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THE DIRECT KINEMATIC MODEL OF THE ARTICULATE 6R ROBOT

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Abstract: *In this paper is presented the direct kinematic model of the articulate 6R robot. By this model the disadvantages of geometric modeling are eliminated due to non-linearity of the geometrical equations and the lack of control over velocity and acceleration on the motion trajectory.*

Key words: *kinematic modeling, geometrical equations, generalized velocities, generalized accelerations,*

1. INTRODUCTION

In the kinematic modeling the static assumption is removed and the generalized and operational coordinates of the robot are functions of time. The kinematic modeling involves solving the two fundamental mutual problems of the robots kinematics: direct and inverse. In the case of kinematic modeling the generalized coordinates, velocities and accelerations are known and are determined the operational velocities and accelerations which define together with $\overline{X}^{(n)0}$ the clamping device movement relative to the fixed system (T_0). For solving the problem of the direct kinematic modeling is used the iterative method. This method is one of the frequently used in the kinematic modeling, according to [1] and [2]. It is based on the introduction in calculus of the position vectors, of the rotation matrices and their derivatives with respect to time.

2. THE DIRECT KINEMATIC MODEL

The direct kinematic modeling of the articulate robots is to determine the operational kinematic parameters of the clamping device in relation with the reference mobile system (T_n) jointly with the clamping device and in relation to fixed reference system (T_0) of the robot, if are known constructive mechanical parameters of the robot and the instantaneous values of the coordinates, generalized velocities and accelerations from the robot couplers. In figure 1 is shown the articulate 6R robot, that was geometric modelated in [3].

The dates obtained from the geometric modeling can be used to the direct kinematic modeling of the robot. The homogeneous transformation matrices can be determined using the rotation matrices and position vectors. Thus, the matrix expression of these matrices is given below:

$$[T]_1^0(t)=\left[\begin{array}{ccc|c} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \tag{1}$$

$$[T]_2^1(t)=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & cq_2 & -sq_2 & 0 \\ 0 & sq_2 & cq_2 & l_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]; \tag{2}$$

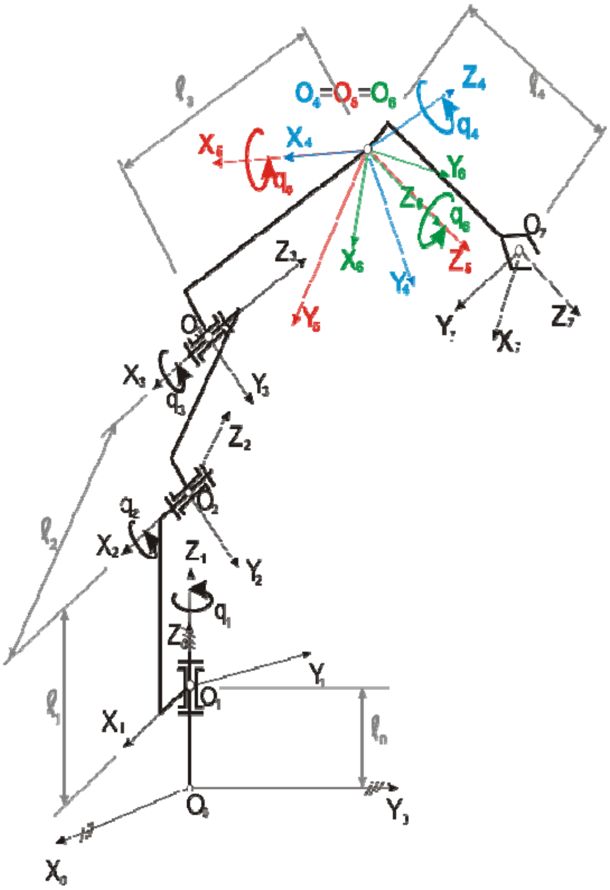

Fig. 1. The kinematic scheme of the articulate 6R robot

$$[T]_3^2(t)=\left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & cq_3 & -sq_3 & 0\\ 0 & sq_3 & cq_3 & l_2\\ \hline 0 & 0 & 0 & 1\end{array}\right]; \quad (3)$$

$$[T]_4^3(t)=\left[\begin{array}{ccc|c}cq_4 & -sq_4 & 0 & 0\\ sq_4 & cq_4 & 0 & 0\\ 0 & 0 & 1 & l_3\\ \hline 0 & 0 & 0 & 1\end{array}\right]; \quad (4)$$

$$[T]_5^4(t)=\left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & cq_5 & -sq_5 & 0\\ 0 & sq_5 & cq_5 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]; \quad (5)$$

$$[T]_6^5(t)=\left[\begin{array}{ccc|c}cq_6 & -sq_6 & 0 & 0\\ sq_6 & cq_6 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]; \quad (6)$$

$$[T]_7^6(t)=\left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & l_4\\ \hline 0 & 0 & 0 & 1\end{array}\right]. \quad (7)$$

The inverse rotation matrices are calculated with the following relation below, according to [2]:

$$[R]_{i-1}^{i}=\left[R_i^{i-1}\right]^{-1}=\left[R_i^{i-1}\right]^{T}. \quad (8)$$

Thus, it resulted:

$$[R]_0^1=\begin{bmatrix}cq_1 & sq_1 & 0\\ -sq_1 & cq_1 & 0\\ 0 & 0 & 1\end{bmatrix};\quad [R]_1^2=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_2 & sq_2\\ 0 & -sq_2 & cq_2\end{bmatrix}; \quad (9)$$

$$[R]_2^3=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_3 & sq_3\\ 0 & -sq_3 & cq_3\end{bmatrix};\quad [R]_3^4=\begin{bmatrix}cq_4 & sq_4 & 0\\ -sq_4 & cq_4 & 0\\ 0 & 0 & 1\end{bmatrix}; \quad (10)$$

$$[R]_4^5=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_5 & sq_5\\ 0 & -sq_5 & cq_5\end{bmatrix};\quad [R]_5^6=\begin{bmatrix}cq_6 & sq_6 & 0\\ -sq_6 & cq_6 & 0\\ 0 & 0 & 1\end{bmatrix}; \quad (11)$$

$$[R]_6^7=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}. \quad (12)$$

The matrix expression of the kinematic axes vectors, respectively of the O_7x_7, is as follows:

$$[\bar{k}]_1^1=\begin{bmatrix}0\\0\\1\end{bmatrix};\quad [\bar{i}]_2^2=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad [\bar{i}]_3^3=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad [\bar{k}]_4^4=\begin{bmatrix}0\\0\\1\end{bmatrix};$$
$$[\bar{i}]_5^5=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad [\bar{k}]_6^6=\begin{bmatrix}0\\0\\1\end{bmatrix};\quad [\bar{k}]_7^7=\begin{bmatrix}0\\0\\1\end{bmatrix}. \quad (13)$$

The kinematic parameters expressed matrix corresponding to the robot base are:

$$[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{v}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{\varepsilon}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{a}]_0^0=\begin{bmatrix}0\\0\\g\end{bmatrix}. \quad (14)$$

Using the inverse matrices and the kinematic parameters the operational angular velocities matrix transposed, of the elements i, i=1÷7. Thus:

$$[\overline{\omega}]_1^1=[R]_0^1\cdot[\overline{\omega}]_0^0+\dot{q}_1\cdot[\overline{k}]_1^1=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}; \tag{15}$$

$$[\overline{\omega}]_2^2=[R]_1^2\cdot[\overline{\omega}]_1^1+\dot{q}_2\cdot[\overline{i}]_2^2=\begin{bmatrix}\dot{q}_2\\\dot{q}_1 sq_2\\\dot{q}_1 cq_2\end{bmatrix}; \tag{16}$$

$$[\overline{\omega}]_3^3=[R]_2^3\cdot[\overline{\omega}]_2^2+\dot{q}_3\cdot[\overline{i}]_3^3=\begin{bmatrix}\dot{q}_2+\dot{q}_3\\\dot{q}_1 sq_2+\dot{q}_1 sq_3\\\dot{q}_1 cq_2+\dot{q}_1 cq_3\end{bmatrix}; \tag{17}$$

$$[\overline{\omega}]_4^4=[R]_3^4\cdot[\overline{\omega}]_3^3+\dot{q}_4\cdot[\overline{k}]_4^4=$$
$$=\begin{bmatrix}cq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1 sq_4 sq_3+\dot{q}_1 sq_2 sq_4\\-sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1 cq_4 sq_3+\dot{q}_1 sq_2 cq_4\\\dot{q}_1 cq_2+\dot{q}_1 cq_3+\dot{q}_4\end{bmatrix}; \tag{18}$$

$$[\overline{\omega}]_5^5=[R]_4^5\cdot[\overline{\omega}]_4^4+\dot{q}_5\cdot[\overline{i}]_5^5=$$
$$=\begin{bmatrix}\dot{q}_2 cq_4+\dot{q}_3 cq_4+\dot{q}_1 sq_4 sq_3+\dot{q}_1 sq_2 sq_4+\dot{q}_5\\\hdashline cq_5(-\dot{q}_2 sq_4-\dot{q}_3 sq_4+\dot{q}_1 cq_4 sq_3+\dot{q}_1 sq_2 cq_4)+\\+sq_5(\dot{q}_1 cq_3+\dot{q}_1 cq_2+\dot{q}_4)\\\hdashline -sq_5(-\dot{q}_2 sq_4-\dot{q}_3 sq_4+\dot{q}_1 cq_4 sq_3+\dot{q}_1 sq_2 cq_4)+\\+cq_5(\dot{q}_1 cq_3+\dot{q}_1 cq_2+\dot{q}_4)\end{bmatrix}. \tag{19}$$

Because of the fact that the operational angular velocities $[\overline{\omega}]_6^6, [\overline{\omega}]_7^7$ are complex, those are no longer written but method of calculation is similar to that used for calculating previous velocities. The antisymetric matrix 3x3, type $\{\overline{\omega}\times\}$, in which $\overline{\omega}$ is the vector angular velocity, can be expressed by relation below, according to [4]:

$$\{\overline{\omega}\times\}=\begin{bmatrix}0&-\omega_z&\omega_y\\\omega_z&0&-\omega_x\\-\omega_y&\omega_x&0\end{bmatrix}. \tag{20}$$

In the following matrix relations, the vector products and double vector that appear are expressed according to (20). The expressions of the operational linear velocity transposed matrix are given by relations below:

$$[\overline{v}]_1^1=[R]_0^1\cdot\{\overline{v}_0^0+\overline{\omega}_0^0\times\overline{r}_1^0\};\quad [\overline{v}]_1^1=\begin{bmatrix}0\\0\\0\end{bmatrix}; \tag{21}$$

$$[\overline{v}]_2^2=[R]_1^2\cdot\{\overline{v}_1^1+\overline{\omega}_1^1\times\overline{r}_2^1\};\quad [\overline{v}]_2^2=\begin{bmatrix}0\\0\\0\end{bmatrix}; \tag{22}$$

$$[\overline{v}]_3^3=[R]_2^3\cdot\{\overline{v}_2^2+\overline{\omega}_2^2\times\overline{r}_3^2\};\quad [\overline{v}]_3^3=\begin{bmatrix}\dot{q}_1 l_2 sq_2\\-\dot{q}_2 l_2 cq_3\\\dot{q}_2 l_2 sq_3\end{bmatrix}; \tag{23}$$

$$[\overline{v}]_4^4=[R]_3^4\cdot\{\overline{v}_3^3+\overline{\omega}_3^3\times\overline{r}_4^3\}; \tag{24}$$

$$[\overline{v}]_4^4=\begin{bmatrix}\dot{q}_1 cq_4(l_2 sq_2+l_3 sq_3+l_3 sq_2)+sq_4(-\dot{q}_2 l_2 cq_3-l_3\dot{q}_2-l_3\dot{q}_3)\\-\dot{q}_1 sq_4(l_2 sq_2+l_3 sq_3+l_3 sq_2)+cq_4(-\dot{q}_2 l_2 cq_3-l_3\dot{q}_2-l_3\dot{q}_3)\\\dot{q}_2 l_2 sq_3\end{bmatrix}; \tag{25}$$

$$[\overline{v}]_5^5=[R]_4^5\cdot\{\overline{v}_4^4+\overline{\omega}_4^4\times\overline{r}_5^4\}; \tag{26}$$

$$[\overline{v}]_5^5=\begin{bmatrix}\dot{q}_1 cq_4(l_2 sq_2+l_3 sq_3+l_3 sq_2)+sq_4(-\dot{q}_2 l_2 cq_3-l_3\dot{q}_2-l_3\dot{q}_3)\\\hdashline cq_5[-\dot{q}_1 sq_4(l_2 sq_2+l_3 sq_3+l_3 sq_2)+\\+cq_4(-\dot{q}_2 l_2 cq_3-l_3\dot{q}_2-l_3\dot{q}_3)+\dot{q}_2 l_2 sq_5 sq_3]\\\hdashline -sq_5[-\dot{q}_1 sq_4(l_2 sq_2+l_3 sq_3+l_3 sq_2)+\\+cq_4(-\dot{q}_2 l_2 cq_3-l_3\dot{q}_2-l_3\dot{q}_3)+\dot{q}_2 l_2 cq_5 sq_3]\end{bmatrix}. \tag{27}$$

The angular operational accelerations can be matrix expressed by the following relations:

$$[\overline{\varepsilon}]_1^1=[R]_0^1\cdot[\overline{\varepsilon}]_0^0+\{[R]_0^1\cdot\overline{\omega}_0^0\times\dot{q}_1\cdot\overline{k}_1^1+\ddot{q}_1\cdot\overline{k}_1^1\};\quad [\overline{\varepsilon}]_1^1=\begin{bmatrix}0\\0\\\ddot{q}_1\end{bmatrix}; \tag{28}$$

$$[\overline{\varepsilon}]_2^2=[R]_1^2\cdot[\overline{\varepsilon}]_1^1+\{[R]_1^2\cdot\overline{\omega}_1^1\times\dot{q}_2\cdot\overline{k}_2^2+\ddot{q}_2\cdot\overline{k}_2^2\};$$
$$[\overline{\varepsilon}]_2^2=\begin{bmatrix}\ddot{q}_2\\\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2\\\ddot{q}_1 cq_2-\dot{q}_1\dot{q}_2 sq_2\end{bmatrix}; \tag{29}$$

$$[\overline{\varepsilon}]_3^3=[R]_2^3\cdot[\overline{\varepsilon}]_2^2+\{[R]_2^3\cdot\overline{\omega}_2^2\times\dot{q}_3\cdot\overline{i}_3^3+\ddot{q}_3\cdot\overline{i}_3^3\}; \tag{30}$$

$$[\bar{\varepsilon}]_3^3=\begin{bmatrix} \ddot{q}_2+\ddot{q}_3 \\ \ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3 \\ \ddot{q}_1 cq_3+\ddot{q}_1 cq_2-\dot{q}_1\dot{q}_2 sq_3-\dot{q}_1\dot{q}_2 sq_2-\dot{q}_1\dot{q}_3 sq_2-\dot{q}_1\dot{q}_3 sq_3 \end{bmatrix}; \tag{31}$$

$$[\bar{\varepsilon}]_4^4=[R]_3^4\cdot[\bar{\varepsilon}]_3^3+\left\{[R]_3^4\cdot\bar{\omega}_3^3\times\dot{q}_4\cdot\bar{k}_4^4+\ddot{q}_4\cdot\bar{k}_4^4\right\}; \tag{32}$$

$$[\bar{\varepsilon}]_4^4=\begin{bmatrix} \ddot{q}_2 cq_4+\ddot{q}_3 cq_4+sq_4(\ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3)+\dot{q}_4(\dot{q}_1 cq_4 sq_3+\dot{q}_1 cq_4 sq_2-\dot{q}_2 sq_4-\dot{q}_3 sq_4) \\ \ddot{q}_2 sq_4+\ddot{q}_3 sq_4+cq_4(\ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3)+\dot{q}_4(\dot{q}_1 sq_4 sq_3-\dot{q}_1 sq_4 sq_2-\dot{q}_2 cq_4-\dot{q}_3 cq_4) \\ \ddot{q}_1 cq_3+\ddot{q}_1 cq_2-\dot{q}_1\dot{q}_2 sq_3-\dot{q}_1\dot{q}_2 sq_2-\dot{q}_1\dot{q}_3 sq_2-\dot{q}_1\dot{q}_3 sq_3+\ddot{q}_4 \end{bmatrix}. \tag{33}$$

$$[\bar{\varepsilon}]_5^5=[R]_4^5\cdot[\bar{\varepsilon}]_4^4+\left\{[R]_4^5\cdot\bar{\omega}_4^4\times\dot{q}_5\cdot\bar{i}_5^5+\ddot{q}_5\cdot\bar{i}_5^5\right\}; \tag{34}$$

$$[\bar{\varepsilon}]_5^5=$$
$$=\begin{bmatrix} \ddot{q}_2 cq_4+\ddot{q}_3 cq_4+sq_4(\ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3)+\dot{q}_4(\dot{q}_1 cq_4 sq_3+\dot{q}_1 cq_4 sq_2-\dot{q}_2 sq_4-\dot{q}_3 sq_4)+\ddot{q}_5 \\ cq_5[-\ddot{q}_2 sq_4-\ddot{q}_3 sq_4+cq_4(\ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3)+\dot{q}_4(-\dot{q}_1 sq_4 sq_3-\dot{q}_1 sq_4 sq_2-\dot{q}_2 cq_4-\dot{q}_3 cq_4)]+sq_5(\ddot{q}_1 cq_3+\ddot{q}_1 cq_2-\dot{q}_1\dot{q}_2 sq_3-\dot{q}_1\dot{q}_2 sq_2-\dot{q}_1\dot{q}_3 sq_2-\dot{q}_1\dot{q}_3 sq_3+\ddot{q}_4)+\dot{q}_5[(\dot{q}_1 cq_5 cq_3+\dot{q}_1 cq_5 cq_2+cq_5\dot{q}_4)+sq_5(\dot{q}_2 sq_4+\dot{q}_3 sq_4-\dot{q}_1 cq_4 sq_3-\dot{q}_1 cq_4 sq_2)] \\ -sq_5[\ddot{q}_2 sq_4+\ddot{q}_3 sq_4+cq_4(\ddot{q}_1 sq_3+\ddot{q}_1 sq_2+\dot{q}_1\dot{q}_2 cq_2+\dot{q}_1\dot{q}_2 cq_3+\dot{q}_1\dot{q}_3 cq_2+\dot{q}_1\dot{q}_3 cq_3)+\dot{q}_4(-\dot{q}_1 sq_4 sq_3-\dot{q}_1 sq_4 sq_2-\dot{q}_2 cq_4-\dot{q}_3 cq_4)]+cq_5(\ddot{q}_1 cq_3+\ddot{q}_1 cq_2-\dot{q}_1\dot{q}_2 sq_3-\dot{q}_1\dot{q}_2 sq_2-\dot{q}_1\dot{q}_3 sq_2-\dot{q}_1\dot{q}_3 sq_3+\ddot{q}_4)+\dot{q}_5[(-\dot{q}_1 sq_5 cq_3-\dot{q}_1 sq_5 cq_2-sq_5\dot{q}_4)+cq_5(\dot{q}_2 sq_4+\dot{q}_3 sq_4-\dot{q}_1 cq_4 sq_3-\dot{q}_1 cq_4 sq_2)] \end{bmatrix}; \tag{35}$$

The expression of the angular operational accelerations $[\bar{\varepsilon}]_6^6$, $[\bar{\varepsilon}]_7^7$, are complex but like the operational angular velocities are determined like the previous angular accelerations. The linear operational accelerations matrix expressed can be determined, as follows:

$$[\bar{a}]_1^1=[R]_0^1\cdot\left\{\bar{a}_0^0+\bar{\varepsilon}_0^0\times\bar{r}_1^0+\bar{\omega}_0^0\times\left(\bar{\omega}_0^0\times\bar{r}_1^0\right)\right\};\ [\bar{a}]_1^1=\begin{bmatrix}0\\0\\g\end{bmatrix}; \tag{36}$$

$$[\bar{a}]_2^2=[R]_1^2\cdot\left\{\bar{a}_1^1+\bar{\varepsilon}_1^1\times\bar{r}_2^1+\bar{\omega}_1^1\times\left(\bar{\omega}_1^1\times\bar{r}_2^1\right)\right\};\quad [\bar{a}]_2^2=\begin{bmatrix}0\\gsq_2\\gcq_2\end{bmatrix}; \tag{37}$$

$$[\bar{a}]_3^3=[R]_2^3\cdot\left\{\bar{a}_2^2+\bar{\varepsilon}_2^2\times\bar{r}_3^2+\bar{\omega}_2^2\times\left(\bar{\omega}_2^2\times\bar{r}_3^2\right)\right\}; \tag{38}$$

$$[\bar{a}]_3^3=\begin{bmatrix} \ddot{q}_1 l_2 sq_2+2\dot{q}_1\dot{q}_2 l_2 cq_2 \\ gs(q_3+q_2)-\ddot{q}_2 l_2 cq_3+1/2\dot{q}_1^2 l_2 s(q_3+2q_2)-1/2\dot{q}_1^2 l_2 sq_3-\dot{q}_2^2 l_2 sq_3 \\ gc(q_3+q_2)+\ddot{q}_2 l_2 sq_3+1/2\dot{q}_1^2 l_2 c(q_3+2q_2)-1/2\dot{q}_1^2 l_2 cq_3-\dot{q}_2^2 l_2 cq_3 \end{bmatrix}; \tag{39}$$

$$[\bar{a}]_4^4=[R]_3^4\cdot\left\{\bar{a}_3^3+\bar{\varepsilon}_3^3\times\bar{r}_4^3+\bar{\omega}_3^3\times\left(\bar{\omega}_3^3\times\bar{r}_4^3\right)\right\}; \tag{40}$$

$$[\bar{a}]_4^4=$$
$$=\begin{bmatrix} cq_4\{\ddot{q}_1 l_2 sq_2+2\dot{q}_1\dot{q}_2 l_2 cq_2+[\ddot{q}_1 s(q_3+q_2)+\dot{q}_1\dot{q}_2 c(q_3+q_2)+\dot{q}_1\dot{q}_3 c(q_3+q_2)]l_3-\dot{q}_1 c(q_3+q_2)(-\dot{q}_3-\dot{q}_2)l_3\}+sq_4[gs(q_3+q_2)-\ddot{q}_2 l_2 cq_3+1/2\dot{q}_1^2 l_2 s(q_3+2q_2)-1/2\dot{q}_1^2 l_2 sq_3-\dot{q}_2^2 l_2 sq_3+(-\ddot{q}_3-\ddot{q}_2)l_3+\dot{q}_1^2 c(q_3+q_2)s(q_3+q_2)l_3] \\ -sq_4\{\ddot{q}_1 l_2 sq_2+2\dot{q}_1\dot{q}_2 l_2 cq_2+[\ddot{q}_1 s(q_3+q_2)+\dot{q}_1\dot{q}_2 c(q_3+q_2)+\dot{q}_1\dot{q}_3 c(q_3+q_2)]l_3-\dot{q}_1 c(q_3+q_2)(-\dot{q}_3-\dot{q}_2)l_3\}+cq_4[gs(q_3+q_2)-\ddot{q}_2 l_2 cq_3+1/2\dot{q}_1^2 l_2 s(q_3+2q_2)-1/2\dot{q}_1^2 l_2 sq_3-\dot{q}_2^2 l_2 sq_3+(-\ddot{q}_3-\ddot{q}_2)l_3+\dot{q}_1^2 c(q_3+q_2)s(q_3+q_2)l_3] \\ gc(q_3+q_2)+\ddot{q}_2 l_2 sq_3+1/2\dot{q}_1^2 l_2 c(q_3+2q_2)-1/2\dot{q}_1^2 l_2 cq_3-\dot{q}_2^2 l_2 cq_3+[-\dot{q}_1^2 s(q_3+q_2)^2+(-\dot{q}_3-\dot{q}_2)(\dot{q}_3+\dot{q}_2)l_3] \end{bmatrix}; \tag{41}$$

$$[\bar{a}]_5^5 = [R]_4^5 \cdot \left\{\bar{a}_4^4 + \bar{\varepsilon}_4^4 \times \bar{r}_5^4 + \bar{\omega}_4^4 \times \left(\bar{\omega}_4^4 \times \bar{r}_5^4\right)\right\}; \tag{42}$$

$$[\bar{a}]_5^5 =$$

$$= \left[\begin{array}{l}
cq_4\{\ddot{q}_1 l_2 sq_2 + 2\dot{q}_1\dot{q}_2 l_2 cq_2 + [\ddot{q}_1 s(q_3+q_2) + \\
\quad + \dot{q}_1\dot{q}_2 c(q_3+q_2) + \dot{q}_1\dot{q}_3 c(q_3+q_2)]l_3 - \\
\quad - \dot{q}_1 c(q_3+q_2)(-\dot{q}_3-\dot{q}_2)l_3\} + \\
\quad + sq_4[gs(q_3+q_2) - \ddot{q}_2 l_2 cq_3 + 1/2\dot{q}_1^2 l_2 s(q_3+2q_2) - \\
\quad - 1/2\dot{q}_1^2 l_2 sq_3 - \dot{q}_2^2 l_2 sq_3 + (-\ddot{q}_3-\ddot{q}_2)l_3 + \\
\quad + \dot{q}_1^2 c(q_3+q_2)s(q_3+q_2)l_3] \\
\hline
cq_5\{-sq_4[\ddot{q}_1 l_2 sq_2 + 2\dot{q}_1\dot{q}_2 l_2 cq_2 + [\ddot{q}_1 s(q_3+q_2) + \\
\quad + \dot{q}_1\dot{q}_2 c(q_3+q_2) + \dot{q}_1\dot{q}_3 c(q_3+q_2)]l_3 - \\
\quad - \dot{q}_1 c(q_3+q_2)(-\dot{q}_3-\dot{q}_2)l_3] + \\
\quad + cq_4[gs(q_3+q_2) - \ddot{q}_2 l_2 cq_3 + 1/2\dot{q}_1^2 l_2 s(q_3+2q_2) - \\
\quad - 1/2\dot{q}_1^2 l_2 sq_3 - \dot{q}_2^2 l_2 sq_3 + (-\ddot{q}_3-\ddot{q}_2)l_3 + \\
\quad + \dot{q}_1^2 c(q_3+q_2)s(q_3+q_2)l_3]\} + sq_5\{gc(q_3+q_2) + \\
\quad + \ddot{q}_2 l_2 sq_3 + 1/2\dot{q}_1^2 l_2 c(q_3+2q_2) - 1/2\dot{q}_1^2 l_2 cq_3 - \\
\quad - \dot{q}_2^2 l_2 cq_3 + [-\dot{q}_1^2 s(q_3+q_2)^2 + \\
\quad + (-\dot{q}_3-\dot{q}_2)(\dot{q}_3+\dot{q}_2)l_3]\} \\
\hline
-sq_5\{-sq_4[\ddot{q}_1 l_2 sq_2 + 2\dot{q}_1\dot{q}_2 l_2 cq_2 + [\ddot{q}_1 s(q_3+q_2) + \\
\quad + \dot{q}_1\dot{q}_2 c(q_3+q_2) + \dot{q}_1\dot{q}_3 c(q_3+q_2)]l_3 - \\
\quad - \dot{q}_1 c(q_3+q_2)(-\dot{q}_3-\dot{q}_2)l_3] + \\
\quad + cq_4[gs(q_3+q_2) - \ddot{q}_2 l_2 cq_3 + 1/2\dot{q}_1^2 l_2 s(q_3+2q_2) - \\
\quad - 1/2\dot{q}_1^2 l_2 sq_3 - \dot{q}_2^2 l_2 sq_3 + (-\ddot{q}_3-\ddot{q}_2)l_3 + \\
\quad + \dot{q}_1^2 c(q_3+q_2)s(q_3+q_2)l_3]\} + cq_5\{gc(q_3+q_2) + \\
\quad + \ddot{q}_2 l_2 sq_3 + 1/2\dot{q}_1^2 l_2 c(q_3+2q_2) - 1/2\dot{q}_1^2 l_2 cq_3 - \\
\quad - \dot{q}_2^2 l_2 cq_3 + [-\dot{q}_1^2 s(q_3+q_2)^2 + \\
\quad + (-\dot{q}_3-\dot{q}_2)(\dot{q}_3+\dot{q}_2)l_3]\}
\end{array}\right]; \tag{43}$$

The operational kinematic parameters can be expressed by the operational velocities and accelerations expressed matrix in reference system (T_n), thus:

$$\left[\dot{X}^n\right] = \left[\left[\bar{v}_7^7\right]^T \ \left[\bar{\omega}_7^7\right]^T\right]^T, \ \left[\ddot{X}^n\right] = \left[\left[\bar{a}_7^7\right]^T \ \left[\bar{\varepsilon}_7^7\right]^T\right]^T. \tag{44}$$

Using the transformation relations, the kinematic operational parameters from the fixed system from the robot's base, can be determined, according to [4]. The matrix expression of these parameters is as follows:

$$[\bar{v}]_7^0 = [R]_7^0 \cdot [\bar{v}]_7^7, \tag{45}$$

$$[\bar{\omega}]_7^0 = [R]_7^0 \cdot [\bar{\omega}]_7^7, \tag{46}$$

$$[\bar{a}]_7^0 = [R]_7^0 \cdot [\bar{a}]_7^7, \tag{47}$$

$$[\bar{\varepsilon}]_7^0 = [R]_7^0 \cdot [\bar{\varepsilon}]_7^7, \tag{48}$$

Using the relations from (45)-(48), the velocity and acceleration of the fixed system (T_0) from the robot base, can be determined. Thus:

$$\left[\dot{X}^0\right] = \left[\left[\bar{v}_7^0\right]^T \ \left[\bar{\omega}_7^0\right]^T\right]^T, \ \left[\ddot{X}^0\right] = \left[\left[\bar{a}_7^0\right]^T \ \left[\bar{\varepsilon}_7^0\right]^T\right]^T. \tag{49}$$

The equations (44) and (49) are the direct kinematic equations, that determine the operational kinematic parameters of the clamping device in relation with the reference systems (T_7) and (T_0).

3. CONCLUSION

The iterative method used to determine the direct kinematic equations, is consists in doing the robot kinematic chain from a fixed base to the clamping device and the determinated by successive iterations of the following kinematic parameters: $\left\{\bar{k}_i^i, \bar{\omega}_i^i, \bar{\varepsilon}_i^i, \bar{v}_i^i, \bar{a}_i^i, i = 1 \div n\right\}$. The mentioned kinematic parameters characterized the movement of each element i, i=1÷n, in relation to the fixed system (T_0) from robot's base.

With the result obtained at the kinematic modeling it can be determined the dynamic model of the 6R using the method of Newton-Euler.

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Modelul cinematic direct al robotului articulat 6R

Rezumat: În această lucrare este prezentat modelul cinematic direct pentru robotul articulat 6R. Prin acest model sunt eliminate dezavantajele modelării geometrice determinate de neliniaritatea ecuaţiilor geometrice şi de lipsa controlului asupra vitezei şi acceleraţiei pe traiectoria de mişcare.

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