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STUDY OF PREDICTIVE MAINTENANCE FOR THE ELASTIC ROTOR IN ELASTIC BEARING WITH DAMPING

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Abstract: *The paper contains theoretical study for machines and equipment having elastic rotors in rotational motion. It analyses the dynamic rotary machines, fitted with elastic rotor in elastic bearings with damping, as well as the rotor symmetrically rotational movement with the elastic bearings with depreciation. The main dynamic rotating equipment defects are removed by static and dynamic balancing, which apply directly to predictive maintenance.*
Key words: *predictive maintenance, elastic rotor, rigid bearing with damping.*

1. INTRODUCTION

Corrective action to eliminate the main defects that occur in dynamic rotating machines are one of the most important components of the implementation of predictive maintenance and these are designed to bring the machines in a dynamic mode compatible with the system and it is necessary to work at a *Good grade* or *Utilisable*. The main defects that occur in dynamic rotating machines are imbalance and misalignment [Cîm 14].

The rotor is a subset of these machines, consisting of a shaft which is one or more discs and executing a movement of rotation around its own axis. As a form, they can be simple or complex, but, regardless of type, being a moving element rotation determines the dynamic properties specific to rotor machines, which do not occur in the other types of machinery or structures.

From great machine rotor class belong the following subclasses of machines: motors, generators, turbines, compressors, pumps and blowers.

In machine operation, the rotor is subjected to vibration of bending and twisting. These vibrations are dependent on the geometry of the rotor and bearing type, and the excitatoare forces. The rotor, the precession, he turns his own Foundation. Complexity of dynamic phenomena is increased if it takes into account the fact that the hydro and aerodynamic forces can act upon the rotor, with variable gradient of temperature and pressure fields, electromagnetic fields etc. [Arg 02].

Types of bearings used in rotor machines are [Don 02]: bearings with ball bearings, bearings, sliding bearings, bearing with gas.

The machines with large power, the most commonly encountered are the berings with sliding, because of their special features: high capacity, high durability, high depreciation, which is the study of this paper.

2. ELASTIC ROTOR SUBJECTED TO ELASTIC BEARING WITH DAMPING

2.1 Theoretical Consideration

Either an elastic rotor, seated in the elastic bearing, in order to simplify calculations, elasticities are considered izotrope, so $k_x=k_y=k_p$. It is also considered that the damping forces are proportional to absolute velocity of the shaft, and the coefficients for the depreciation in the camp shall be independent of direction of movement.

There is a disk of mass "m" fixed on a shaft that rotates with constant angular velocity in two elastic bearings (Fig. 1). Elastic constant of the shaft is considered to be less than 10% of the constant elasticity of the concentration bearings.

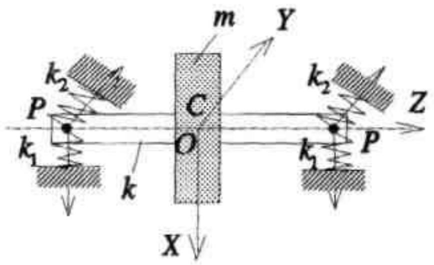

Fig. 1. The elastic rotor on the elastic bearing

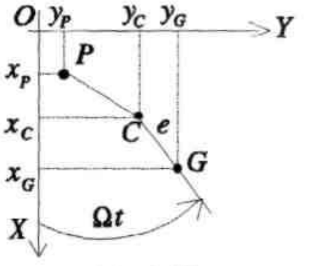

Fig. 2. The points possitions

It is considered also that the gravity centre of the disc, the G point, does not coincide with its geometric center, the point C, which coincides with the centre of the cross section of the shaft. It neglects the mass of the shaft and all the forces of friction; when Ω=0 (the shaft does not rotate), the rotor axes does not deformează; the elastic constant of the shaft is k.

The C point is geometrical center of the shaft; the G point is the gravity centre of the disc ; ē is the excentricity, hence the distance between them is a constant, and $r_c = OC$ is the arrow shaft in the disc plane. The first of these forces depends on the elasticity of the shaft and is proportional to the arrow. She does so the value "kr_c", turning to the rotation center, the O point.

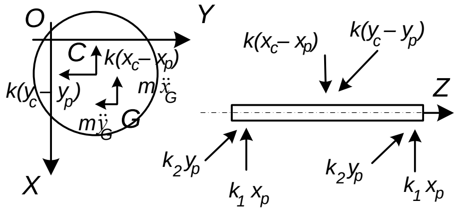

Fig. 3. Mechanical scheme of the rotor and shaft charge [Cîm 14]

The principle of D'Alembert's is applyed for the fictional dynamic equilibrium and obtain the scalar differential equations system:

- for the disc:

$$\begin{cases} m\,\ddot{x}_G + k(x_C - x_P) = 0 \\ m\,\ddot{y}_G + k(y_C - y_P) = 0 \end{cases} \quad (1)$$

- for the shaft they are:

$$\begin{cases} 2c\,\dot{x}_P + 2k_P x_P = k(x_C - x_P) \\ 2c\,\dot{y}_P + 2k_P y_P = k(y_C - y_P) \end{cases} \quad (2)$$

- But there are the linkage relations, using the symbols as in [Cîm 14]:

$$\begin{cases} x_G = x_C + \bar{e}\cos\Omega t \\ y_G = y_C + \bar{e}\sin\Omega t \end{cases} \quad (3)$$

If they are working with compexe variables, the notations are:

$$\begin{cases} r_P = x_P + i y_P \\ r_C = x_C + i y_C \\ r_G = x_G + i y_G \end{cases} \quad (4)$$

Using the (4) relations, the (1) becomes, and gives the disc mass centre motion equation:

$$m\,\ddot{r}_C + k(r_C - r_P) = m\,\bar{e}\,\Omega^2 e^{i\Omega t} \quad (5)$$

With (4) the (2) becomes, and contains the shaft mass centre motion equation:

$$2c\,\dot{r}_P + 2k_P r_P + k(r_P - r_C) = 0 \quad (6)$$

It can noted $\omega = \sqrt{k/m}$ the rotor owner pulsation, and $N = k/(2k_P)$ is the ration between the shaft elasticity constant and the elasticity constant of the bearing. If $\zeta = 2c/c_{cr}$ where the critical constant is $c_{cr} = 2\sqrt{km}$, and the result becomes:

$$\begin{cases} \ddot{r}_c + \omega^2 (r_c - r_p) = \bar{e}\Omega^2 e^{i\Omega t} \\ 2\zeta\omega\,\dot{r}_P + \dfrac{1}{N}\omega^2 r_P + \omega^2 (r_P - r_C) = 0 \end{cases} \quad (7)$$

a) The general solution to the homogeneous equation

For the rotor in equilibrium $\bar{e}=0$, the solution of the (7) has the form: $r_C=R_Ce^{\lambda t}$ for the mass center, and $r_P=R_Pe^{\lambda t}$ for the disc. Using them the (7) will be a homogeneous algebraic equation system:

$$\begin{cases}(2\zeta\omega\lambda+\frac{1}{N}\omega^2+\omega^2)R_P-\omega^2 R_C=0 \\ -\omega^2 R_P+(\lambda^2+\omega^2)R_C=0\end{cases} \quad (8)$$

To remove the trivial solution, the determinant of the system is equivalent to the zero, hence:

$$\begin{vmatrix} 2\zeta\omega\lambda+\omega^2(1+\frac{1}{N}) & -\omega^2 \\ -\omega^2 & \omega^2+\lambda^2 \end{vmatrix}=0 \quad (9)$$

which leads to the characteristic equation:

$$\left[\left(\frac{\lambda}{\omega}\right)+A\right]\cdot\left[\left(\frac{\lambda}{\omega^2}\right)^2+2B\left(\frac{\lambda}{\omega}\right)+B^2+C^2\right]=0 \quad (10)$$

This equation has a negative real root: $(\lambda/\omega)_1=-A$, and has two complex conjugate roots with negative real part $(\lambda/\omega)_{2,3}=-B\pm iC$, in this situation the the system has a stable motion, because $-B>0$ during the motion.

Free movement of C point shall be described by a solution of the form:

$$r_C(t)=R_{C1}e^{-A\omega t}+R_{C2}e^{-B\omega t}+R_{C3}e^{-B\omega t}e^{-iC\omega t} \quad (11)$$

The owner pulsation of the free vibration (which is also a perfectly balanced rotor $\bar{e}=0$), is $\omega_{owner}=C\,\omega$, where C is the imaginary part of the complex roots of the characteristic equation.

b) The particular solution to the homogeneous equation

If you are studying only stationary movement by mass imbalance of the rotor, it will seek solutions with the form:

$$r_C(t)=\tilde{r}_C e^{i\Omega t} \qquad r_P(t)=\tilde{r}_{P}e^{i\Omega t} \quad (12)$$

where: $\tilde{r}_C$ and $\tilde{r}_P$ are the vector in the complex plane of the corresponding amplitude of the points C and P movement.

3. CONCLUSIONS

In the study of the elastic rotor subjected to elastic bearing with damping, there are the following conclusions:

1. Points P, C, G describe circular orbits with its centre at the point a, but this time, they are not collinear, there is always short-circuit between vectors OP, OG and OC (Fig. 4.).

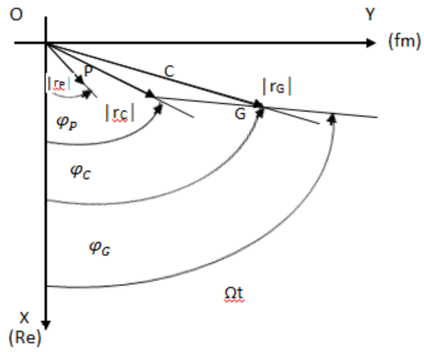

Fig. 4. The OP, OG, and OC vectors

2. Maximum amplitude of speeds for P and G appear at the different angular velocities ω_P and ω_C , they are different as the corresponding angular velocities demonstrated in our previous study [Cîm 14] (for isotropic bearings).
3. The angular velocities ω_P and ω_C for which the rotor has the maximum amplitudes have the names of *angular speeds critical imbalance*. Sometimes they vary considerably depending on the rotor critical speeds capital, being much older than they.
4. **A calculation so that it neglects the depreciation of the bearings will give erroneous results of critical pulsations!**

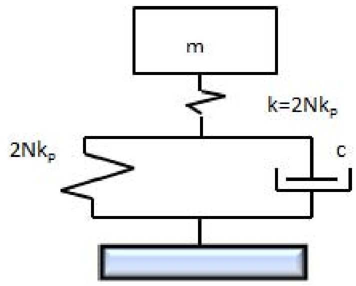

Fig. 5. Mechanical Model

5. The study can have the mechanical model as in figure 5 with one freedom degree, depending of the system mechanical characteristics.
6. The above observations can be summarized using the model shown in Figure 5. In Figure 6 it can be seen that the pulsations of the system increase with the increase in depreciation (damping) from the bearings ($0 < c < \infty$).

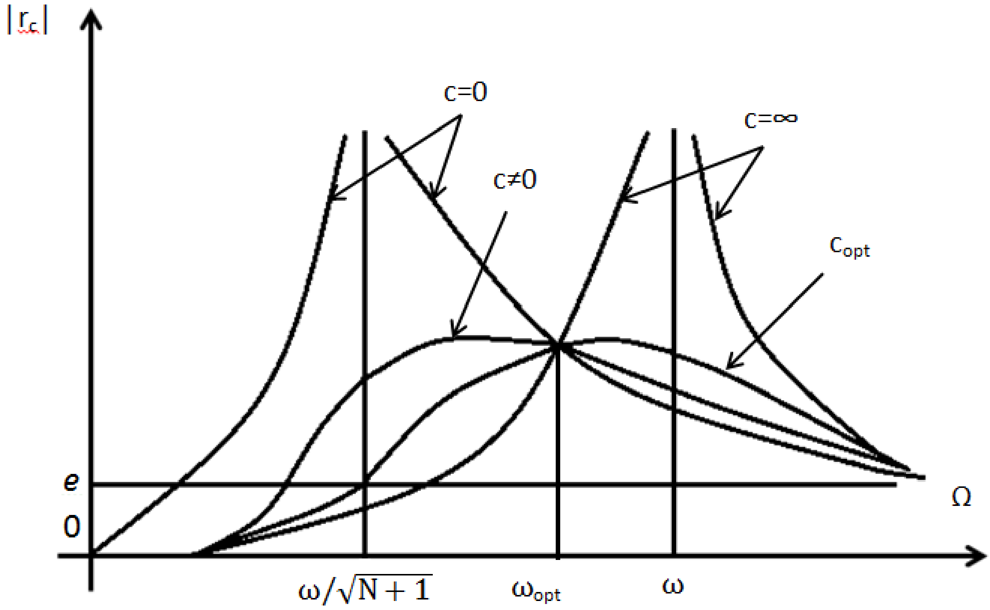

Fig. 6. The system owner pulsations

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Studiu asupra mentenanței predictive pentru un rotor elastic în lagăre elastic cu amortizare

Rezumat: *Lucrarea conține fundamentarea teoretică pentru masinile si utilajele, cu organe de masini in miscare de rotatie. Se analizează masinile dinamice rotative, prevazute cu rotor elastic in lagare elastic cu amortizare, precum si rotorul simetric cu miscarea de rotatie in lagare elastic cu amortizare. Principalele defecte la echipamentele dinamice rotative sunt eliminate prin echilibrarea statica si dinamica, prin care se aplica direct mentenanta predictivă.*

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