TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue I, March, 2013

CREATING A MATHEMATICAL MODEL FOR THE COST OF SUPPLY

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Abstract: *In this paper have made a detailed presentation of the concept of mathematical modeling in production systems, by highlighting the purpose and the benefits. Own contributions are: a mathematical model to determine the minimum cost of supply depending on transport cost and value of the quantity supplied. Note that the creation model is based on two classical models existing in the literature plus the contribution of the author; Identify solutions for the supply model was done using MATLAB working environment 2011 in two variants namely fixed transportation charge and variable charge transport. It was highlighted by a graph, the importance (weight) transport cost in the total cost of supply; Identify areas for cost transport solutions through "response surface method" which includes an analysis of ANOVA type; Achieving the above was performed using Design-Expert program 8*
Key words: *modeling, model, supply, MATLAB and Design Expert.*

1. THE CONCEPT OF MODELING

Modeling is a scientific and technical research that is an exact reproduction of the system investigated in the form of mathematical equations or other forms, to study the properties and transformations taking place within the analyzed.

We know that modeling is a new activity, the literature states that "science is generally based on the notion of modeling" and "scientific progress is progress modeling natural phenomena and ideas".

The purpose of modeling a production system:

- highlights all the phenomena that take place within the production system analysis;
- with the model can be approximated to be the results if the application of policies and management measures, based on these results and make decisions as proper conduct profitable business;
- an exact description of the elements they represent.

Making a model is an activity that requires a lot of subtlety and skill in the modeling process. Following the results in a model must meet two important conditions.

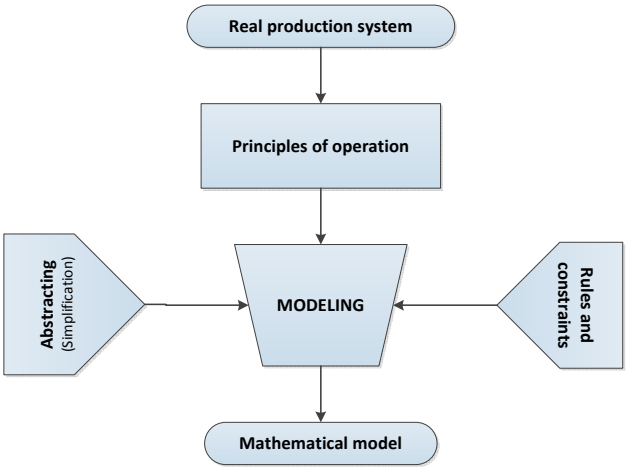

Fig. 1. Representation of the concept of modeling

The first condition is that the model be simple enough to use in its implementation, which implies that he is a simplified representation (abstract) system of production. The second condition is that the model is an exact representation of the system of production, involving a complication of the model.

Building a model representative of a production system can be achieved through an iterative process starting from the simplest

existing model. Achieving this goal is only possible by defining the limits of the model in order to use only the features essential to the aim pursued.

By lowering the complexity of the model considered, it offers several advantages such as cost savings arising from the analysis and evaluation of results obtained with the model by applying a set of policies and / or action, getting results in a very short time compared to real situations to avoid damaging critical or extremely likely to result in the case of experiments on a real production system.

2. STAGES OF MODELING

Modeling is a research tool and aims to play real situations to allow a better understanding of the processes undertaken to achieve a product.

It can be said that the "model" is a true representation of reality and at the same time be a simplified representation of productive practice to offer more efficient solutes. A model must emphasize an intuitive picture, rigorous, logical structure for the purposes of process modeling, the detection of links and laws that cannot be identified by other methods.

Work towards the model is to select one of various existing modeling tools classic, provided it is compatible with the problem formulated or in developing new models. In this activity, one that made the model to analyze, establish links between production system and use modeling tool.

Instruments used in the modeling of a production system can be of two types, namely:

- mathematical models from the classic combination of existing literature;
- creating new models, which involves holding the analyst has a "solid bag" of mathematical knowledge.

A model can be characterized in several ways, namely: simplicity, accessibility and adaptability.

Modeling process includes the following steps:

- detailed definition and purpose modeling system;
- data collection and analysis;
- actual construction of the model;
- model checking;
- model validation.

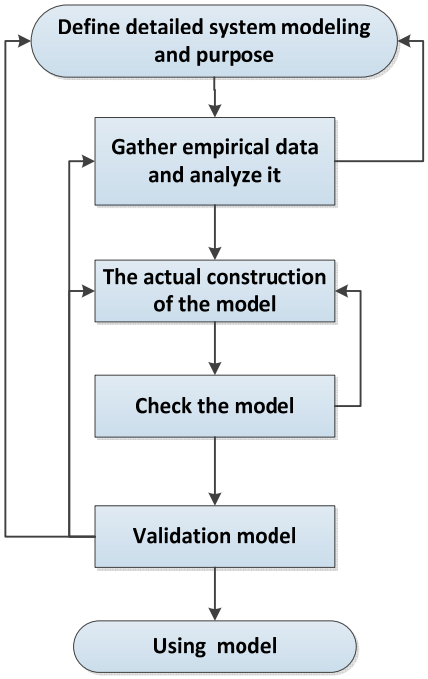

Fig. 2. Stages of making a model [4]

Step 1 Define detailed system modeling and purpose, is made taking into account the structure of production in question and requires the following specifications:

- delimitation process and observing their behavior in order to understand their functioning and be able to control;
- develop questions to be answered by the model (which are links, constraints, how to produce the show is performed and studied);
- identification of parameters and variables of interest.

Step 2 Gather empirical data and analyze it, is the determination and collection of relevant data to estimate the parameters used and their interaction due to the restrictions on implementation.

Step 3 The actual construction of the model, is the stage in which the grounding assumptions required for the process studied and a statement of preliminary findings to reveal the nature of relationships between variables used in this process.

Step 5 Validation involves comparing model predictions obtained with existing empirical

evidence for accepting or rejecting the model developed.

Trying model is recommended to be "in vitro" by applying descriptive or normative model business practice and its effectiveness in finding descriptive / normative. This testing model will be extremely small samples, because it may involve considerable risk.

3. CLASSIFICATION OF MODELS

After the type of data used to create models for production systems, they can be classified into several types, namely:

Scale models - this category includes models with small or enlarged, "identical" to the object it represents.

Analog models - models created are considered substitutes and analogous systems of study. After the problem is resolved substitute system, transfer the solution obtained on the original system, relative to the size or properties of that system (eg non-electrical quantities measuring electrically).

Mathematical models - such models are general and are also more abstract models. Through the mathematical modeling any object, item or production system can be written in the form of mathematical relationships. Besides the description of objects or elements and we can put conditions limiting the use. With these methods we can assign a symbol such as x or α, a feature: a quantity, distance, a load, amount of benefit, etc.

Symbolic models - they are used only when logical or mathematical expression of relationships, can be qualitative and quantitative, which involves the logical or mathematical links between inputs and outputs selected.

After nature awareness of variables that are used to create models for production systems, they can be classified into several types, namely:

Deterministic models are those models whose inputs are known with certainty. For example, you can apply to programming model the operations within a manufacturing technology for a product because in this case is aware of all material resources comprising the product and all human resources participating in the property, during activities performance on each step and where each activity is performed. Deterministic models will always give the same solution for the same input data that are supposed to be known with certainty.

Knowing with certainty that all data entering the implementation model is a simplifying hypothesis, these deterministic models are important for:

- can be used in a variety of management activities, which are used in decision making;
- be solved through analytical solutions of the optimal decision variables. With the help of mathematical relationships or by applying an optimization algorithm against established performance criteria, can identify the optimal values of decision variables;

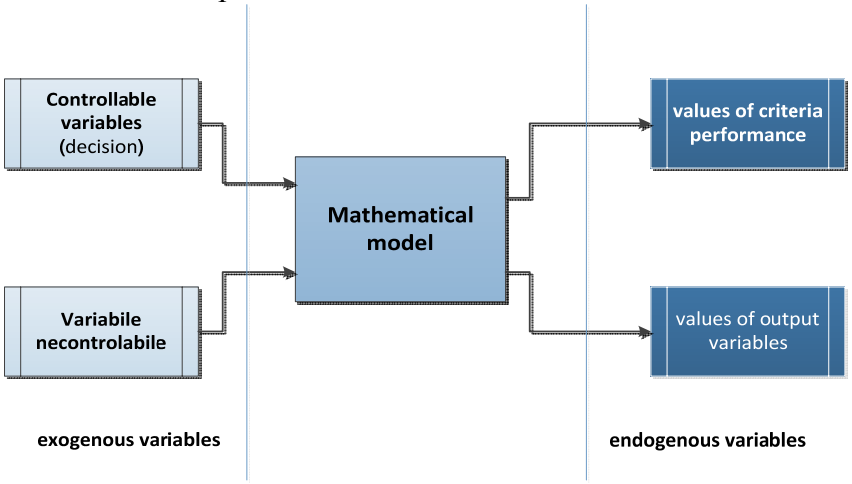

Fig. 3. Classification of models based on inputs and outputs [2]

- quickly find an optimal solution for deterministic type models can be done using specialized software, such software programming involving linear function, which can be used to optimize inventory.
- repeated use of mathematical models helps users to develop their skills to create deterministic models.

Probabilistic or stochastic models are those whose input types are not known with certainty. Variability data to enter in the model can be created by defining a mathematical function or a table containing the probabilities associated input variables.

Solving probabilistic models is achieved by using process simulation, which cannot be solved by analytical. For some types of probabilistic models is to obtain analytical solutions only.

4. MODEL FOR DETERMINING THE MINIMUM COST OF SUPPLY FOR AN ORDER (PROPOSED BY THE AUTHORS)

In the literature there are two ways to create models first requires a sound knowledge of mathematics, that is creating the model from scratch, and the second way is to create models based on the existing classical literature.

The model developed is based on two classic models have been described above, namely: the method of transport and transport costs.

The model was created to use in making purchasing decisions. The solution offered is based on two factors which directly influence the total cost of supply, namely: transport cost + the quantity transported. Transportation cost consists of: price * distance * quantity transported, and the quantity transported is equal to the total amount recorded on the invoice.

$$C_{ij} = (T_{ij} * x_{ij} * D_{ij}) + P_{ij}$$
$$x_{ij} \leq a_i \qquad i=1,m \qquad (1)$$
$$x_{ij} \geq b_j \qquad j=1,n$$
$$\text{f. obiectiv} \quad \min C_{ij}$$

where:

C_{ij} = total cost of supply from supplier „i” to „j”;

T_{ij} = transport rate lei / unit (1 kg) * distance (1 km);

D_{ij} = distance transport provider „i” to „j”;

x_{ij} = quantity transported from supplier „i” to „j”;

a_{ij} = maximum amount that a supplier can deliver an order;

b_{ij} = supplying the required amount of the beneficiary „j”;

Pij = value of goods transported.

To determine the value of goods transported can use the relation 2, which allows for the introduction of a discount factor if necessary, taking it between 0 and 1. Note that the value of goods purchased is equal to the total bill.

$$P_{ij} = \sum_{k=1}^{p} (c_{ik} * m_{ik}) * d_{ik}; \quad k=1,p \qquad (2)$$
$$0 \leq d_{ij} \leq 1$$

where:

c_{ik} = material cost “k” to “i”;

m_{ik} = amount of material „k” to „i”;

d_{ik} = discount offered by the company “i” for material “k”;

k = type of material.

5. IDENTIFYING SOLUTIONS USING MATLAB

In this paper is how to use the model presented by two cases. In the first case, the transport rate is constant, and the second rate is variable, depending on the distance transported, to supply raw materials necessary for manufacture.

Start by defining the maximum quantities that can be delivered on an order by each supplier. To test the model were taken fictitious data, namely: F1=300 kg, F2=350 kg, F3=420 kg and F4=385 kg, (fig. 4., line 9). These values can be identified by closer collaboration with suppliers.

I can assure the maximum amount for delivery. Usually, these amounts differ from provider to provider because of size and liquidity deposits locked in stocks.

```
m=4;
n=1;
a=zeros(m); % definirea dimensiuni matricei frunizorilor
b=zeros(n); % definirea dimensiuni matricei beneficiarului
T=zeros(m,n); % definirea dimensiuni matricei tarifelor
D=zeros(m,n); % definirea dimensiuni matricei distantelor
x=zeros(m,n); % definirea dimensiuni matricei a cantitatii transportata
P=zeros(m,n); % definirea dimensiuni matricei de pret
a=[300 350 420 385]; % definirea valorii cantitatilor disponibile la furnizori
b=[300] ; % definirea valorii necesara de aprovizionat pt. beneficiar
T=[0.1 0.1 0.1 0.1]'; % definirea valorii tarifului de transport lei/km
D=[350 350 500 350]'; % definirea valorii distantelor in Km de la "m" la "n".
x=[300 300 300 300]'; % definirea valorii cantitatilor transportate de la "m" la "n"
P=[1000 1300 1000 900]'; % definirea valorii pretului de achizitie
C=zeros(m,n);  % definirea dimensiuni matricei costurilor totale
minim=10^10;
for i=1:m
    for j=1:n
        C(i,j)=T(i,j)*x(i,j)*D(i,j)+P(i,j);  % calcularea costului de aprovizionare

        if C(i,j)<minim  % se verifica costul

            if x(i,j)<=a(i) && x(i,j)>=b(j) % definirea conditilor de aprovizionare

            imin=i;
            jmin=j;
            minim=C(i,j);
            end
        end
    end
end
clc; % curatirea ecranului
disp([' Costul minimi este = ' num2str(minim) ' lei ' ] );
disp([' Costul total de aprovizionare minim s-a obtinut de la furnizorul  = ' num2str(imin)]);
```

Fig. 4. The case when the transport rate is fixed

The solution offered the model is as follows: Minimum cost of supply is 11.400 lei from provider 4.

For a clear analysis of the full cost of supply, Matlab provides facility to achieve a schedule to detect more easily obtained from each supplier costs. As can be seen in Figure 5, the minimum cost of supply was obtained from the supplier F4, followed by F1, F2 and F3.

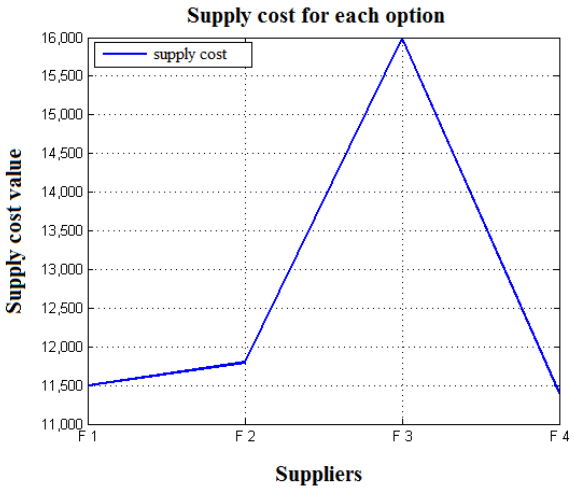

Fig. 5. The cost of supply, if the fixed

Case 2, when goods are transported by a company which offers discount for some distance, in this case the 10 – 400 km rate is 0.1 lei/km from 401 – 800 km the rate is 0.07 lei/km.

There is a route that has value greater than 401 km, ie between the holder and 3, in consequence of the transport rate for this route will take the value 0.07 lei/km.

The remaining data used in the model will remain the same, in order to highlight the importance of transport costs in the supply.

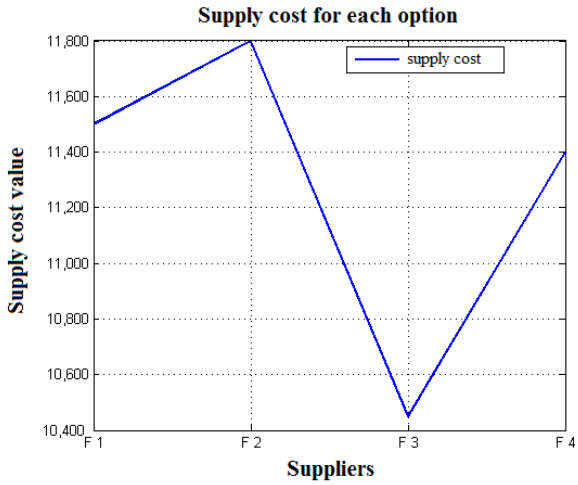

Fig. 6. The cost of supply for various rates

In this case, the minimum cost of supply is 10,450 lei and was obtained from the supplier F3.

As can be seen from the graph, the minimum cost of supply for variable-rate version of the provider to obtain 3, followed by F4, F1 and F2 (figure 6).

6. IDENTIFICATION OF SURFACE TRANSPORTATION SOLUTIONS FOR THE COST OF USING "RESPONSE SURFACE METHOD"

Response surface method in the literature is the acronym for RSM (Response Surface Methodology) is a statistical and mathematical techniques meeting used to identify, develop, improve and optimize production processes.

Applying this method involves defining several input variables that affect the performance or quality characteristic of a process. These input variables are called independent variables.

Statistical modeling of a process tries to determine a relationship between output variables and input variables.

ANOVA dispersion analysis is used in the "response surface method" to check the total variance can be decomposed into the sum of variation between groups and variation within groups. F statistical test (Fisher) for dispersion analysis is the ratio indicators of variability for the two sources of variation: variation between groups and variability within groups.

The authors propose to conduct an analysis of the cost of delivery using response surface method for identifying surface graphics solutions and determine the decisive factors. For this analysis will use computer software, namely Design Expert, version 8, the company offered State-Ease, Inc. .. This software can be used in the demo for 45 days.

After the analysis phase, the program made a diagnosis, which is the correlation output variables predicted output variables calculated in the table variable (figure 7).

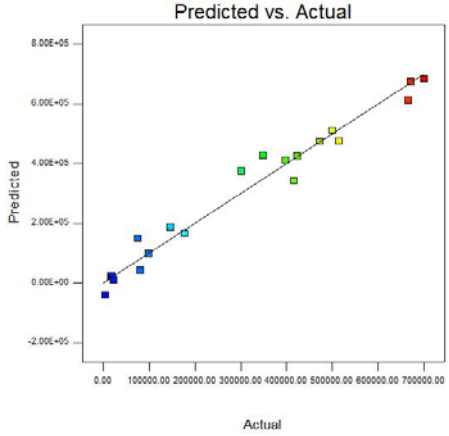

Fig. 7. The cost of transportation according to the values calculated and predicted right

By keeping constant the amount of transport variable name to value 7000 kg, can be identified with the graph, surface transportation solutions for the cost, depending on the other two input variables. If you change the quantity, response surface will remain the same but will move down to the value defined (figure 8).

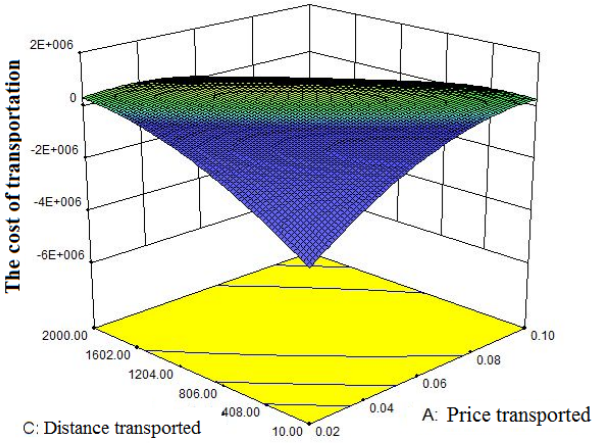

Fig. 8. Transport cost analysis based on price and distance

The program allows a 2D analysis, the cost of transport depending on price and distance transported (figure 9).

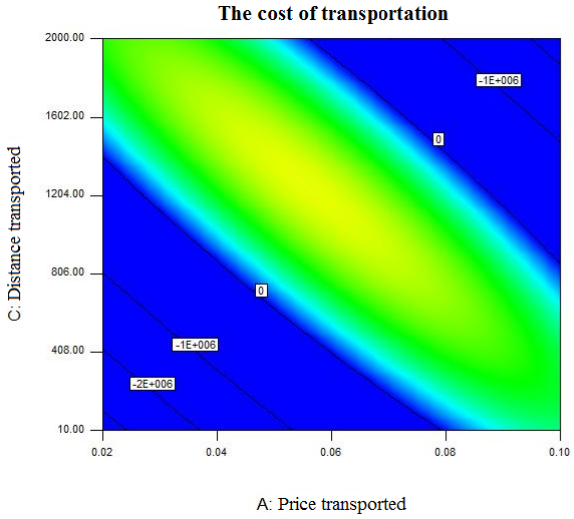

Fig. 9. The cost of transportation depending on price and distance

Case 2 Transportation tariff is fixed and has a value of 0.1 RON / km, regardless of the distance transported. Using surface analysis

will identify solutions for transport costs, in terms of: the two independent variables (distance, amount) and response variable (transport costs). To identify surface response, the program must identify the type of relationship existing between the input variables and output.

After the analysis phase, the program made a diagnosis, which is the correlation output variables predicted output variables calculated in the table of variables (figure 10).

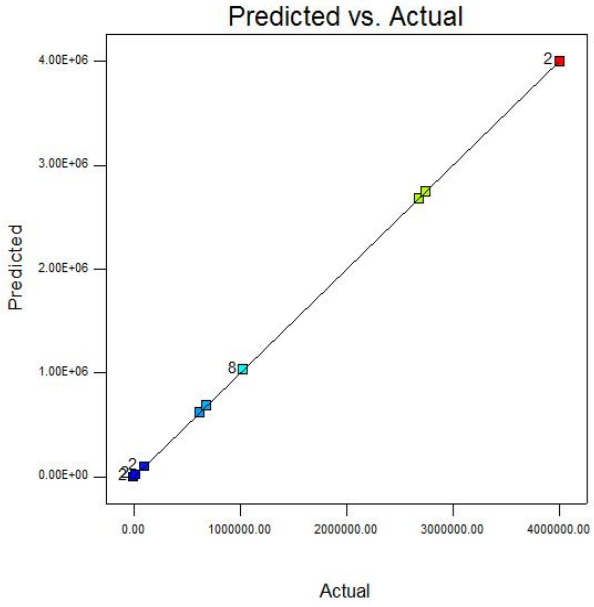

Fig. 10. The cost of transportation according to the values calculated and predicted right

In this case, the variable rate transport name has a value of 0.1 RON / km and to identify the cost of transport from the chart, depending on the other two input variables, namely: quantity and distance. As in previous versions to change the tariff, the solution remains the same, but will move vertically depending on the value defined (figure 11).

Response surface analysis of the cost of transport depending on quantity and distance transported (figure 12).

7. CONCLUSION

In conclusion, the minimum purchase cost does not always involve the minimum cost of supply because of two graphical analysis (figure 5 and 6) can highlight the importance of transport costs in the supply. The next section will be a decisive factor analysis to determine the total cost of transport.

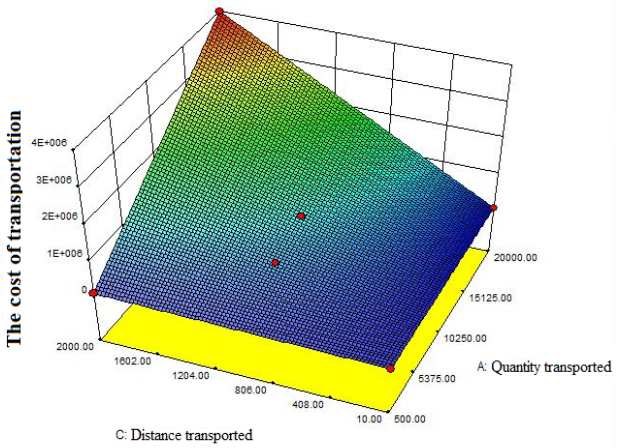

Fig. 11. Analysis of transport cost depending on quantity and distance

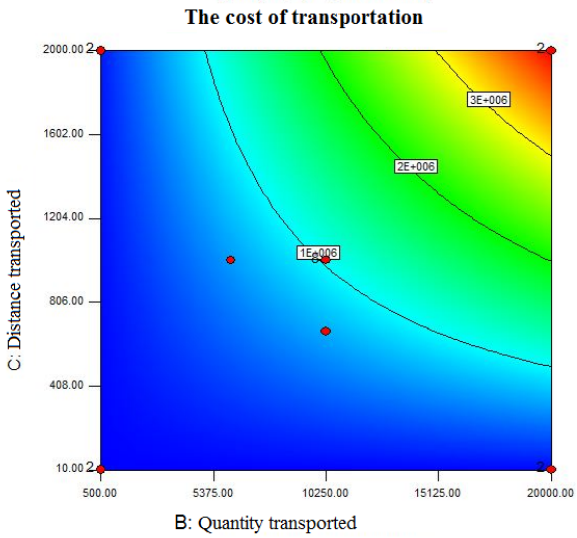

Fig. 12. Transport cost depending on quantity and distance

8. REFERENCES

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Crearea unui model matematic pentru determinarea costului de aprovizionare

Rezumat: In acesta lucrare s-a realizat o prezentare detaliată a conceptului de modelare matematică în cadrul sistemelor de producție, prin evidențierea scopului si avantajelor oferite de acestea. Contribuțiile proprii sunt: un model matematic pentru determinarea costului minim de aprovizionare în funcție de costul de transport şi valoarea cantității aprovizionate. De menționat că, crearea modelului se bazează pe două modele clasice existente în literatura de specialitate plus aportul autorilor; Identificarea soluțiilor modelului de aprovizionare s-a realizat cu ajutorul mediului de lucru MATLAB 2011, în două variante şi anume: cu tarif de transport fix şi cu tarif de transport variabil. S-a pus în evidență cu ajutorul unui grafic, importanța (ponderea) costului de transport în costul total de aprovizionare; Identificarea suprafețelor soluțiilor pentru costul de transport prin „Metoda suprafeței de răspuns" ce include o analiză de tip ANOVA. Realizarea celor menționate anterior s-a efectuat cu ajutorul programului Design-Expert 8, specializat în acest domeniu.

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