ;
\end{aligned} \tag{47}$$

$$\overline{M}^1_{l_{O_1}} = [R]^1_2 \cdot \overline{M}^2_{l_{O_2}} + \overline{r}^1_{c_1} \times \overline{F}^1_1 + \overline{r}^1_2 \times [R]^1_2 \cdot \overline{F}^2_{l_2} + \overline{M}^1_{c_1}, \tag{48}$$

where from the following scalar components are obtained:

$$\begin{aligned}
M^1_{l_x} = &\ 2cq_2 \dot{q}_5 J^{*5}_z c^2 q_5 \dot{q}_2 - cq_2 l_3 M_4 \ddot{q}_2 l_4 - cq_2 q_3 M_5 \ddot{q}_2 q_4 - 2cq_2 q_3 M_5 \dot{q}_4 \dot{q}_2 + sq_2 q_4 M_4 g + \\
&+ cq_2 \dot{q}_5 J^{*5}_x \dot{q}_2 - cq_2 J^{*5}_z \dot{q}_5 \dot{q}_2 + cq_2 sq_5 l_6 F^6_{l_y} + sq_2 l_5 M_5 g - sq_2 l_3 M_5 \ddot{q}_4 - sq_2 l_3 M_4 \ddot{q}_4 + \\
&+ sq_2 q_4 M_5 g - 2 l_1 cq_2 M_5 \dot{q}_4 \dot{q}_2 - cq_2 l_3 F^6_{l_y} - cq_2 l_2 F^6_{l_y} + cq_2 cq_5 M^6_{l_x} - cq_2 q_3 F^6_{l_y} + \\
&+ cq_2 sq_5 M^6_{l_z} - sq_2 J^{*5}_y \ddot{q}_5 + sq_2 l_6 F^6_{l_z} - l_1 cq_2 F^6_{l_y} - l_1 cq_2 M_5 \ddot{q}_2 l_5 - l_1 cq_2 M_4 \ddot{q}_2 q_4 - \\
&- l_1 cq_2 M_4 \ddot{q}_2 l_4 - 2 l_1 cq_2 M_4 \dot{q}_4 \dot{q}_2 - l_3 M_4 \ddot{q}_1 - l_3 M_5 \ddot{q}_1 - q_3 M_5 \ddot{q}_1 - q_3 M_4 \ddot{q}_1 - l_2 M_5 \ddot{q}_1 - \\
&- q_3 M_3 \ddot{q}_1 - l_2 M_4 \ddot{q}_1 - l_2 M_3 \ddot{q}_1 + sq_2 l_3 M_4 \dot{q}_2^2 q_4 + sq_2 q_4 M_5 \ddot{q}_3 + sq_2 q_4 M_4 \ddot{q}_3 + \\
&+ sq_2 l_4 M_5 g + sq_2 l_4 M_5 \ddot{q}_3 + sq_2 l_4 M_4 g + sq_2 l_4 M_4 \ddot{q}_3 + sq_2 l_5 M_5 \ddot{q}_3 + sq_2 q_4 cq_5 F^6_{l_z} + \\
&+ sq_2 l_4 cq_5 F^6_{l_z} + sq_2 l_5 cq_5 F^6_{l_z} - sq_2 q_3 cq_5 F^6_{l_x} - sq_2 q_3 sq_5 F^6_{l_z} - sq_2 q_3 M_5 \ddot{q}_4 - \\
&- sq_2 q_3 M_4 \ddot{q}_4 - sq_2 l_2 cq_5 F^6_{l_x} - sq_2 l_2 sq_5 F^6_{l_z} - sq_2 l_2 M_5 \ddot{q}_4 - sq_2 l_2 M_4 \ddot{q}_4 - sq_2 l_3 cq_5 F^6_{l_x} - \\
&- sq_2 l_3 sq_5 F^6_{l_z} - sq_2 q_4 sq_5 F^6_{l_x} - sq_2 l_4 sq_5 F^6_{l_x} - sq_2 l_5 sq_5 F^6_{l_x} - l_1 sq_2 cq_5 F^6_{l_x} - l_1 sq_2 sq_5 F^6_{l_z} - \\
&- l_1 sq_2 M_5 \ddot{q}_4 - l_1 sq_2 M_4 \ddot{q}_4 - l_1 M_5 \ddot{q}_1 - l_1 M_4 \ddot{q}_1 - l_1 M_3 \ddot{q}_1 - l_1 M_2 \ddot{q}_1 - cq_2 q_3 M_4 \ddot{q}_2 q_4 - \\
&- cq_2 q_3 M_4 \ddot{q}_2 l_4 - cq_2 q_3 M_5 \ddot{q}_2 l_4 - 2cq_2 q_3 M_4 \dot{q}_4 \dot{q}_2 - cq_2 l_2 M_5 \ddot{q}_2 l_5 - cq_2 l_2 M_4 \ddot{q}_2 q_4 - \\
&- cq_2 l_2 M_5 \ddot{q}_2 q_4 - cq_2 l_2 M_5 \ddot{q}_2 l_4 - 2cq_2 l_2 M_5 \dot{q}_4 \dot{q}_2 - 2cq_2 l_3 M_5 \dot{q}_4 \dot{q}_2 - cq_2 l_3 M_5 \ddot{q}_2 l_4 - \\
&- cq_2 l_3 M_5 \ddot{q}_2 q_4 - cq_2 l_2 M_4 \ddot{q}_2 l_4 - 2cq_2 l_2 M_4 \dot{q}_4 \dot{q}_2 + cq_2 sq_5 J^{*5}_z cq_5 \ddot{q}_2 - cq_2 l_3 M_5 \ddot{q}_2 l_5 - \\
&- 2cq_2 J^{*5}_x \dot{q}_5 c^2 q_5 \dot{q}_2 - cq_2 l_3 M_4 \ddot{q}_2 q_4 - 2cq_2 l_3 M_4 \dot{q}_4 \dot{q}_2 - cq_2 cq_5 J^{*5}_x sq_5 \ddot{q}_2 +
\end{aligned}$$

$$
\begin{aligned}
&+sq_2l_3M_4\dot{q}_2^2l_4+sq_2l_3M_5\dot{q}_2^2l_5+sq_2l_3M_5\dot{q}_2^2l_4-sq_2sq_5\dot{q}_2^2J_z^{*5}cq_5+sq_2cq_5\dot{q}_2^2J_x^{*5}sq_5+\\
&+sq_2l_3M_5\dot{q}_2^2q_4+sq_2q_3M_5\dot{q}_2^2l_4+sq_2q_3M_5\dot{q}_2^2l_5+sq_2q_3M_4\dot{q}_2^2q_4+sq_2l_2M_5\dot{q}_2^2q_4+\\
&+sq_2l_2M_5\dot{q}_2^2l_4+sq_2l_2M_5\dot{q}_2^2l_5+sq_2l_2M_4\dot{q}_2^2q_4+sq_2l_2M_4\dot{q}_2^2l_4+l_1sq_2M_5\dot{q}_2^2q_4+\\
&+l_1sq_2M_5\dot{q}_2^2l_4+l_1sq_2M_5\dot{q}_2^2l_5+l_1sq_2M_4\dot{q}_2^2q_4+l_1sq_2M_4\dot{q}_2^2l_4-l_1cq_2M_5\ddot{q}_2q_4-\\
&-l_1cq_2M_5\ddot{q}_2l_4-sq_2M_{l_y}^6+sq_2q_3M_5\dot{q}_2^2q_4-cq_2q_3M_5\ddot{q}_2l_5+sq_2q_3M_4\dot{q}_2^2l_4-\\
&-cq_2\dot{q}_2J_y^{*5}\dot{q}_5\ ;
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
M_{l_y}^1=&-sq_2q_3M_5\ddot{q}_2l_4-2sq_2q_3M_5\dot{q}_4\dot{q}_2-sq_2q_3M_5\ddot{q}_2l_5-cq_2l_3M_5\dot{q}_2^2l_5-cq_2J_y^{*5}\ddot{q}_5-cq_2l_6F_{l_z}^6-\\
&-l_1sq_2F_{l_y}^6+sq_2sq_5M_{l_z}^6+sq_2cq_5M_{l_x}^6-sq_2l_2F_{l_y}^6-sq_2q_3F_{l_y}^6+cq_2l_2M_4\ddot{q}_4-cq_2l_3M_5\dot{q}_2^2l_4-\\
&-2sq_2q_3M_4\dot{q}_4\dot{q}_2-sq_2l_2M_4\ddot{q}_2q_4-2sq_2J_x^{*5}\dot{q}_5c^2q_5\dot{q}_2-sq_2cq_5J_x^{*5}sq_5\ddot{q}_2+\\
&+2sq_2\dot{q}_5J_z^{*5}c^2q_5\dot{q}_2-2sq_2l_3M_5\dot{q}_4\dot{q}_2-sq_2l_2M_5\ddot{q}_2q_4-sq_2l_2M_5\ddot{q}_2l_4-sq_2l_2M_5\ddot{q}_2l_5-\\
&-2sq_2l_2M_5\dot{q}_4\dot{q}_2+l_1cq_2sq_5F_{l_z}^6+l_1cq_2M_5\ddot{q}_4+l_1cq_2M_4\ddot{q}_4+cq_2l_3M_4\ddot{q}_4-sq_2l_3M_5\ddot{q}_2l_4-\\
&-sq_2l_3M_5\ddot{q}_2q_4-sq_2l_2M_4\ddot{q}_2l_4-2sq_2l_3M_4\dot{q}_4\dot{q}_2-sq_2l_3M_4\ddot{q}_2l_4-2sq_2l_2M_4\dot{q}_4\dot{q}_2-\\
&-sq_2l_3M_5\ddot{q}_2l_5-sq_2l_3M_4\ddot{q}_2q_4-cq_2q_3M_5\dot{q}_2^2q_4-cq_2q_3M_5\dot{q}_2^2l_4-cq_2q_3M_5\dot{q}_2^2l_5-\\
&-cq_2l_4M_5g-cq_2q_4M_5g-cq_2q_4M_4g-cq_2q_4M_5\ddot{q}_3-cq_2q_4M_4\ddot{q}_3+cq_2l_3M_5\ddot{q}_4-\\
&-cq_2l_5M_5g-cq_2l_4M_5\ddot{q}_3-cq_2l_4M_4g-cq_2l_4M_4\ddot{q}_3-cq_2l_5M_5\ddot{q}_3+l_1cq_2cq_5F_{l_x}^6-\\
&-cq_2l_5cq_5F_{l_z}^6-cq_2l_4cq_5F_{l_z}^6+cq_2l_2sq_5F_{l_z}^6+cq_2l_2cq_5F_{l_x}^6+cq_2M_{l_y}^6+cq_2q_3sq_5F_{l_z}^6+\\
&+cq_2q_3cq_5F_{l_x}^6+cq_2l_4sq_5F_{l_x}^6+cq_2q_4sq_5F_{l_x}^6+cq_2l_3sq_5F_{l_z}^6+cq_2l_3cq_5F_{l_x}^6+cq_2l_5sq_5F_{l_x}^6-\\
&-cq_2q_4cq_5F_{l_z}^6-cq_2l_2M_5\dot{q}_2^2l_4-cq_2l_2M_4\dot{q}_2^2l_4-l_1cq_2M_5\dot{q}_2^2q_4-sq_2l_3F_{l_y}^6+\\
&+cq_2sq_5\dot{q}_2^2J_z^{*5}cq_5-cq_2q_3M_4\dot{q}_2^2q_4-cq_2q_3M_4\dot{q}_2^2l_4-cq_2l_2M_5\dot{q}_2^2q_4+cq_2q_3M_5\ddot{q}_4+\\
&+cq_2q_3M_4\ddot{q}_4-cq_2cq_5\dot{q}_2^2J_x^{*5}sq_5-cq_2l_3M_4\dot{q}_2^2q_4-cq_2l_3M_4\dot{q}_2^2l_4-cq_2l_2M_5\dot{q}_2^2l_5-\\
&-cq_2l_2M_4\dot{q}_2^2q_4-l_1cq_2M_4\dot{q}_2^2l_4-l_1sq_2M_5\ddot{q}_2q_4-l_1sq_2M_5\ddot{q}_2l_4+sq_2\dot{q}_5J_x^{*5}\dot{q}_2-\\
&-l_1cq_2M_5\dot{q}_2^2l_4-l_1cq_2M_5\dot{q}_2^2l_5-l_1cq_2M_4\dot{q}_2^2q_4-sq_2J_z^{*5}\dot{q}_5\dot{q}_2-2l_1sq_2M_5\dot{q}_4\dot{q}_2-\\
&-l_1sq_2M_5\ddot{q}_2l_5-l_1sq_2M_4\ddot{q}_2q_4-sq_2\dot{q}_2J_y^{*5}\dot{q}_5-l_1sq_2M_4\ddot{q}_2l_4-2l_1sq_2M_4\dot{q}_4\dot{q}_2+\\
&+sq_2sq_5J_z^{*5}cq_5\ddot{q}_2+sq_2sq_5l_6F_{l_y}^6-sq_2q_3M_5\ddot{q}_2q_4-sq_2q_3M_4\ddot{q}_2q_4-sq_2q_3M_4\ddot{q}_2l_4+\\
&+cq_2l_2M_5\ddot{q}_4\ ;
\end{aligned}
\tag{50}
$$

$$
\begin{aligned}
M_{l_z}^1=&-2sq_5\dot{q}_5J_z^{*5}cq_5\dot{q}_2+2sq_5J_x^{*5}\dot{q}_5cq_5\dot{q}_2-J_x^{*5}\ddot{q}_2c^2q_5+cq_5l_6F_{l_y}^6+J_z^{*5}c^2q_5\ddot{q}_2+M_5\ddot{q}_2l_5^2+\\
&+M_5\ddot{q}_2q_4^2+M_4\ddot{q}_2q_4^2+M_5\ddot{q}_2l_4^2+M_4\ddot{q}_2l_4^2+2q_4M_4\ddot{q}_2l_4+J_x^{*5}\ddot{q}_2+\\
&+J_z^{*3}\ddot{q}_2+l_5F_{l_y}^6-sq_5M_{l_x}^6+cq_5M_{l_z}^6+q_4F_{l_y}^6+l_4F_{l_y}^6+2q_4M_5\dot{q}_4\dot{q}_2+\\
&+2q_4M_5\ddot{q}_2l_4+2l_5M_5\dot{q}_4\dot{q}_2+J_z^{*2}\ddot{q}_2+2l_5M_5\ddot{q}_2q_4+2l_5M_5\ddot{q}_2l_4+\\
&+q_4M_5cq_2\ddot{q}_1+l_5M_5cq_2\ddot{q}_1+q_4M_4cq_2\ddot{q}_1+l_4M_5cq_2\ddot{q}_1+l_4M_4cq_2\ddot{q}_1+\\
&+J_z^{*4}\ddot{q}_2+2l_4M_4\dot{q}_4\dot{q}_2+2l_4M_5\dot{q}_4\dot{q}_2+2q_4M_4\dot{q}_4\dot{q}_2\,.
\end{aligned}
\tag{51}
$$

The generalized driving forces have the following expressions:

$$Q_m^1 = \left[\overline{F}_{l_1}^1\right]^T \cdot \overline{j}_1^1 = \left[F_{l_x}^1 \quad F_{l_y}^1 \quad F_{l_z}^1\right] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = F_{l_y}^1; \tag{52}$$

$$\begin{aligned} Q_m^1 = {} & M_5\ddot{q}_1 + M_4\ddot{q}_1 + M_3\ddot{q}_1 + sq_2M_5\ddot{q}_4 + sq_2sq_5F_{l_z}^6 + sq_2cq_5F_{l_x}^6 + M_1\ddot{q}_1 - sq_2M_5\dot{q}_2^2q_4 - \\ & - sq_2M_5\dot{q}_2^2l_4 - sq_2M_5\dot{q}_2^2l_5 + cq_2F_{l_y}^6 + M_2\ddot{q}_1 + cq_2M_5\ddot{q}_2l_5 + cq_2M_4\ddot{q}_2q_4 + \\ & + cq_2M_4\ddot{q}_2l_4 + 2cq_2M_4\dot{q}_4\dot{q}_2 + 2cq_2M_5\dot{q}_4\dot{q}_2 - sq_2M_4\dot{q}_2^2q_4 - sq_2M_4\dot{q}_2^2l_4 + \\ & + cq_2M_5\ddot{q}_2q_4 + cq_2M_5\ddot{q}_2l_4 + sq_2M_4\ddot{q}_4; \end{aligned} \tag{53}$$

$$Q_m^2 = \left[\overline{M}_{l_{O_2}}^2\right]^T \cdot \overline{k}_2^2 = \left[M_{l_x}^2 \quad M_{l_y}^2 \quad M_{l_z}^2\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = M_{l_z}^2; \tag{54}$$

$$\begin{aligned} Q_m^2 = {} & -2sq_5\dot{q}_5J_z^{*5}cq_5\dot{q}_2 + 2sq_5J_x^{*5}\dot{q}_5cq_5\dot{q}_2 - J_x^{*5}\ddot{q}_2c^2q_5 + cq_5l_6F_{l_y}^6 + J_z^{*5}c^2q_5\ddot{q}_2 + M_5\ddot{q}_2l_5^2 + \\ & + M_5\ddot{q}_2q_4^2 + M_4\ddot{q}_2q_4^2 + M_5\ddot{q}_2l_4^2 + M_4\ddot{q}_2l_4^2 + 2q_4M_4\ddot{q}_2l_4 + J_x^{*5}\ddot{q}_2 + \\ & + J_z^{*3}\ddot{q}_2 + l_5F_{l_y}^6 - sq_5M_{l_x}^6 + cq_5M_{l_z}^6 + q_4F_{l_y}^6 + l_4F_{l_y}^6 + 2q_4M_5\dot{q}_4\dot{q}_2 + \\ & + 2q_4M_5\ddot{q}_2l_4 + 2l_5M_5\dot{q}_4\dot{q}_2 + J_z^{*2}\ddot{q}_2 + 2l_5M_5\ddot{q}_2q_4 + 2l_5M_5\ddot{q}_2l_4 + \\ & + q_4M_5cq_2\ddot{q}_1 + l_5M_5cq_2\ddot{q}_1 + q_4M_4cq_2\ddot{q}_1 + l_4M_5cq_2\ddot{q}_1 + l_4M_4cq_2\ddot{q}_1 + \\ & + J_z^{*4}\ddot{q}_2 + 2l_4M_4\dot{q}_4\dot{q}_2 + 2l_4M_5\dot{q}_4\dot{q}_2 + 2q_4M_4\dot{q}_4\dot{q}_2; \end{aligned} \tag{55}$$

$$Q_m^3 = \left[\overline{F}_{l_3}^3\right]^T \cdot \overline{k}_3^3 = \left[F_{l_x}^3 \quad F_{l_y}^3 \quad F_{l_z}^3\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = F_{l_z}^3; \tag{56}$$

$$Q_m^3 = -sq_5F_{l_x}^6 + M_5g + M_5\ddot{q}_3 + cq_5F_{l_z}^6 + M_4g + M_4\ddot{q}_3 + M_3g + M_3\ddot{q}_3; \tag{57}$$

$$Q_m^4 = \left[\overline{F}_{l_4}^4\right]^T \cdot \overline{i}_4^4 = \left[F_{l_x}^4 \quad F_{l_y}^4 \quad F_{l_z}^4\right] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = F_{l_x}^4; \tag{58}$$

$$\begin{aligned} Q_m^4 = {} & cq_5F_{l_x}^6 + sq_5F_{l_z}^6 + M_5sq_2\ddot{q}_1 - M_5\dot{q}_2^2q_4 - M_5\dot{q}_2^2l_4 + M_5\ddot{q}_4 - M_5\dot{q}_2^2l_5 + M_4sq_2\ddot{q}_1 - \\ & - M_4\dot{q}_2^2q_4 - M_4\dot{q}_2^2l_4 + M_4\ddot{q}_4; \end{aligned} \tag{59}$$

$$Q_m^5 = \left[\overline{M}_{l_{O_5}}^5\right]^T \cdot \overline{j}_5^5 = \left[M_{l_x}^5 \quad M_{l_y}^5 \quad M_{l_z}^5\right] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = M_{l_y}^5; \tag{60}$$

$$Q_m^5 = M_{l_y}^6 - l_6F_{l_y}^5 + J_y^{*5}\ddot{q}_5 - cq_5\dot{q}_2^2J_x^{*5}sq_5 + sq_5\dot{q}_2^2J_z^{*5}cq_5. \tag{61}$$

3. CONCLUSION

The expressions of these generalized driving forces (53), (55), (57), (59), (61), represent the system of dynamic differential equations, describing the dynamic model of the TRTTR1 modular serial robot.

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MODELAREA DINAMICĂ A ROBOTULUI SERIAL MODULAR TRTTR1

Rezumat: În lucrare, autorii efectuează un studiu dinamic al unui robot serial modular cu cinci grade de libertate de tip TRTTR1. Metoda folosită este formalismul Newton-Euler. La început sunt prezentate câteva consideraţii generale asupra metodei iterative Newton-Euler, care apoi sunt aplicate pentru deducerea ecuaţiilor dinamice ale robotului TRTTR1. Modelarea dinamică a robotului s-a făcut cu un program de modelare simbolică, *Robot_Symbolic*, modulul *Robot_Dynamics*. Acest program presupune cunoaşterea parametrilor de distribuţie a maselor şi a parametrilor geometrici şi cinematici obţinuţi prin modelările geometrică şi cinematică ale robotului. Pentru început se parcurge structura mecanică a robotului prin iteraţii spre exterior, obţinând astfel torsorul forţelor exterioare. Apoi, se parcurge structura mecanică a robotului prin iteraţii spre interior, pentru a determina torsorul forţelor de legătură, respectiv forţele generalizate motoare din cuplele robotului. Expresiile acestor forţe generalizate constituie sistemul de ecuaţii diferenţiale de mişcare a robotului.

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