TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue I, March, 2013

THE ALLOCATION OF CUSTOMERS TO WAREHOUSES OPTIMIZATION USING GENETIC ALGORITHMS

Ciprian CRISTEA

Abstract: *The requirements for a new generation of manufacturing systems come from business objectives. Furthermore the organizations which understand the needs and expectations of the customers will do everything possible to meet their needs. To this end, the new manufacturing systems must change to stay competitive in today's rapidly changing business climate. In terms of the topics approached, this study follows the current national and international researches regarding the implementation of genetic algorithms in the production systems domain. The allocation of customers to capacitated warehouses is a current issue. The supply management of deposits represents a big challenge for the organizations which target the costs reduction and, at the same time, the improvement of customer services. This paper presents the allocation of customers to warehouses optimization using genetic algorithms.*
Keywords: genetic algorithms, supply chain network, supply chain management, warehouse scheduling, supply.

1. INTRODUCTION

Today's market place presents more and more challenge for firms. Probably the biggest challenges being faced are the increase demand from customer for higher quality and service. The achievement of competitive advantage through service comes from a combination of carefully thought-out strategy for service and the development of adequate delivery systems.

Supply Chain Management (SCM) represents a set of decisions and activities used to integrate suppliers, manufacturers, warehouses, transporters, retailers and customers. The right product is distributed at the right quantities, to the right location, and at the right time, in order to minimize system-wide costs. Furthermore the customer service level requirements have to be met and surpassed [3].

Distribution management is a part of SCM and refers to the storage and flows from the final production point to the customer. Distribution management is based on deeper, more sophisticated understanding of markets. By selling the products to the customer, the enterprise closes its economic cycle (Figure 1).

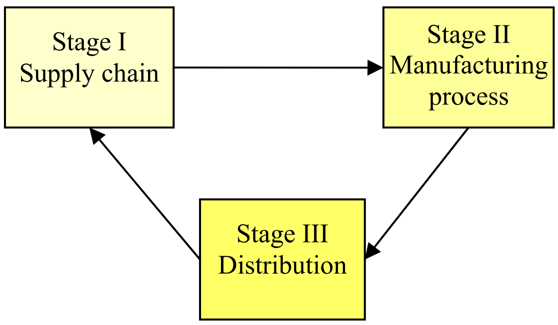


Fig. 1 Economic cycle of the enterprise

In today's customer-centric business environment, the ability of firms to allocate customer to their available warehouse can be translated into a competitive advantage. With its increasing meaning, the problem of allocating a set of customers to multiple warehouses started to draw attention from academicians and professionals too. Efficient allocation will lead to the best exploitation of resources and lower costs.

Warehouses play a significant role in upholding the uninterrupted flow of goods and materials between the enterprise and customers. Frequently the strategic role of warehouses is determined by their ability to serve as many customers as possible without incurring additional costs. A balanced allocation problem occurs when constructing a supply chain network for a set of customers by more than one distributor (allocation of customers to nearby warehouses). A balanced allocation problem to the distribution centers can be helpful in avoiding underutilization and overcrowding of distribution centers [4].

One of the most important issue in the SCM is to find the network strategy that can give the least cost of the physical distribution flow. This paper deals the problem of sequencing requests of products to fulfill several customers' orders. The performance criterion that is evaluated represents the minimization of shipping cost.

2. PROBLEM DESCRIPTION

Firm Beta produces and distributes chain link fences and their related hardware items to a total of twenty-one customers across the U.S. Beta has one factory which ship its output to three warehouses, denoted as warehouses I, II and III (Fig. 2). The maximum storage capacity of each warehouse is 880000 units. Each customer is supplied from warehouses according to their demand. All shipping costs data between these warehouses and their customers are in table 1[6].

Table 1

Unit Shipping Costs [$/unit]

Customer *i*	**Unit Shipping Cost at Warehouse *j* [$/unit]**			**Demand (in unit)**
	I	**II**	**III**	
1	2.9	3.2	3.5	113644
2	3.9	4.0	4.3	25360
3	3.5	3.6	3.5	82507
4	3.5	3.6	3.6	80159
5	3.3	3.4	3.0	75274
6	3.3	3.4	3.1	116064
7	3.1	3.3	3.8	329263
8	2.5	2.9	3.0	162106
9	3.2	3.3	3.0	151417
10	4.0	4.1	4.6	40833
11	3.1	3.3	3.4	97758
12	3.1	3.4	4.0	63643
13	2.8	3.0	3.2	367379
14	3.0	3.2	3.6	276387
15	3.1	3.2	2.6	85180
16	3.2	3.3	2.8	79662
17	2.7	3.0	3.1	122560
18	2.9	3.0	2.4	106198
19	3.5	3.6	3.5	57305
20	2.7	3.0	3.2	119524
21	3.6	3.7	3.8	60096

Many organizations have a warehouse management characterized by delivering goods at their location to a given set of fixed point or customers. Usually all customer's demand is served by the warehouse to which customer is assigned. But it is not always the case in practice. Frequently customer orders cannot be filled entirely from one warehouse (primary warehouse) inventory due to stock outs because a 100% in-stock policy for all possible demand levels would involve too much safety stock to be feasible. To deal this problem a customer is supplied with part of goods from an alternative warehouse (secondary warehouse). Although it surely results in supplementary transportation costs, it can maintain a high level of stock availability to the customers [6].

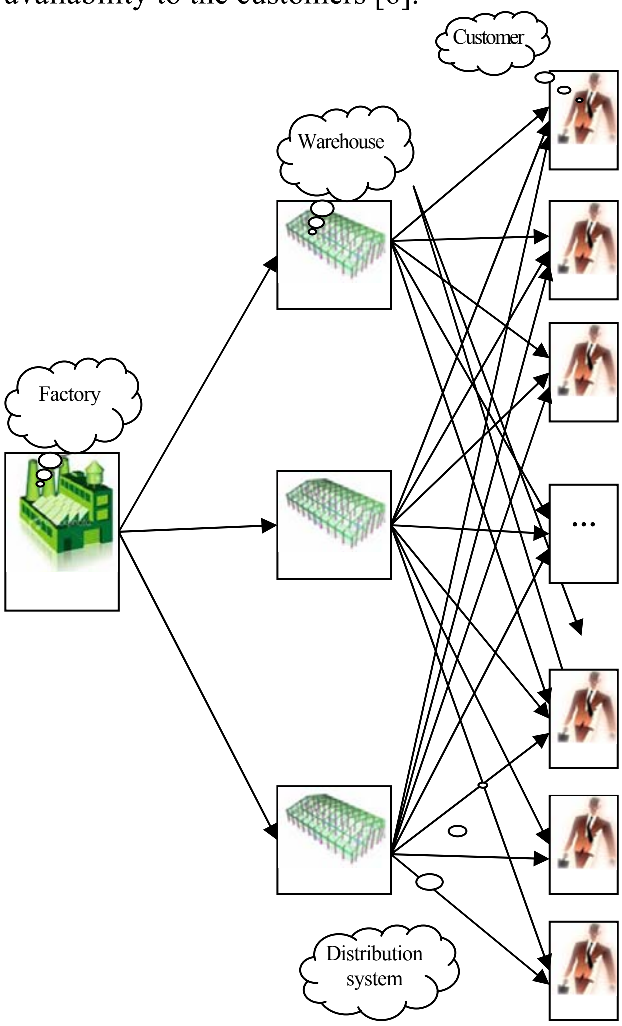


Fig. 2 Distribution system of firm Beta

In [6] the distribution of firm Beta was optimized by using a genetic algorithm approach. It was assumed that all 90%

customers' demands are supplied by their primary warehouses and the rest 10% demands are supplied by their secondary warehouses. The total shipping cost was 8021597 $.

3. GENETIC ALGORITHMS

The genetic algorithm (GA) is an optimization and search technique based on the biological evolution process that is found in natural evolution. A GA allows a population composed of many individuals to evolve under particular selection rules to a state that maximizes the "fitness" (i.e., maximizes the profit function) [2]. The individuals or population's members, generation after generation, compete with each other to survive (Darwinian selection).

A GA assumes that a potential solution of a specified problem is an individual (chromosome) that can be represented by a set of parameters. Every chromosome is formed by a set of genes (parameters) which represents the coded value of problem variables. These variables form together the potential solution of the problem.

The next step is to define the entire range of possible solutions (the search space). Then has to be defined the objective function.

The GA starts from a set of randomly generated solutions (initial population). Then is computed the objective function, resulting the fitness of each individual (to see whether they will survive). Depending on their fitness, the individuals are selected to become parents in the next generation. When a constraint is violated, a penalty is imposed for unfeasible solutions. The population members' fitness depends on the penalty imposed by the constraint being violated [5].

Then genetic operators are used to alter the genetic composition of offspring. The main genetic operators are the following:

- Crossover – produces a new offspring by interchanging subparts of the selected parents (chromosomes);
- Mutation – randomly alters one or more gene values (features) of a selected chromosome from its initial state to introduce extra variability.

Also have to be defined specific parameters values:

- Population size – how many individuals constitute the population;
- Crossover rate – the probability that the individual would be crossover;
- Mutation rate – the probability that a gene would be mutated.

Each generation consists of individuals who survive from the previous generations. Habitually, the population size does not change from one generation to the next generation.

The generational process is repeated until a termination condition is reached. The main terminating conditions are:

- A fixed number of generation is reached;
- A solution that satisfied minimum criteria is found;
- A limited evolution time;
- When the fitness is deemed as convergent;
- Combinations of the above.

4. MODELLING FORMULATION

The optimization proposed in this paper starts from [6], and is focused on the firm Beta allocation of the customers to warehouses by shipping costs minimizing. Each customer demand is supplied by a primary and a secondary warehouse. Each warehouse may serve as both a primary and a secondary warehouse for some customers. Given m customers and r warehouses the objective function is:

$$\min f = \sum_{i=1}^{m}\sum_{j=1}^{r} t_{ij} \cdot P_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{r} t_{ij} \cdot S_{ij} \qquad (1)$$

subject to:

$$\sum_{j=1}^{r} x_{ij} = 1, \ i = 1,2, \ldots, m \qquad (2)$$

$$\sum_{j=1}^{r} z_{ij} = 1, \ i = 1,2, \ldots, m \qquad (3)$$

$$\sum_{i=1}^{m} P_{ij} + S_{ij} \le q_j, \quad j = 1, 2, \ldots, r \qquad (4)$$

where:

i – index for customers;

j – index for warehouses;

d_i – demand of customer i [units];

α_i – proportion of the demand of each customer that is served by the primary warehouse assigned to it, $\alpha \in [0,1]$;

t_{ij} – unit shipping costs for customer i when the primary warehouse assigned to it is warehouse j [\$/unit];

q_j – capacity of warehouse j [units];

$P_{ij} = [\alpha_i \cdot d_i \cdot x_{ij}]$ – goods supplied to customer i from the primary warehouse j [units];

$S_{ij} = [(1-\alpha_i) \cdot d_i \cdot x_{ij}] + 1$ – goods supplied to customer i from the secondary warehouse j [units];

$$x_{ij} = \begin{cases} 1, \textit{ if warehouse j serves customer i as} \\ \textit{the primary warehouse;} \\ 0, \textit{ otherwise.} \end{cases}$$

$$z_{ij} = \begin{cases} 1, \textit{ if warehouse j serves customer i as} \\ \textit{the secondary warehouse;} \\ 0, \textit{ otherwise.} \end{cases}$$

The objective function minimizes the total shipping costs related to satisfying the primary and secondary demand for all customers. Constraints set (2) and (3) assert that for every customer there should be one primary and one secondary warehouse to supply goods. Constraint set (4) guarantees that a warehouse is able to satisfy the primary and secondary customers demand (its capacity guarantees 100% the demand of customers).

5. PENALIZATION AND EVALUATION

Given that the capacity is specified for each of the warehouses, some situation may arise in which the demand of some customers can not be satisfied or partially satisfied by such capacities. In these situations the fitness value is penalized.

If the capacity limit of warehouse is exceeded, the function of punishment is defined as follows:

$$g_j = \begin{cases} 0, \; if \sum_{i=1}^{m} P_{ij} + S_{ij} > q_j \\ \left[\frac{1}{q_j} \cdot \left(\sum_{i=1}^{m} P_{ij} + S_{ij} \right) \right] - 1, \; if \sum_{i=1}^{m} P_{ij} + S_{ij} \le q_j \end{cases} \quad (5)$$

The fitness function is defined as follows:

$$C = \sum_{i=1}^{m} \sum_{j=1}^{r} t_{ij} \cdot P_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{r} t_{ij} \cdot S_{ij} + C_{pen} \cdot \sum_{j=1}^{r} pen_j \quad (6)$$

$$pen_j = \begin{cases} g_j, \; if \; g_j > 0; \\ 0, \; if \; g_j \le 0. \end{cases} \quad (7)$$

where:

C_{pen} – punishment coefficient;

pen – total punishments.

6. GENETIC ALGORITHM CODIFICATION

Genetic algorithms represent the warehouse allocation of customers as individuals or a population member. The codification of individuals is a very important issue, because an efficient representation scheme is decisive for a genetic algorithm to solve a given problem.

The main data regarding the allocation of each customer are the following:

- The primary warehouse (warehouse I = 0, warehouse II = 1, warehouse III = 2);
- The secondary warehouse (warehouse I = 0, warehouse II = 1, warehouse III = 2);
- Proportion α of the demand of each customer that is served by the primary warehouse assigned to it;

$$\alpha = \begin{cases} 0{,}51 \ldots 0{,}99, \textit{ if the no. of primary warehouse} \neq \\ \textit{the number of secondary warehouse;} \\ 1, \textit{ if the no. of primary warehouse} = \\ \textit{the number of secondary warehouse.} \end{cases} \quad (8)$$

So, 21x3=63 variable should be used which should enlarge considerably the entire range of possible solutions (the search space). For this reason, it was used a particular codification which allowed the decrease of variable from 63 to 21, so all the data regarding a customer are contained in a single code.

Firstly the possible allocation schemes for a customer were identified (see in Table 2). For the interpretation of a scheme the following rules were used:

- The primary warehouse number is the quotient of the scheme number divided by 3;
- The secondary warehouse number is the remainder of the scheme number divided by 3.

For completing the code structure, the proportion α of the demand of each customer that is served by the primary warehouse assigned to it must be included – equation (9).

$$cod = sch \cdot 10^6 + \alpha \cdot 100 = (3 \cdot ndp + nds) \cdot 10^6 + \alpha \cdot 100 \quad (9)$$

where:
cod – code;
sch – scheme number;
ndp – primary warehouse number;
nds – secondary warehouse number.

For a better understanding of potential solutions codification, the following example is presented. If scheme number 5 (warehouse number 1 is the primary warehouse and warehouse number 2 is the secondary warehouse) is used and the demand proportion of each customer that is served by the primary warehouse assigned to it is 72% (α=0.72), then the code of the possible solution is the following:

$$5000072 = 5 \cdot 10^6 + 0.72 \cdot 100 = (3 \cdot 1 + 2) \cdot 10^6 + 0.72 \cdot 100$$

Table 2

Scheme codification

Scheme	Decoding	The meaning
0	00	Warehouse 0 – primary warehouse Warehouse 0 – secondary warehouse
1	01	Warehouse 0 – primary warehouse Warehouse 1 – secondary warehouse
2	02	Warehouse 0 – primary warehouse Warehouse 2 – secondary warehouse
3	10	Warehouse 1 – primary warehouse Warehouse 0 – secondary warehouse
4	11	Warehouse 1 – primary warehouse Warehouse 1 – secondary warehouse
5	12	Warehouse 1 – primary warehouse Warehouse 2 – secondary warehouse
6	20	Warehouse 2 – primary warehouse Warehouse 0 – secondary warehouse
7	21	Warehouse 2 – primary warehouse Warehouse 1 – secondary warehouse
8	22	Warehouse 2 – primary warehouse Warehouse 2 – secondary warehouse

The next step was to generate, considering the constraints, the list of possible codes (297 codes). For a code decoding equations (10) and (11) were used.

$$sch = \left[\frac{cod}{1000000} \right] \quad (10)$$

$$\alpha = \frac{(cod - sch \cdot 1000000)}{100} \quad (11)$$

For a better understanding of potential solutions decoding of code 5000072 is presented:

$$sch = \left[\frac{5000072}{1000000} \right] = 5 = 1 \cdot 3 + 2 \rightarrow primary\ warehouse = 1;\ secondary\ warehouse = 2.$$

$$\alpha = \frac{(5000072 - 5 \cdot 1000000)}{100} = 0.72$$

The 21 genes which form the chromosome represent the 21 customer which have to be allocated to the warehouses. The genetic simulation has to identify the optimal solution which consists in specifying for each of the 21 customers (genes) the way of allocating to the primary and secondary warehouse, the proportion of allocating the customer from the primary warehouse, so that the shipping costs are minimized.

Because the chromosome is formed by 21 genes, and each gene could be represented by 297 possible codes, the genetic simulation for this problem uses a search space of 297^{21}.

7. RESULTS

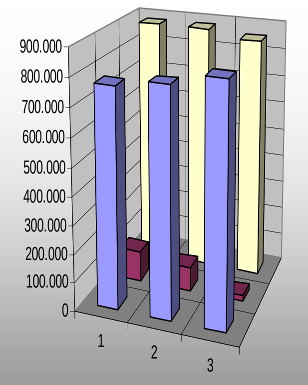
For genetic simulation, the genetic algorithms software of the TUC-N Center of Consultancy and Optimal Design was used.

Figure 3 gives out an illustration of warehouse allocations of customers. The quantity of goods delivered by each warehouse is very close to their maximum capacity (i.e. warehouse I is almost 100% supplied with goods). The total shipping cost is 7932714.3 $.

8. CONCLUSIONS

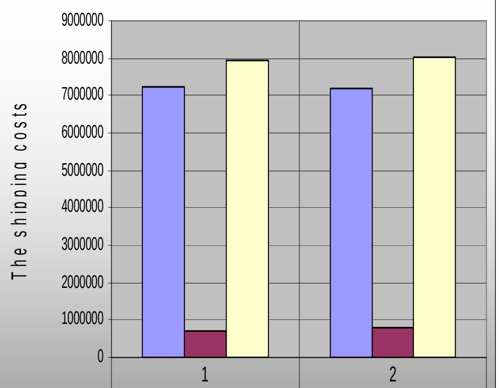
Figure 4 shows a comparison between the total shipping costs obtained with the proposed genetic algorithm and costs obtained in [6]. There is an improvement of almost 90000 $ obtained after genetic simulation.

I consider that, by using a variable proportion of the demand of each customer that is served by the primary warehouse assigned to



	1	2	3
Primary warehouse deliveries	770.355	793.626	833.766
Secondary warehouse deliveries	109.635	85.887	19.050
Total deliveries	879.990	879.513	852.816

Fig. 3 The warehouse allocation of customers



	1	2
Shipping costs for primary warehouses	7247026,8	7206575
Shipping costs for secondary warehouses	685687,5	815022
Total shipping costs	7932714,3	8021597

Cases studied (1-genetic algorithm proposed; 2 - [6])

Fig 4 The shipping costs

it for all the customers, a weakness of the model [6] is removed. So any subjectivity regarding the proportion selection of the demand for each customer that is served by the primary warehouse assigned to it is eliminated.

Another advantage of genetic algorithms comes from the fact that genetic simulation generates a number of local optima (beside the global one). This is very useful when the best solution isn't accepted by different reasons.

REFERENCES

[1] Cristea C., *Cercetări privind proiectarea şi managementul sistemelor de producţie*, Teză de doctorat, UTC-N, Cluj-Napoca, 2008.

[2] Haupt R. L., Haupt S. E., *Practical Genetic Algorithms*, John Wiley&Sons Publishing, New-Jersey, 2004.

[3] Li L., *Supply Chain Management: Concepts, Techniques and Practices. Enhancing Value Through Collaboration*, World Scientific Publishing Co. Pte. Ltd., Singapore, 2007.

[4] Rajesh R., Pugazhendhi S., Gabesh K., *Simulated annealing for balanced allocation problem*, International Journal of Advanced Manufacturing Technology, Volume 61, Issue 5-8, pp. 431-440, ISSN 0268-3768, Springer-Verlag Publishing, 2012.

[5] Tudose L., Pop D., Proiectare *optimală cu algoritmi genetici,* Editura Mediamira, Cluj-Napoca, 2002.

[6] Zhou G., Min H., Gen M., *Capacitated warehouse allocation of customers with a primary and a secondary warehouse: a genetic algorithm approach*, The 5th International Conference on Engineering Design& Automation, Las Vegas, pp. 599-604, 2001.

OPTIMIZAREA CU AJUTORUL ALGORITMILOR GENETICI A APROVIZIONĂRII CLIENŢILOR

Rezumat: Cerinţele pentru o nouă generaţie de sisteme de producţie derivă din obiectivele afacerii. Mai mult, organizaţiile care urmăresc cerinţele clienţilor vor face tot ceea ce le stă în putinţă pentru a le îndeplini aşteptările. În acest scop, noile sisteme de producţie trebuie să se schimbe pentru a rămâne competitive în mediul de afaceri actual schimbător. Prin prisma tematicii abordate această lucrare se înscrie pe linia preocupărilor existente pe plan naţional şi mondial de implementare a algoritmilor genetici în domeniul sistemelor de producţie. Aprovizionarea clienţilor de la depozite cu capacitate limitată este o problemă de actualitate. Managementul aprovizionării depozitelor reprezintă o mare provocare pentru organizaţiile care urmăresc reducerea costurilor şi, în acelaşi timp, îmbunătăţirea serviciului acordat clienţilor. Această lucrare prezintă optimizarea cu ajutorul algoritmilor genetici a aprovizionării clienţilor.

Ciprian CRISTEA, PhD Eng., Technical University Cluj-Napoca, The Faculty of Electrical Engineering, Department of Electrical Machines and Drives, 26-28 G. Bariţiu Street, Cluj-Napoca, e-mail: cristeaciprian1@gmail.com