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A MATHEMATICAL MODEL APPLIED TO AN ECONOMICAL PROCESS

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Abstract: *The main research methods used in NPL (new product launch) in this case are: economic statistical methods, optimal control theory and minimum time problem. In this paper we used a mathematical model using optimal control theory minimum time. For this purpose we assume there is provided a time frame within which to achieve the new product launch. We considered a time proposed, but the problem this time mathematics was thus minimizing pair status (good impression, investment in the process), determining the optimal trajectory, is the optimal launch have evolved. NPL problem of economic analysis, revealed a situations in which the optimal control problem is with fixed ends, meaning that we assume from the beginning a good impression fixed and a fixed budget, ie points from which we come and to which are fixed and are not included in the intervals.*

Key words: *mathematical modeling, control function, optimal state path, The Hamiltonian function.*

1. INTRODUCTION

The first objective of maximizing the "good impression" and release time is motivated by the consideration of the effects of company management integrated marketing communications and advertising campaign on consumer behavior. On the other hand, the effect of the advertising policy until T, depend on the good impressions at time T: the higher the value of good impressions at time T with the higher sales volume next, regardless of the chosen advertising policy after time T.

The second objective is to minimize time-to-T, which reflects the importance of reducing "time-to-market" (time to market) product. Time plays a crucial role in product introduction problem, both for industry products with "fast cycle" and for those products that are new to the market and the firm.

A third objective is to minimize the total costs for advertising, which is a natural requirement of efficiency.

Advertising activity plays an important role in launching a new product. We will address the issue of determining advertising plan for preparing the product launch, with objectives to maximize product image during launch advertising campaign length minimizing and minimizing total advertising spend in a fixed time interval.

Company must carefully plan their marketing various actions to be done to introduce a new product on the market. Such coordinated actions must be taken both before sales start to increase and after. Marketing communications company has an important role to play, as can be seen in the automotive industry, where effective advertising campaign usually precedes the distribution of a new car model. You must be formulated and discussed the problem of determining the advertising policy to prepare the introduction of a new product.

We assume that the company has 3 objectives: the first objective is to maximize the product image (good impression) Release time T, to minimize the cost of advertising. The second objective means to determine the release (or entry) - launching time T, which will begin placing and the third objective, planning the launch and advertising campaign associated with the interval [0, T]. [3]

We formulate an optimal control problem where the good impression evolves depending on the firm's advertising policy. On the other hand we do not take into account direct sales because they start after start time T, when the launch political advertising stops. In fact assume that advertising policy launch is the first part of a more complex advertising: a second part of the start time T and serves a different purpose depending on the sales. We focus only on the first part of which is restricted by a range of pre-launch until at time T. We want to get some qualitative information about the optimal advertising policies and to refer to a concept of good impression which is analogous to the Nerlove-Arrow. [15]

2. THE FORMULATING OF OPTIMAL CONTROL PROBLEM WITH ONE COMMAND

Control problem formulation for Release of new products (LPN) Consider three unknown functions: x (t), y (t), u (t), $t \in [0,T]$ where:

- t is the time,
- x(t) is a function of "level of good impression" about the new product to be launched, the size (value) to measuring the percentage
- y(t) is the function that shows "the investment at time t"
- u(t) is the control function, the function showing the intensity, increase or decrease the cost y (t) of the investment at time t, u (t) can be interpreted as a means pumping costs.

The curve: $\Gamma : \begin{cases} x = x(t) \\ y = y(t) \end{cases} \; t \in [0,T] \qquad (1)$

is "the state curve" and the pair (x (t), y (t)) state "dynamic system" generated by the new product launch (NPL). t is the maximum time that can be done to launch.

State equations, the equations governing the system state is a generalization of equations Nerlowe-Arrow. [15]

The state equations with initial conditions are:

$$\begin{cases} \mathrm{x}'(\mathrm{t}) = \mathrm{b} \cdot [\mathrm{u}(\mathrm{t})]^{\alpha} - \mathrm{c} \cdot \mathrm{x}(\mathrm{t}), \mathrm{b}, \mathrm{c} > 0 \\ \mathrm{y}'(\mathrm{t}) = \mathrm{u}(\mathrm{t}) \cdot \mathrm{e}^{-\mathrm{pt}}, \mathrm{p} > 0 \\ \mathrm{x}(0) = \mathrm{x}_0 \geq 0 \\ \mathrm{y}(0) = 0 \\ 0 \leq \mathrm{u}(\mathrm{t}) \leq \mathrm{u}_0, \forall \mathrm{t} \in [0,\mathrm{T}], \alpha \in (0,1] \end{cases} \qquad (2)$$

Pentru $\alpha = 1$ se obțin chiar ecuațiile propuse de Nerlowe-Arrow, [3] $\alpha \in (0,1]$ şi sunt date de:

$$\begin{cases} x'(t) + c \cdot x(t) = b \cdot u(t), b, c > 0 \\ y'(t) = u(t) \cdot e^{-pt}, p > 0 \\ t \in [o, T] \end{cases} \qquad (3)$$

The positive constants b, c, p are adjustable release behavior of the process have the following meanings:

b = coefficient representing the influence of increasing or decreasing the control function u (t) determined by increasing or decreasing function of "good impression" x (t)

c = coefficient to decrease (degradation) of "good impression" x (t)

p = coefficient of variation that balances costs exponentially to promote y (t) expressed through control function u (t) so that there is a correlation between the function x (t) a "good impression" and the function y (t) expenses promotion at time t.

The solutions of the system (1) are subject of the behavior's restrictions. At initial time t=0 "good impression" can have a $x_0 \geq 0$ value, so $x(0) = x_0$, and the investment function is zero, y(0)=0. For the control function we have a finite interval $[0, u_0]$, so $0 \leq u(t) \leq u_0$, $\forall t \in [0,T]$.

At the final time $t = t^* \leq T$, is described a state $(\bar{x}, \bar{y})$, ie gives $\bar{x}, \bar{y} \in \Re$, so as to $x(t^*) = \bar{x}$ and $y(t^*) = \bar{y}$. In conclusion, the optimal control problem with fixed extremes is to determine an optimal trajectory:

$\Gamma^* : \begin{cases} x = x^*(t) \\ y = y^*(t) \end{cases} \; t \in [0, t^*]$ so as to, under the conditions laid during the point $(x(t), y(t)) \in \Gamma^*$ to move from A(x(0), y(0)) in $B(\bar{x}, \bar{y})$ to be minimum. This time is $t^* \le T$, and A and B are the extremities.

Formaly, this control problem is expressed by:

$$\begin{cases} x'(t) + c \cdot x(t) = b \cdot [u(t)]^{\alpha}, b, c > 0 \\ y'(t) = u(t) \cdot e^{-pt}, p > 0 \\ x(0) = x_0, y(0) = 0, x_0 \cup \Re \\ 0 \le u(t) \le u_0, t \in [0,T] \\ x(t^*) = \bar{x}, y(t^*) = \bar{y}, t^* \le T \\ t^* = \min\{t\} \\ t \in [o,T] \end{cases} \quad (4)$$

3. THE HAMILTONIAN MAXIMISING AND THE CONTROL FUNCTION DETERMINATION

The optimal control problem (4) solved using the "principle of maximum PONTREAGUINE". For this purpose, first we introduce the Hamilton's function:

$$H(x,y,u) = \Psi_1(t)\cdot(b\cdot u^{\alpha}(t) + \Psi_2(t)\cdot(e^{-pt}\cdot u(t)) - \Psi_1(t)\cdot c\cdot x(t) \quad (5)$$

where Ψ_1 and Ψ_2 general solution of the system are the components of Hamilton, which is given by:

$$\begin{cases} \Psi_1'(t) = -\left[\dfrac{\partial H}{\partial x}\right] \\ \Psi_2'(t) = -\left[\dfrac{\partial H}{\partial y}\right] \end{cases} \quad (6)$$

In this case (6) is:

$$\begin{cases} \Psi_1'(t) = c \cdot \Psi_1(t) \\ \Psi_2'(t) = 0 \end{cases} \quad (7)$$

Integrating the system (7) it obtain $\Psi_2(t) = c_2, \Psi_1(t) = c_1 \cdot e^{ct}, c_1, c_2 \in \Re$. In conclusion, the general solution of the systemsoluția generală a sistemului (7) is:

$$\begin{cases} \Psi_1(t) = C_1 \cdot e^{ct} \\ \Psi_2(t) = C_2 \end{cases} t \in [0,T] \quad (8)$$

With these results, The Hamiltonian becames:

$$H(x,y,u) = \left[b\cdot u^{\alpha}(t) - c\cdot x(t)\right]\cdot C_1\cdot e^{ct} + C_2\cdot e^{-pt}\cdot u(t) = \left[b\cdot u^{\alpha}(t) - c\cdot x(t)\right]\cdot C_1\cdot e^{ct} + C_2\cdot e^{-pt}\cdot u(t) \quad (9)$$

Based on the principle of maximum PONTREAGUINE's still determine maximum function H (u), the variable u, the function H (u) is:

$$H(u) = A \cdot u^{\alpha} + B \cdot u + C \quad (10)$$

where:

$$A = b \cdot C_1 \cdot e^{ct}$$
$$B = C_2 \cdot e^{-pt}$$
$$C = -c \cdot C_1 \cdot x(t) \cdot e^{ct} \quad (11)$$

Determining the maximum of H (u) involves the calculation of critical points for the function H (u), ie solving the equation: H'(u)=0, which leads to: $\alpha \cdot A \cdot u^{\alpha-1} + B = 0, \alpha \in (0,1)$ results:

$$B \cdot u^{\alpha-1} = -\alpha \cdot A \quad \text{or}$$

$$u^{\alpha-1} = -\frac{\alpha \cdot A}{B} \Leftrightarrow u^{\alpha-1} = -\frac{C_1}{C_2} \cdot b \cdot \alpha \cdot e^{(c+p)\cdot t}$$

At this point a discussion is necessary according to the sign constants C_1 şi C_2.

a) If C_1 and C_2 are zero, it follows that $\Psi_1 = 0, \Psi_2 = 0, \forall t \in [0,T]$. This approach is inconsistent with the principle of PONTREAGUINE.

b) If $C_1 > 0, C_2 > 0$, results $H'(u) > 0$

In this case the optimal control function (optimal control) is:

$$u^*(t) = u_0, t \in [0,T] \quad (12)$$

c) If $C_1 < 0, C_2 < 0$, results $H'(u) < 0$.

It is noted that in this case: $\max_{u \in [o,u_0]} H(u) = H(0) = C$ so:

$$u^*(t) = 0, t \in [0,T] \quad (13)$$

In this case the control problem does not make sense.

d) If $C_1 < 0, C_2 > 0$ we are led to (12) or (13).

e) If $C_1 > 0, C_2 < 0$

$$u^*(t) = \lambda \cdot e^{\mu t}, \lambda > 0, \mu > o \quad \text{with}$$

$$\lambda = \left(-\alpha \cdot b \cdot \frac{C_1}{C_2}\right)^{\frac{1}{1-\alpha}}, \mu = \frac{c+p}{1-\alpha} \quad \text{so:}$$

$$u^*(t) = \lambda \cdot e^{\mu t}, t \in [0,T] \quad (14)$$

We take Hamilton's system solution corresponding to $\frac{C_1}{C_2} = -1$, so $\lambda = (\alpha \cdot b)^{\frac{1}{1-\alpha}}$

4. DETERMINING THE OPTIMAL STATE PATH

If the optimal control function $u^*(t)$ was determined, we can determine the optimal path Γ^* integrating the system (4).

We observe the case (13) which leads to:

$$\begin{cases} x'(t) + c \cdot x(t) = 0 \\ y'(t) = 0 \\ x(0) = x_0, y(0) = 0 \end{cases} \quad (15)$$

$y(t) = K_2, y(0) = 0 \Rightarrow K_2 = 0$, so in this case y(t)=0, $\forall t \in [0,T]$.

In conclusion, launching new product at no cost. Consequence of this leads us to exclude this case.

For the case (12), equations of state are: $\begin{cases} x'(t) + c \cdot x(t) = b \cdot u_0{}^{\alpha} \\ y'(t) = u_0 \cdot e^{-pt} \end{cases}$

$$\Rightarrow y(t) = -\frac{u_0}{p} \cdot e^{-pt} + K_2 \quad (16)$$

From y(0)=0 $\Rightarrow K_2 = \frac{u_0}{p}$ and we obtain the cost function:

$$y(t) = \frac{u_0}{p}\left(1 - e^{-pt}\right), t \in [0,T]$$

For $t = t^*$, $y(t^*) = \bar{y}$ so $\bar{y} = \frac{u_0}{p}\left(1 - e^{-pt^*}\right)$ and we obtain the minimum time:

$$t^* = \frac{1}{p} \cdot \ln \frac{u_0}{u_0 - p \cdot \bar{y}} \quad (17)$$

With the conditions : $t^* \in (0,T]$ and $\bar{y} < \frac{u_0}{p}$ or $0 < p < \frac{u_0}{\bar{y}}$.

For the function x(t), we integrate:

$$x' + c \cdot x = \upsilon \quad (18)$$

where: $\upsilon = b \cdot u_0{}^{\alpha}$

The general solution of the equation (18) is:

$$x(t) = K_1 \cdot e^{-ct} + \frac{b}{c} \cdot u_0{}^{\alpha} \quad (19)$$

From the initial condition $x(0) = x_0$ we obtain the integrated constant K_1

$$K_1 = x_0 - \frac{b}{c} \cdot u_0^{\alpha}$$

So the solution is Cauchy problem:

$$x^*(t) = \left[x_0 - \frac{b}{c} \cdot u_0^{\alpha} \right] \cdot e^{-c \cdot t} + \frac{b}{c} \cdot u_0^{\alpha} \quad (20)$$

Finally, the final condition $x(t^*) = \bar{x}$ leads to $\bar{x} - \gamma = (x_0 - \gamma) \cdot e^{-ct^*}$, where $\gamma = \frac{b}{c} \cdot u_0{}^{\alpha}$. Of equality $e^{ct^*} = \frac{x_0 - \gamma}{\bar{x} - \gamma} > 1$ results:

$$t^* = \frac{1}{c} \cdot \ln \frac{x_0 - \gamma}{\bar{x} - \gamma} > 0 \Leftrightarrow t^* = \frac{1}{c} \cdot \ln \frac{b \cdot u_0^{\alpha} - c \cdot x_0}{b \cdot u_0^{\alpha} - c \cdot \bar{x}} \quad (21)$$

But the innegality $\frac{x_0 - \gamma}{\bar{x} - \gamma} > 1$ $\Leftrightarrow \frac{x_0 - \bar{x}}{\bar{x} - \gamma} > 0$

Knowing that: $x_0 - \bar{x} < 0$ so $\bar{x} - \gamma < 0$ şi so $x_0 < \bar{x} < \frac{b}{c} \cdot u_0{}^{\alpha}$

Finally, another limitation of the input data is:

$$x_0 < \bar{x} < \frac{b}{c} \cdot u_0{}^{\alpha} \quad (22)$$

Since the current point (x(t), y(t)) arrive at the same time $(\bar{x}, \bar{y})$, the t^* value from (16) and (17) is the same, so we have the equation:

$$\frac{1}{p} \cdot \ln \frac{u_0}{u_0 - p \cdot \bar{y}} = \frac{1}{c} \cdot \ln \frac{x_0 - \frac{b}{c} \cdot u_0^{\alpha}}{\bar{x} - \frac{b}{c} \cdot u_0^{\alpha}} \Leftrightarrow \left(\frac{b \cdot u_0^{\alpha} - c \cdot x_0}{b \cdot u_0^{\alpha} - c \cdot \bar{x}} \right)^p = \left(\frac{u_0}{u_0 - c \cdot \bar{x}} \right)^c$$

equation whose solution must obtain the assumption that:

$$p \in \left[0, \frac{u_0}{\bar{y}} \right) \text{and} \frac{b}{c} \cdot u_0^{\alpha} > \bar{x} \quad (23)$$

Problem to new product launch on the market in this case is more laborious and see that it has solution if all that emerged during resolution of the input data: $T, u_0, x_0, \bar{x}, \bar{y}$ and the adjustment coefficients b, c, p, α is checked. Finally, the case of (14) is treated similarly. The state equations for this case are:

$$\begin{cases} x'+c\cdot x=b\cdot\lambda^{\alpha}\cdot e^{\alpha\mu t} \\ y'=\lambda\cdot e^{(\mu-p)t} \\ t\in(0,T) \end{cases} \quad (24)$$

plus the same initial and final conditions. In this case too, the constants λ and μ are given by :

$$\lambda=\left(-\alpha\cdot b\cdot\frac{C_1}{C_2}\right)^{\frac{1}{1-\alpha}}, \qquad \mu=\frac{c+p}{1-\alpha},$$

and $C_1>0$ and $C_2<0$, results:

$-\alpha\cdot b\cdot\frac{C_1}{C_2}>0$, with $\frac{C_1}{C_2}=-1$, so $\lambda=(\alpha\cdot b)^{\frac{1}{1-\alpha}}$

Integrating (23) we obtain: $y(t)=K_2+\frac{\lambda}{\mu-p}\cdot e^{(\mu-p)t}$, with t=0, results y(0)=0, so: $K_2=\frac{\lambda}{p-\mu}$, which leds to:

$$y^*(t)=\frac{\lambda}{p-\mu}\cdot(1-e^{(\mu-p)t}) \quad (25)$$

And for $t=t^*$, $\bar{y}=\frac{\lambda}{p-\mu}\cdot(1-e^{(\mu-p)t^*})$, whence t^* is:

$$t^*=\frac{1-\alpha}{c+p\cdot\alpha}\cdot\ln\frac{(1-\alpha)\cdot\lambda+\bar{y}(c+p\cdot\alpha)}{(1-\alpha)\cdot\lambda} \quad (26)$$

Because $t^*>0$, it is necessary the condition:

$$\frac{(1-\alpha)\cdot\lambda+\bar{y}(c+p\cdot\alpha)}{(1-\alpha)\cdot\lambda}>1 \quad (27)$$

Or equivalent with $1+\frac{\bar{y}\cdot(c+p\cdot\alpha)}{(1-\alpha)\cdot\lambda}>1$, which is always true because $\bar{y}\cdot(c+p\cdot\alpha)>0$ and $(1-\alpha)\cdot\lambda>0$.

Function x (t), we integrate the first equation in (23) and obtain the general solution:

$$x(t)=K_1\cdot e^{-ct}+\frac{b\cdot\lambda^{\alpha}}{c+\mu\cdot\alpha}\cdot e^{\mu\alpha t} \quad (28)$$

Using the initial condition $x(0)=x_0$ we obtain the integrate constant K_1 so $x^*(t)$ becomes:

$$x^*(t)=\frac{b\cdot\lambda^{\alpha}}{c+\mu\cdot\alpha}\cdot e^{\mu\alpha t}+\frac{x_0\cdot(c+\mu\cdot\alpha)-b\cdot\lambda^{\alpha}}{c+\mu\cdot\alpha}\cdot e^{-ct} \quad (29)$$

If impose and final condition $x(t^*)=\bar{x}$, (28) becames:

$$(c+\mu\cdot\alpha)\cdot\bar{x}=b\cdot\lambda^{\alpha}\cdot e^{\mu\alpha t^*}+[x_0\cdot(c+\mu\cdot\alpha)-b\cdot\lambda^{\alpha}]\cdot e^{-ct^*}$$

or equivalent with:

$$b\cdot\lambda^{\alpha}\cdot e^{(c+\mu\alpha)t^*}-\bar{x}\cdot(c+\mu\cdot\alpha)\cdot e^{ct^*}+x_0\cdot(c+\mu\cdot\alpha)-b\cdot\lambda^{\alpha}=0 \quad (30)$$

This equation can not be solved in general, it involves the application of a numerical method to find an approximate minimum time t^*. It is proposed to Newton's method for this purpose.

Finally

$$\Gamma^*:\begin{cases} x^*(t)=\frac{b\cdot\lambda^{\alpha}}{c+\mu\cdot\alpha}\cdot e^{\mu\alpha t}+\frac{x_0\cdot(c+\mu\cdot\alpha)-b\cdot\lambda^{\alpha}}{c+\mu\cdot\alpha}\cdot e^{-ct} \\ y^*(t)=\frac{\lambda}{p-\mu}\cdot(1-e^{(\mu-p)t}) \end{cases}, t\in[0,t^*] \quad (31)$$

We addressed the new product launch (NPL) by using optimal control in both cases (12) or (14) if there is a single control function. This is achieved if we use the initial condition to determine the constant of integration. The final condition was used resulting in optimal time t^*. A more general and more control functions is obtained if the integration constant is determined from the final condition.

5. CONCLUSION

The practical value of the mathematical model is to provide complete and thorough information companies interested in commercial success of a new product and to provide an overview of very well defined on the whole process of release. Also, specialists in new product development are able to decide when they can to stop investing in promotional or longer allocate the amounts necessary for the success of the new product have planned.

Extensive coverage due to problems of optimal control problems attached LPN I appreciated that research conducted is sufficiently complete. This statement is the fact that a company wants to introduce a new product category at baseline t = 0 did any expense, ie y (0) = 0.

This mathematical modeling is entirely an original contribution [14] with a great practical value for both management and manufacturing companies for marketers, who may have an image and an effective control possibilities to minimize the time in which the new product launch.

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Un model matematic aplicat unui proces economic

Rezumat: Principalele metode de cercetare utilizate în NPL (lansarea unui nou produs), în acest caz sunt: metode statistice economice, teoria controlului optimal şi problema de timp minim. În această lucrare am utilizat un model matematic folosind teoria de control optimal al timpului minim. În acest scop, vom presupune că este prevăzut un interval de timp în care să realizeze lansarea unui nou produs. Am considerat un timp propus, dar de data aceasta problema matematica a fost minimizarea perechii (impresie bună, investiţii în proces), determinarea traiectoriei optime, evolutia optima. Problema de analiză relevata in problema de control optimal este cu capete fixe, ceea ce înseamnă că ne asumăm de la început o impresie bună stabilita şi un buget fix, adică punctele de la care am ajuns şi care sunt fixate, nu sunt incluse în intervale.

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