TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue I, March, 2013

THE APPLICATION OF THE OPTIMAL CONTROL PROBLEM WITH MOBILE EXTREMES ON THE NEW PRODUCT LAUNCH PROCESS

Corina Adriana DOBOCAN, Ioan BLEBEA, Daniela POPESCU

Abstract: *Problem launching new products on the market can mathematically model the problem of optimal control with a minimum of time. In this paper we solve such a problem in the initial and final conditions are not fixed. This reached an optimal control problem with the "extreme mobile. After applying transversality conditions we concluded that such a problem cannot have both ends mobile, that it makes sense only if more than one end is mobile. limit ourselves to resolving cases in which one end is fixed and a mobile, because if both ends are fixed is solved in detail in works [1] and [2].*
Key words: *optimal control, The Hamiltonian, optimal strategy, transversality conditions.*

1. INTRODUCTION

Mathematical modeling of the process to new products market launching process can be shortened by several mathematical theories. Such models are: optimal control theory, nonlinear mathematical programming, stochastic theory. We will limit the given model approach "optimal control" that the theoretically is more laborious but provide efficient numerical methods and easier to interpret in terms of applicants.

We deal therefore necessary condition of existence, condition formulated as "PRINCIPLE OF MAXIMUM PONTREAGUINE" [16] which leads to an algorithm that determines the optimal control analytic solution.

In the presence of optimal control problems it is necessary to study several issues, including:
- The existence of transfer strategies
- Existence strategy (strategies) optimal
- Uniqueness of optimal strategy
- Methods for determining the optimal strategy
- Methods for approximating the optimal strategy if the exact determination is difficult.

Pontreaguine's maximum principle gives necessary conditions for a strategy to be optimal.

The mathematical model originally developed based on optimal control theory to a minimum, model, due to the complexity and variety release process of factors involved in this process we considered appropriate to reflect in a real importance as opinion consumers about the new product launched (the "function of good impressions") and how to invest in this process. (the cost function)

The main research methods used in LPN (new product launch) in this case are: economic statistical methods, optimal control theory and minimum time problem.

In the LPN we used a mathematical model using optimal control theory minimum time. For this purpose we assume there is provided a time frame within which to achieve the new product launch. We considered a time proposed, but the problem this time mathematics was thus minimizing pair status (good impression, investment in the process), go through optimal trajectory, ie the optimal release have evolved.

In this formulation x (t) is "function good impressions" which is validated by the processes of economic statistics survey also

function y (t) represents the "cost function", ie the number y (t) represents the costs incurred by a firm release for the new product by the time proposed. We noted with t^* minimum time determined by the principle of Pontreaguin, which managed to launch the new product, ie the minimum time this process for a category of products.

In these circumstances, we consider some initial data at t = 0, ie at the beginning of the final data release and certain. At the initial time t = 0, professionals advertising campaign appreciates the percentage of "good impression" on the category of new products to be launched in value. Value is determined also by survey method.

• In the LPN, the cost function is controlled by another function u(t), which is intended to represent the rate of expenditure at the time. This control function is determined by the "Principle of the Hamiltonian Maxim" axiomatic that it goes through a process, a process that depends on all the data involved in the mathematical model governing the LPN process.

• Non-autonomous system state equations which define the optimal control problem of a coercive process play an LPN. These equations form a system of two differential equations in general nonlinear, three unknown functions: x (t), y(t) and u(t). Formally, this system of state equations are not within the classical theory of systems of differential equations, such as discussed above, his equations representing only restrictions optimization problem.

• This optimization problem is very optimal control problem LPN attached to a process which is expressed as follows: to determine the control function u(t) and optimal trajectory determined by the state vector (x (t), y (t)) which end up through the minim time. In this work we have studied if defined mobile extremities and good impression and promotion budget at a timeare fixed, when driving real axis interval or browse intervals parallel to the axis. This case leads to an optimal control problem with mobile extremities.

The New Product Launch Problem can be solved in several ways, such as linear or nonlinear mathematical programming, or a stochastic approach, but the most suggestive and appropriate way is given by the optimal control theory [1] and [2].

2. THE FORMULATING OF OPTIMAL CONTROL PROBLEM WITH MOBILE EXTREMITIES

Extremities status issue launch new products on the market will be those given in [1].

$$\begin{cases} x'(t)+c\cdot x(t)=b\cdot u^{\alpha}(t) \\ y'(t)=u(t)\cdot e^{-p\cdot t} \end{cases}, t\in[0,T] \qquad (1)$$

where $\alpha\in(0,1)$, b, c, p >0.

The function x(t) is the goodwill about the products launched on time $t\in[0,T]$, and y (t) represents expenditure until the time of $t\in[0,T]$, u(t) is the function of control which represents the rate of spending time $t\in[0,T]$.

Solving the problem arises if the system equations of salt (1) and add conditions at baseline t = 0 and the final time $t=t^*\le T$.

In our case the two conditions is formulated as follows:

At time t = 0, $(x(0),y(0))\in A_0$ where a variety defined by: $A_0\subset\Re^2$ is given positive numbers : $x_0 < x_0'$ So that

$$A_0=\{(x,y)\in\Re^2 / x\in[x_0,x_0'], y=0\} \qquad (2)$$

For the final time $t=t^*\le T$ we define the variety $B_0\subset\Re^2$, which is defined as: are given positive numbers $\bar{x}$ and $\bar{y}<\bar{y}'$

$$B^*=\{(x,y)\in\Re^2 / x=\bar{x}, y\in[\bar{y},\bar{y}'] \qquad (3)$$

Finally the problem of optimal control with movable extremities attached market launch of new products is the date by:

$$\begin{cases} x'+c\cdot x=b\cdot u^{\alpha}(t) \\ y'=u(t)\cdot e^{-p\cdot t}, t\in[0,T] \\ (x(0),y(0))\in A_0 \\ (x(t^*),y(t^*))\in B^*, \alpha\in(0,1), b,c,p>0 \\ x,y\in C'(0,T) \end{cases} \qquad (4)$$

The problem (4) requires to determine the minimum time noted by $t^*, 0<t^*\le T$ and optimal strategy $s:\{t^*,\Gamma^*]$ where the path is

defined by the functions of state x(t) and y(t) which verifies the equations of state (1) and has ends on A_0 respectively B^* (fig.1).

3. SOLVING OF THE PROBLEM (4) USING THE PONTREAGHIN MAXIMUM PRINCIPLE

For this purpose it is necessary to maximize the Hamilton function, which in our case is given by:

$$H(t,x,y,u)=[b\cdot u^{\alpha}(t)-c\cdot x(t)]\Psi_1(t)+[e^{-pt}\cdot u(t)]\Psi_2(t), t\in[0,T] \quad (5)$$

Where (Ψ_1,Ψ_2) is the solution of Hamilton's system given by:

$$\begin{cases}\Psi_1'(t)=-\dfrac{\partial H(t)}{\partial x}\\ \Psi_2(t)=-\dfrac{\partial H(t)}{\partial y}\end{cases},t\in[0,T] \quad (6)$$

If account is taken of (5) the system (6) becomes:

$$\begin{cases}\Psi_1'(t)=c\cdot\Psi_1(t)\\ \Psi_2(t)=0\end{cases},t\in[0,T] \quad (7)$$

Integrating the system (7) we obtain:

$$\begin{cases}\Psi_1(t)=C_1\cdot e^{c\cdot t}\\ \Psi_2(t)=C_2\end{cases} \quad (8)$$

In this case C_1 and C_2 are arbitrary integration constant $C_1,C_2\in\Re$.

The Hamiltonian given by (5) is now the form

$$H(u)=A\cdot u^{\alpha}+B\cdot u+C \quad (9)$$

where

$$A=C_1\cdot b\cdot e^{c\cdot t};B=C_2\cdot e^{-p\cdot t};C=-C_1\cdot c\cdot e^{c\cdot t} \quad (10)$$

It should be noted that maximizing the Hamiltonian is done in relation to the variable "u", reason was given as (9). More than this maximization will lead to determining the control function u(t). It will now use transversality conditions. In the case of varieties A_0 and B^* from $\Re$ have the property that the tangent plane at any coincides with the very point of the variety of variety, see Fig 1.

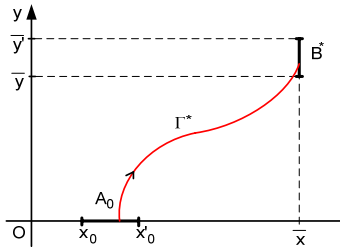


Fig. 1. The geometrically picture of the mobile extremities A_0 and B^*, with the optimal path Γ^*

If we note by T_0 and T^* the tangential planes A_0 and B^* in paragraphs $(x(0),y(0))\in A_0$ respectively $(x(t^*),y(t^*))\in B^*$ then the transversely conditions are:

$$<\vec{\Psi}(0),\vec{w}_0-\vec{v}_0>0,\forall\vec{w}_0\in A_0=T_0 \quad (11)$$

Respectively:

$$<\vec{\Psi}(0),\vec{w}^*-\vec{v}^*>0,\forall\vec{w}^*\in T^*=B^* \quad (12)$$

Where $<\cdot,\cdot>$ is the inner product of $\Re^2$ and the vectors involved in (11) and (12) are given by:

$$\vec{\Psi}(t)=\Psi_1(t)\cdot\vec{i}+\Psi_2(t)\cdot\vec{j} \quad (13)$$

$$\vec{v}_0=x(0)\cdot\vec{i}+y(0)\cdot\vec{j}=x(0)\cdot\vec{i},x(0)\in A_0 \quad (14)$$

$$\vec{v}^*=x(t^*)\cdot\vec{i}+y(t^*)\cdot\vec{j}=\vec{x}\cdot\vec{i}+\vec{y}\cdot\vec{j},y\in[\bar{y},\bar{y}'] \quad (15)$$

$$\vec{w}_0=s\cdot\vec{i},s\in[x_0,x_0'],\vec{w}_0\subset T_0 \quad (16)$$

$$\vec{w}^*=x\cdot\vec{i}+l\cdot\vec{j},l\in[\bar{y},\bar{y}'],\vec{w}^*\subset T^* \quad (17)$$

With these specifications (11) and (12) become:

$$(s-x_0)\cdot\Psi_1(0)=0,\forall s\in[x_0,x_0'] \quad (18)$$

Respectively:

$$(l-\bar{y})\cdot\Psi_2(t)=0,\forall s\in[\bar{y},\bar{y}'] \quad (19)$$

From (18) we deduce the cases:

$$\Psi_1(0)=0 \quad \text{or} \quad s=x_0 \quad (20)$$

respectively (19) provides:

$$\Psi_2(t^*)=0 \quad \text{or} \quad l=\bar{y} \quad (21)$$

If we take into consideration (8) $\Psi_1(0)=0$ involved $C_1=0$ and $\Psi_2(t^*)=0$ involve $C_2=0$, which means the vector $\vec{\Psi}=\vec{0}$, what leads to a contradiction with the Maximum Principle of Pontreaghin. Consequently optimal control problems with extreme mobile of the new products launch on the market we have the sum of the form:

$$A_0=[x_0,x_0'] \quad \text{and} \quad B^*=\{(\bar{x},\bar{y})\} \quad (22)$$

Respectively:

$$B^*=[\bar{y},\bar{y}'] \quad \text{and} \quad A=\{x_0\} \quad (23)$$

All of (20) and (21) is also obtained an optimal control problem with fixed extremities t=0 results $x(0)=x_0$ and y(0)=0, respectively $t=t^*$ results $x(t^*)=\bar{x}$ and $y(t^*)=\bar{y}$, problems studied in [1] and [2]. For problems (22) and (23) it can give a geometrical picture as is show in Fig 2 and Fig 3.

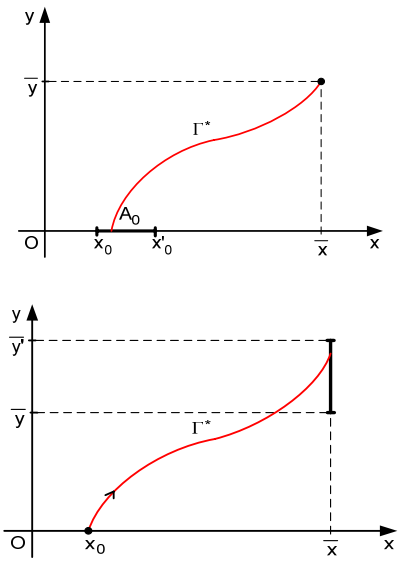


Fig. 2 and Fig. 3 The geometrically picture of the semimobile extremes and their optimal paths

In this paper we solve the problem defined by conditions (23).

From (23) we deduce that $C_2=0$ so $H(u)=A\cdot u^{\alpha}+C$ that increases in $[[0,u_0]$ if A>0 equivalent with $C_1>0$, in this case $\max H(u)=H(u_0)$ so the optimal control function is:

$$u(t)=u_0, t\in[0,T] \qquad (24)$$

With this results the state equations are:

$$\begin{cases} x'+c\cdot x=b\cdot u_0^{\alpha} \\ y'=u_0\cdot e^{-p\cdot t} \end{cases} \qquad (25)$$

By integrating this system we obtain:

$$\begin{cases} x(t)=K_1\cdot e^{-c\cdot t}+\dfrac{b}{c}\cdot u_0^{\alpha} \\ y(t)=K_2-\dfrac{u_0}{p}\cdot e^{-p\cdot t} \end{cases} \qquad (26)$$

where K_1 and K_2 are constants of integration which will determine the initial condition t = 0, which leads to:

$$\begin{cases} K_1=x_0-\dfrac{b}{c}\cdot u_0^{\alpha} \\ K_2=\dfrac{u_0}{p} \end{cases} \qquad (27)$$

In consequence, the path which leaving from $(x_0,0)$ is;

$$\Gamma:\begin{cases} x(t)=(x_0-\dfrac{b}{c}\cdot u_0^{\alpha})\cdot e^{-c\cdot t}+\dfrac{b}{c}\cdot u_0^{\alpha} \\ y(t)=\dfrac{u_0}{p}\cdot(1-e^{-p\cdot t}) \end{cases}, t\in[0,T] \qquad (28)$$

From the condition $(x(t^*),y(t^*))\in[\bar{y},\bar{y}']$ it will result the minimum time and the optimal path Γ^*

$$\Gamma^*:\begin{cases} x^*(t)=(x_0-\dfrac{b}{c}\cdot u_0^{\alpha})\cdot e^{-c\cdot t}+\dfrac{b}{c}\cdot u_0^{\alpha} \\ y^*(t)=\dfrac{u_0}{p}\cdot(1-e^{-p\cdot t}) \end{cases}, t\in(0,t^*) \qquad (29)$$

If:

$$p>\frac{2\cdot u_0}{\bar{y}} \qquad (30)$$

It is obtained the minimum time "t^*" given by:

$$t^*=\frac{1}{p}\cdot\ln\frac{p\cdot\bar{y}-u_0}{u_0} \qquad (31)$$

Assuming that $x(t^*)=\bar{x}$ condition equivalent with:

$$\left(\frac{u_0}{p\cdot\bar{y}-u_0}\right)^p=\left(\frac{1}{p}\cdot\ln\frac{p\cdot\bar{y}-u_0}{u_0}\right)^c \qquad (32)$$

It is noted that equation (32) can be seen in unknown p or c.

4. CONCLUSION

The second case is given by using the control function of a two-spline functions. One such issue is still possible because the four parameters.

• Determination of the splines is as follows: it starts with a command and determine a trajectory passing through, but do not ask to go through. Next take the other command and control function require that appropriate path to go through, but do not ask her to go through. Denote by and the two paths, two developments that appropriate release process.

• Finally, we conducted a study to the LPN with mobile extremities. The approach of this study was entirely possible because the optimal control theory there is such a problem called optimal control problem with mobile extremities or "optimal control problem with transversality conditions".

The practical value of the mathematical model is to provide complete and thorough information companies interested in commercial success of a new product and to provide an overview of very well defined on the whole process of release. Also, specialists in new product development are able to decide when they can to stop investing in promotional or longer allocate the amounts necessary for the success of the new product have planned.

Extensive coverage due to problems of optimal control problems attached LPN we appreciated that research conducted is sufficiently complete. This statement is the fact that a company wants to introduce a new product category at baseline t =0 did any expense, ie y (0) = 0.

Analysis and use of numerical models and graphs obtained from them is natural if one takes into account the significance of functions x(t) and y(t) for a control function u(t) that generates the state vector (x(t), y(t)) of the new products to market. Each simulation we present can give the user (proprietary companies products to be launched on the market) information on how the proposed project is developing a specific marketing specialists budget effort and time that will be launched.
Finally, a schedule determined by a "model number" given may lead to a decision to stop the launch before it is finished, depending on how it evolves x(t) or the period by changing the control function u(t) and the maximum period for launching budget if the graph indicate that good impression x(t) has an unsatisfactory behavior.

We note that the problem with mobile extremities does not make sense in general. Analyses of cases that make sense and were not treated in detail in this work are presented in the thesis of the author. We note, however, that these compatible cases lead to the problem of control with a single command that presented in [2].

5. AKNOWLEDGMENT

This paper was supported by the project "Progress and development through post-doctoral research and innovation in engineering and applied sciences – PRIDE – Contract no. POSDRU/89/1.5/S/57083", project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

6. REFERENCES

[1] Dobocan, C.: *Optimal control problem in new products launch,* Annals of DAAAM for 2009 & Proceedings: "Intelligent Manufacturing & Automation: Focus on Theory, Practice & Education", p. 1033, 25-28th November 2009, Vienna, Austria, ISSN 1726-9679

[2[Blebea, I.,Dobocan, C., Morariu-Gligor R.: *Optimal control problem in new products launch-Optimal path using a single command*- New Trends in Mechanisms Science, ISBN 978-90-481-9688-3, p. 467-473.

[3] Buratto, A.; Viscolani, B. (2002). *An optimal control student problem and a marketing counterpart,* Mathematical and Computer Modelling, 20:19-33, University of Padova, Italy

[4] Buratto, A.; Viscolani, B. (2002). *New product introduction: goodwill, time and advertising cost,* Available from: http://www.springerlink.com, *Accessed:* 2007-06-11.

[5] Dobocan, C., Blebea, I., Florea, R., Pop, C.A., *„Management of a new product development"*, Quality-Innovation-European Integration, ISBN 3-901888-41-3, Sibiu, 18-20 sept. 2008.

[6] Sarkadi, K., Vincze, I., „*Mathematical methods of statistical quality control*", Budapest, Akademia Klade, 1974.

[7] Shimp, T.A., "*Advertising Promotion Supplemental Aspects of Integrated*

Marketing Communications", Hercourt College Publishers, The Dryden Press, ISBN 0-03-021113-1, 2000.
[8] Smirnov, V. I., "*A Course of Higher Mathematics*", V. 4, Moscow, Fizmatgiz, 1958 (Russian edition). The English translation: "*A Course of Higher Mathematics*", Reading, Mass., 1961.
[9] Smith, R.E., and Fowley, J.W., "*New product organisations: high-performance team management for a changing environment*", New Food Products for a Changing Marketplace, Brody, A.L. and Lord, J.B. (Eds) (Lancaster, PA: Technomic), 2000.
[10] Uhlig, F., Mullges, G.E., "*Numerical algorithms With C*", Springer, 1986.
[11] Ulrich, T. K., Eppinger, D. S., "*Product Design and Development*", Irwin McGraw-Hill, ISBN 0-07-229647-X, 2000.
[12] Zabreiko, P.P., Koshekv, şi alţii, "*Integral Equations*", Reference Book, Moscow, Nauka, 1968 (Russian edition).
[13] Zeldovich, Y., MySkis, A.D., "*Elements of Applied Mathematics*", 3rd Edition, Moscow, Nauka, 1972 (Russian edition).
[14] Dobocan, C.;(2009). *Theoretical researches and contributions about new products development and promotion*, Doctoral Thesis, Technical University of Cluj-Napoca, Faculty of Machines Building, Romania
[15] Kantorovici, L.V.; Akilov, G.P. (1986). *Functional analysis,* Scientific and Encyclopedic Ed., Bucharest, Romania
[16] Pontreaguine, L.; Boltianski, V. (1974). *Théorie mathématique des processifs optimaux,* MIR Ed., Moscow
[17] Potra, T.G.; Rosa, I.; Toader, G.; Toader, S. (2005). *Differential geometry; differential equations and optimal control,* Transilvania Press Ed., ISBN 973-98180-1-1, Cluj-Napoca, Romania

Aplicarea problemei controlului optimal cu extreme mobile asupra noului process de lansare a produsului

Rezumat: Problema de lansare pe piata a produselor noi poate fi modelata matematic cu ajutorul problemei de control optimal cu timp minim. În această lucrare vom rezolva o astfel de problemă în condiţiile iniţiale şi finale care nu sunt fixate. Acest lucru determina o problemă de control optimal cu extremitati mobile. După aplicarea condiţiilor de transversalitate am ajuns la concluzia că o astfel de problemă nu poate avea ambele capete mobile si are sens numai în cazul în care doar una dintre extremitati este mobila. Ne limităm la cazurile de rezolvare a problemei în care un capăt este fixat şi un capat este mobil, acest lucru deoarece cazul in care ambele capete sunt fixate este rezolvată în detaliu în lucrările [1] şi [2].

DOBOCAN Corina Adriana, Dr. Ing., Lecturer, Technical University of Cluj-Napoca, Design Engineering and Robotics, corina2dobocan@yahoo.com , +4064401664, Cluj-Napoca, 7 Fabricii Street, appt. 52, +40723520259.
BLEBEA Ioan, Dr. Ing., Professor, Technical University of Cluj-Napoca, Design Engineering and Computing, blebea@muri.utcluj.ro , +40401664, Cluj-Napoca, 3 Putna Street, appt. 10, +40763606151.
POPESCU Daniela, Dr. Ing., Professor, Technical University of Cluj-Napoca, Design Engineering and Robotics, daniela.popescu@muri.utcluj.ro , +4064401615, Cluj-Napoca, 103-105 Muncii Blvd., +40745656725.