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KINEMATICAL CONTROL FUNCTIONS FOR A MOBILE ROBOT

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Abstract: *The mobile robotic field has greatly expanded in the recent years and currently has applications in the most areas of activity. The main objective of robots, in order to achieve a task is to describe motion trajectories, based on control functions, which are consisting in displacements from a point, to a programmed position. Within the paper, will be presented mathematical considerations about a mobile structure, proposed for implementation, for which, will be determined the equations of direct kinematic model.*

Key words: *mobile robot, direct kinematic model, constraints, motion trajectories.*

1. INTRODUCTION

The human access in difficult areas has opened new lines of research which addresses issues with multiple destinations, resulting in achievement of complex devices to cover areas where safety of life is threatened by various factors. Therefore these systems architecture is varied and dependent on their destination. Thus, the mobile robots is the most spectacular and representative group of mechatronic systems, especially due to the attempt to copy and move closer to the model of the living world.

2. GEOMETRICAL MODELING OF THE MOBILE ROBOTS

The direct geometry equations expresses the position and orientation of the mobile robot structure with respect to fixed reference system $\{0\}$. Mobile robots are performing a plane-parallel motion, so that the law of motion, relative to the fixed reference system is:

$$\underset{(3\times1)}{{}^{0}\overline{X}} = \begin{bmatrix} \overline{r}_p \\ \theta \end{bmatrix} = \begin{bmatrix} x_p(t) = q_1(t) \\ y_p(t) = q_2(t) \\ \theta(t) = q_3(t) \end{bmatrix} \tag{1}$$

The above relation contains three independent parameters, characterizing the position and orientation of the mobile robot, therefore the mobile robot in finite displacement has three degrees of freedom, as can be observed from Fig. 1.

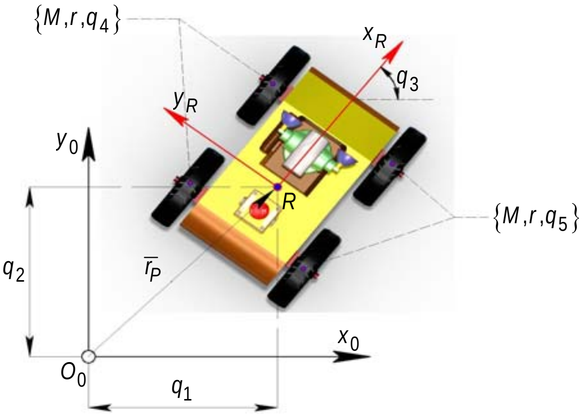

Fig.1 The independent parameters in finite displacements

The rotation around z axis, characterizes the mobile robot orientation, the orientation of the system having the origin in point P, attached to the mobile robot with respect to $\{0\}$. The inverse matrix representing the orienting of fixed reference frame with respect to mobile frame, is presented according to [1], [2] as:

$$R(\overline{z},q_3) = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^{R}_{0}[R] = {}^{0}_{R}[R]^{-1} = \begin{bmatrix} cq_3 & sq_3 & 0 \\ -sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

where $cq_i = \cos q_i$, and $sq_i = \sin q_i$.

3. KINEMATICAL MODELING OF THE MOBILE ROBOTS

The unknowns in direct kinematic model are the linear and angular velocity of the mobile structure with respect to the fixed reference system: ${}^{0}\dot{\overline{X}}=\begin{bmatrix}\dot{x}_p & \dot{y}_p & \dot{\theta}\end{bmatrix}^T=\begin{bmatrix}\dot{q}_1 & \dot{q}_2 & \dot{q}_3\end{bmatrix}^T$; if there are known the angular speeds of the wheels. In order to determine the kinematic equations, there is studied the locomotion of the mobile platform and are determined the kinematic constraints. According to Fig.1, or Fig.2 the independent parameters which are characterizing the geometry of the robot in finite displacements are:

$$\overline{X}(t)=\left[q_i(t);\ i=1\rightarrow 5\right]^T. \qquad (3)$$

The column vector of operational velocities, which expresses the absolute movement of the mobile robot is:

$${}^{0}\dot{\overline{X}}=\begin{bmatrix}\dot{x}_p & \dot{y}_p & \dot{\theta}\end{bmatrix}^T=\begin{bmatrix}\dot{q}_1 & \dot{q}_2 & \dot{q}_3\end{bmatrix}^T; \qquad (4)$$

The transfer of column vector of operational velocities from the fixed system $\{0\}$ in the system attached to the mobile robot, denoted $\{R\}$ is described by the matrix equation, written in the following form:

$${}^{R}\dot{\overline{X}}=\begin{bmatrix}{}^{R}\dot{x}_p\\ {}^{R}\dot{y}_p\\ {}^{R}\omega\end{bmatrix}={}^{0}_{R}[R]^{-1}\cdot{}^{0}\dot{\overline{X}}=\begin{bmatrix}cq_3 & sq_3 & 0\\ -sq_3 & cq_3 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix} \qquad (5)$$

$${}^{R}\dot{\overline{X}}=\begin{bmatrix}\dot{q}_1\cdot cq_3+\dot{q}_2 sq_3\\ -\dot{q}_1\cdot sq_3+\dot{q}_2\cdot cq_3\\ \dot{q}_3\end{bmatrix};$$

If the movement of mobile structure is achieved only after x_R axis, resulting that the sliding on the y_R axis is not possible in infinitesimal displacements, the velocity vector has the following form: ${}^{R}\dot{\overline{X}}=\begin{bmatrix}{}^{R}\dot{x}_p & 0 & {}^{R}\omega\end{bmatrix}^T$, and as a result, it appears the sliding constraint along y_R axis of the mobile robot, presented as:

$$-\dot{q}_1\cdot sq_3+\dot{q}_2\cdot cq_3=0,:\ \frac{dq_2}{dq_1}=\mathrm{tg}\,q_3. \qquad (6)$$

The sliding constraint along the y_R axis can be expressed in differential form, as results from the following equation:

$$-dq_1\cdot sq_3+dq_2\cdot cq_3+0\cdot dq_3=0 \qquad (7)$$

There is introduced the hypothesis that mobile robot wheels are driven by the same engine, having identical kinematic parameters. On the basis of these considerations the two wheels in the kinematic study are replaced with a single fictitious wheel, as can be seen from Fig.2, which is midway between the two, with the same features.

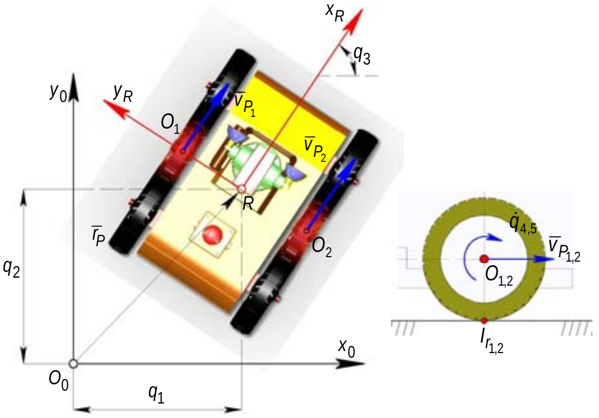

Fig.2 – Linear speed of the points P_1 and P_2

To obtain the linear velocity of the center of the two fictive wheels P_1 and P_2, the position vectors, with respect to fixed reference frame, having the form presented below:

$$\overline{r}_{P_1}=\begin{pmatrix}x_{P_1}\\ y_{P_1}\\ z_{P_1}\end{pmatrix}=\begin{pmatrix}q_1+l\cdot sq_3\\ q_2-l\cdot cq_3\\ r\end{pmatrix};\ \overline{r}_{P_2}=\begin{pmatrix}x_{P_2}\\ y_{P_2}\\ z_{P_2}\end{pmatrix}=\begin{pmatrix}q_1-l\cdot sq_3\\ q_2+l\cdot cq_3\\ r\end{pmatrix}. \qquad (8)$$

There are derivate with respect to time the equations (8) of the position vector and thus are obtained the linear velocities projected onto $\{0\}$ fixed reference system axes, as shown by:

$$\begin{aligned}\overline{v}_{P_1}&=\begin{pmatrix}\dot{x}_{P_1} & \dot{y}_{P_1} & \dot{z}_{P_1}\end{pmatrix}^T=\begin{pmatrix}\dot{q}_1+l\cdot\dot{q}_3\cdot cq_3\\ \dot{q}_2+l\cdot\dot{q}_3\cdot sq_3\\ 0\end{pmatrix};\\ \overline{v}_{P_2}&=\begin{pmatrix}\dot{x}_{P_2} & \dot{y}_{P_2} & \dot{z}_{P_2}\end{pmatrix}^T=\begin{pmatrix}\dot{q}_1-l\cdot\dot{q}_3\cdot cq_3\\ \dot{q}_2-l\cdot\dot{q}_3\cdot sq_3\\ 0\end{pmatrix};\end{aligned} \qquad (9)$$

Using the Fig.2, if it is imposed the condition that the robot wheels to rotate without slipping, I_{r1} and I_{r2} become instant centers of rotation, in this case being valid the relations:

$${}^{R}\overline{v}_{P_1}=\begin{pmatrix}r\cdot\dot{q}_4 & 0 & 0\end{pmatrix}^T;\ {}^{R}\overline{v}_{P_2}=\begin{pmatrix}r\cdot\dot{q}_5 & 0 & 0\end{pmatrix}^T \qquad (10)$$

Projecting the linear velocity equations (10) from the mobile reference system $\{R\}$, on the fixed reference system and then multiplying with the rotation matrix ${}^{0}_{R}[R]$ it results:

$$\begin{aligned}\overline{v}_{P_1}&=\begin{pmatrix}r\cdot\dot{q}_4\cdot cq_3 & r\cdot\dot{q}_4\cdot sq_3 & 0\end{pmatrix}^T;\\ \overline{v}_{P_2}&=\begin{pmatrix}r\cdot\dot{q}_5\cdot cq_3 & r\cdot\dot{q}_5\cdot sq_3 & 0\end{pmatrix}^T.\end{aligned} \qquad (11)$$

By equating equations (9) and (11) there is obtained the following system of equations:

$$\begin{cases} \dot{q}_1 + l\cdot\dot{q}_3\cdot cq_3 = r\cdot\dot{q}_4\cdot cq_3 \\ \dot{q}_2 + l\cdot\dot{q}_3\cdot sq_3 = r\cdot\dot{q}_4\cdot sq_3 \\ \dot{q}_1 - l\cdot\dot{q}_3\cdot cq_3 = r\cdot\dot{q}_5\cdot cq_3 \\ \dot{q}_2 - l\cdot\dot{q}_3\cdot sq_3 = r\cdot\dot{q}_5\cdot sq_3 \end{cases} \tag{12}$$

Solving the system of equations(12), there is resulting the speed of characteristic point P projected onto the fixed reference system as:

$$\dot{\bar{X}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \omega = \dot{q}_3 \end{bmatrix} = \begin{bmatrix} cq_3 & 0 \\ sq_3 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{r}{2}\cdot\left(\dot{q}_4 + \dot{q}_5\right) \\ \frac{r}{2\cdot l}\cdot\left(\dot{q}_4 - \dot{q}_5\right) \end{bmatrix}. \tag{13}$$

Based on the relation(13), in order to the structure to perform a linear movement without slipping then results:

$$\dot{x}_R = \dot{q}_1\cdot cq_3 + \dot{q}_2\cdot sq_3 = \frac{r}{2}\left(\dot{q}_4 + \dot{q}_5\right);$$

$$q_{ji}\left(\tau_{i-1}\right) = q_{ji-1}; \quad q_{ji}\left(\tau_i\right) = q_{ji} \tag{14}$$

According to (14), it can be seen that:

$$\dot{q}_4 = \dot{q}_5 + \frac{2\cdot l}{r}\cdot\dot{q}_3, \quad \dot{q}_5 = \dot{q}_4 - \frac{2\cdot l}{r}\cdot\dot{q}_3 \tag{15}$$

Thus, substituting $\dot{q}_4$ and $\dot{q}_5$ from (15), in the expression of $\dot{x}_R$, there are obtained:

$$\begin{aligned} \dot{q}_1\cdot \mathrm{c}q_3 + \dot{q}_2\cdot \mathrm{s}q_3 - \dot{q}_4\cdot r + \dot{q}_3\cdot l = 0 \\ \dot{q}_1\cdot \mathrm{c}q_3 + \dot{q}_2\cdot \mathrm{s}q_3 - \dot{q}_5\cdot r - \dot{q}_3\cdot l = 0 \end{aligned} \tag{16}$$

which are expressing the conditions for pure rotation of wheels to run without slipping.

The constraint defined by expression (7) is also a requirement for rolling without slipping (16), obtaining the kinematic constraints for mobile structure expressed in differential form:

$$\begin{aligned} -sq_3\cdot dq_1 + cq_3\cdot dq_2 + 0\cdot dq_3 + 0\cdot dq_4 + 0\cdot dq_5 = 0 \\ cq_3\cdot dq_1 + sq_3\cdot dq_2 + l\cdot dq_3 - r\cdot dq_4 + 0\cdot dq_5 = 0 \\ cq_3\cdot dq_1 + sq_3\cdot dq_2 - l\cdot dq_3 + 0\cdot dq_4 - r\cdot dq_5 = 0 \end{aligned} \tag{17}$$

The equations (17), don`t satisfy the Cauchy`s conditions, and as a result the relations (17) are nonintegrable, thus the mobile robot has in infinitesimal displacements only two degrees of freedom. So, the mobile robot is a nonholonomic mechanical system. [1]

After the kinematical study in Cartesian coordinates, there are determined the kinematic equations in polar coordinates, in order to implement the kinematic control functions. In expressing the kinematic control functions, the input data are the coordinates between two points A, B, between the mechanical system has to move, and the orientation of the robot in point A:

$${}^{0}\bar{X} = \left\{ {}^{A}\bar{X}; {}^{B}\bar{X} \right\} = \left\{ \left(x_p^A \quad y_p^A \quad \theta_A \right)^T ; \left(x_p^B \quad y_p^B \quad - \right)^T \right\}$$

respectively the time in which the robot has to run the distance between the previous mentioned points: $\Delta t = t_B - t_A$. Depending on the input data, there will be determined the velocities and coordinates of the wheels.

4 DYNAMIC STUDY OF THE MOBILE PLATFORM

According to relation (7), there is at least one linkage relationship between the number of parameters which are characterizing the movement of mobile robot, two parameters are independent in infinitesimal displacements, hence, the mobile robots are nonholonomic mechanical systems, so the dynamic study is in accordance with specific equations for that model. In case of holonomic mechanical systems, according to Hamilton's variational principle, highlighted in [1], [3], [4]:

$$\int_{t_0}^{t_1} \left(\delta E_C + \sum_{i=1}^{n} \bar{F}_i \cdot \delta \bar{r}_i \right) \cdot dt = 0, \ \delta t = 0; \tag{18}$$

The above equation contains the term δE_C representing virtual differentiation of kinetic energy. The generalized inertia force, can be determined using the principle based on acceleration energy presented as:

$$\int_{t_0}^{t_1} \left[\int_{t_0}^{t_1} \left[\int_{t_0}^{t_1} \delta E_A \cdot dt \right] \cdot dt + \sum_{i=1}^{n} \bar{F}_i \cdot \delta \bar{r}_i \right] \cdot dt = 0 \tag{19}$$

So, the generalized inertia force, based on two principles mentioned above, is as follows:

$$Q_{i\mathscr{F}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_C}{\partial \dot{q}_i} \right) - \frac{\partial E_C}{\partial q_i} = \frac{\partial E_A}{\partial \ddot{q}_i}, \quad i = 1 \to 5; \tag{20}$$

The term E_C from (20) representing the total kinetic energy, having the explicit form:

$$E_C^i = \frac{1}{2}\cdot M_i \cdot {}^{i}\bar{v}_{C_i}^T \cdot {}^{i}\bar{v}_{C_i} + \frac{1}{2}\cdot {}^{i}\bar{\omega}_i^T \cdot {}^{i}I_i^* \cdot {}^{i}\bar{\omega}_i; \tag{21}$$

and E_A denotes the total acceleration energy, which is determined for each component by:

$$\begin{aligned} E_A^i = \frac{1}{2}\cdot M_i \cdot {}^{i}\dot{\bar{v}}_{C_i}^T \cdot {}^{i}\dot{\bar{v}}_{C_i} + \frac{1}{2}\cdot {}^{i}\dot{\bar{\omega}}_i^T \cdot \left\{ {}^{i}I_i^* \cdot {}^{i}\dot{\bar{\omega}}_i + \right. \\ \left. + \left[{}^{i}\bar{\omega}_i \times {}^{i}I_i^* \cdot {}^{i}\bar{\omega}_i \right] \right\} + \frac{1}{2}\cdot {}^{i}\dot{\bar{\omega}}_i^T \cdot \left[{}^{i}\bar{\omega}_i \times {}^{i}I_i^* \cdot {}^{i}\bar{\omega}_i \right] \end{aligned} \tag{22}$$

In the case of nonholonomic systems [5], the above principle is completed by applying Lagrange multipliers denoted λ_i, and after reaching some transformations there is obtained the Lagrange-Euler equation for nonholonomic systems in the form given below as follows:

$$Q_{i\mathcal{F}} + Q^i_f + Q^i_g = Q^i_m + \sum_{j=1}^{3} \lambda_j \cdot a_{ji}, \quad i = 1 \to 5 \quad .(23)$$

In the equation obtained above Q^i_f, Q^i_g and Q^i_m are: the generalized friction forces, the generalized gravity forces and the generalized driving forces, and a_{ji} are the coefficients of displacements $\left(dq_i, i = 1 \to 5\right)$ from the equations of restrictions. Based on expression (23), there can be determined the differential equations of motion that characterizes mobile structure.

5. CONCLUSIONS

In the above paper, are presented general aspects of mathematical modeling for mobile structures of robots, which are performing plane-parallel movements. The mathematical model of the mobile structure proposed for implementation, is the starting point for direct geometry equations, equations for determining direct kinematic model. Based on the geometrical model, it has been made a general description of the kinematic model, description which will be the basis for determining the dynamic equations or differential equations of motion of the mobile robot. From the study of the geometric model of the structure, are resulting kinematic constraints which show that the mobile robot is subjected to nonholonomic links. In the sane paper, there is presented a few considerations about dynamic study of mobile platforms, based on Lagrange-Euler equation for nonholonomic systems.

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Funcții de comandă cinematică pentru un robot mobil

Rezumat: Domeniul roboților mobili a cunoscut o dezvoltare majoră în ultimii ani şi are în prezent aplicații în majoritatea domeniilor de activitate umană. Obiectivul principal al roboților, în scopul de a realiza o sarcină, este de a descrie traiectorii de mişcare, bazate pe funcții de control, care constau în deplasări de la un punct, la o poziție programată. În cadrul lucrării, vor fi prezentate considerații matematice generale cu privire la o structură mobilă, propusă pentru dezvoltare, pentru care, vor fi stabilite ecuațiile de modelului cinematic direct.

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