TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue I, March, 2013

COMPUTATION MODEL FOR THE DESIGN OF THE ROTOR OF SMALL WIND TURBINES WITH HORIZONTAL AXIS

Nichifor TODOR

Abstract: *Currently in the field of production of electricity from wind power, Romania has a prevailing role in Central and Eastern Europe. The energy production is realised mainly in ferms composed of high power wind turbines. In the same time, there are a lot of houses which are not connected to the national power grid due to the high costs involved. A more acceptable sollution in this case would be provided by the building of small power wind turbines near the consumer. In the following i propose a computational model for the rotor of a small wind turbine.*
Key words: *SWT (Small Wind Turbine) blade, hub.*

1. THE ESTABLISHMENT OF THE INITIAL PROJECT DATA

In order to begin the designing of a wind turbine (no matter of what type), first a processing of the meteorological data provided by the long term measurements of the weather stations in the area must be realized. This information must then be translated to the location where the wind turbine is meant to be built. Based on this data the insurance curves (v ≥)=f(t) and frequency curves v = f(t) are plotted in order to assess the wind resource of the site of demand.

Small power wind turbines (SWT) on the other hand, are used mostly in order to supply isolated consumers or to compensate for part of the use of electricity from the power grid, with the use of wind power. From this aspect stems the great importance of their emplacement in areas as close as possible to the consumers in need. A constructive solution is needed in order to facilitate the construction of such small power wind turbines. The aim of this study is to develop a simple model of calculating the blades of a small wind turbine, needed for the design or verification before purchase of the characteristics of such a turbine. The model would help ensure that an investment is worth making for a specific site.

For an identified site, it is indicated to first determine the wind parameters such as average annual speed, maximum speed and periods with very low wind currents, air density or the predominant wind direction. These indicators should be measured throughout a period of at least a year and at different heights from the ground. This can be done by installing an anemometric pole with measuring devices arranged at different levels. The resulting data would help determine the potential of the site more accurately and while also considering financial aspects, would help establish the type of wind turbine and the appropriate rotor size.

2. THE DETERMINATION OF THE CONSTRUCTION PARAMETERS OF THE WIND TURBINE

Given the initial data on the wind speed v_{hub} at the height the turbine will be installed, the power at the root of the turbine must be correlated with the consumption need calculated in order to be covered by the energy produced through the wind turbine's functioning is given by [2]:

$$P_{hub}=C_{Phub}\cdot\rho\cdot\frac{v_{hub}^{3}}{2}\cdot\frac{\pi D^2}{4}\cdot\left(1-\frac{d^2}{D^2}\right) \quad (1)$$

where:

$C_{Phub}\approx 0,43$ in the case of simplified turbines

$\rho = 1,225$ kg/m^3 - air density

v_{hub} [m/s] – wind speed of the hub level

D [m] - the size of the rotor at peripheral level

d [m] - the diameter of the turbine's hub rotor

Usually in practice we have:

$d/D = 0,2$ (2) and thus

$$\left(1-\frac{d^2}{D^2}\right)=0.96. \quad (3)$$

For turbines with a horizontal axis, based on the frequency and insurance curves, a design speed v_N is established. This is then used to compute the P_{hubN} through the above calculation. The turbine calculus is usually made using the normal functioning program, where the functional performances are optimal.

The speed at which the wind turbine starts spinning is $v_{in} = \frac{v_N}{2}$, and the maximum utilization speed is usually $v_{max,shutdown} = 3\cdot v_N$, depending on the energy conversion type and the chosen oversaturation system.

In order to determine the normal peripheral speed u_N we use: $u_N = \lambda\cdot v_N$, where λ – turbine tip speed ratio.

For the rotors with a low number of blades (two or three blades) we have $\lambda\geq 4$, which defines high speed turbines.

The nominal functioning point for the turbine is characterized by a value of the speed λ_0 for which we have a maximum power coefficient $C_{Phub\ max}$ and thus a value $P_{hub\ max}$.

The installation point for maximum functioning power is characterized by the following sizes: $\lambda_i \neq \lambda_0$,

$C_{Phub\ i} < C_{Phub\ max}$, $v_i > v_N$

Thus the wind turbines will work at partial capacity for $v < v_i$ when we also have $P_{hub} < P_{hub\ i}$ or at the installation power for $v > v_i$ when $P_{hub\ i}= P_{hub\ max}$. For values of $v_i > v_{max\ shutdown}$ the turbine enters the protection phase.

The turbine tip speed ratio λ is in strict relation with its solidity:

$$s=\frac{S_p}{S_b}, \quad (4)$$

where:

S_p - surface (area) of the blade

S_b – swept area.

The more the rapidity increased, the more the solidity s decreases and we have a slimmer blade. This dependency at the horizontal ax turbines can be seen in Table 1 [1].

The data in the table are obtained as a result of the statistical studies made by Hutter on a large number of blades. The option for the use of the value λ_0 in design and construction must be made carefully, as it can be observed that solidity decreases once with the increase in tip speed ratio. This leads to high aerodynamic charges on the mechanical resistance structure and thus safer materials and technologies must be used.

s [%]	20	15	10	7	6
λ_0	4	5	6	7	8

Tab. 1. Speed ratio and λ_0 value

Having a determined value of the ratio $S_p/S_b = f(\lambda)$, we can determine the medium length l_m of the chord of a profile of a blade with R radius, for a rotor with z blades. Thus:

$$S_p = (R-0,2\cdot R)\cdot z\cdot l_m \quad (5)$$

aut of which:

$$l_m=\frac{S_p}{(R-0,2R)\cdot z}=\frac{S_p\cdot\frac{S_b}{S_b}}{(R-0,2R)\cdot z}=\frac{S_b\cdot\left(\frac{S_p}{S_b}\right)}{(R-0,2R)\cdot z} \quad (6)$$

In order to determine the normal rotor rotational speed of a wind turbine with the radius R:

$$u_R=\omega\cdot R=\frac{\pi\cdot n}{30}\cdot R, \quad (7)$$

where: rotor rotational speed is:

$$n=\frac{u_R\cdot 30}{\pi\cdot R}=\frac{u_R\cdot 60}{\pi\cdot D} \quad (8)$$

Thus can determined the nominal values for $u_{R\ N}$ (at $u_R = u_N$), n_N and ω_N.

3. THE ANALYTICAL DETERMINATION OF THE PRESSURE DISTRIBUTION ALONG THE RADIUS

Due to the turbine's hub and to the open field at the tip of the blade the loading of the blade is uneven and thus a distribution of the pressure drop along the radius is allowed. After [5] these distributions have the shape of an arc of a parabola or an ellipse arc and can be defined as:

$$y = a + b \cdot x - c \cdot x^k , \qquad (9)$$

where:

$$y = \frac{\Delta p_{local}}{\Delta p_{global}} , \qquad (10)$$

with:
Δp – the turbine pressure drop
$a, b, c \in R$
$x = \frac{r}{R} \in [\, 0{,}2 \div 1]$
$k \subset \{ 2, 4, 6, 8, 10 \}$
r – current radius
R – rotor radius

At the hub, where $x = r/R = 0{,}2$ we note $y = y_{02}$.

On the exerior of the rotor $x = 1$ and we have $y = 0$.

The real constants a, b and c are determined by taking into account:

-The need to realize the prescribed power based on the sum of the partial powers from along the radius. Considering Δp_{global} as the total pressure fall on the rotor, the following global power relation results, depending on the local pressure fall Δp_{loc}.

$$\Delta p_{global} \cdot Q = \int \Delta p_{loc} \cdot v \cdot 2 \cdot \pi \cdot dr, \qquad (11)$$

where Q – flow in m/s.

- The loading of the end blade is null.

We thus have [5]:

$$a = \frac{(0{,}68 - f_1)\cdot y_{02} - 0{,}2 + 0{,}2^k}{0{,}31 \cdot 0{,}2^k + 0{,}48 - 0{,}8 \cdot f_1} \qquad (12)$$

$$b = \frac{1 - 0{,}2^k - (1 - f_1)\cdot y_{02}}{0{,}31 \cdot 0{,}2^k + 0{,}48 - 0{,}8 \cdot f_1} \qquad (13)$$

$$c = \frac{0{,}8 - 0{,}31 \cdot y_{02}}{0{,}31 \cdot 0{,}2^k + 0{,}48 - 0{,}8 \cdot f_1} \qquad (14)$$

where $f_1(k) = \frac{1 - 0{,}2^{k+2}}{0{,}48 \cdot (k+2)}$.

In order to obtain through computation a pressure distribution as close as possible to the theoretical one, the angular and kinematic elements are calculated. We have [4]:

$$\Delta p_{global} = k_{\Delta p} \cdot \frac{\rho \cdot v^2}{2} \ [\text{N/m}^2] , \qquad (15)$$

where: $k_{\Delta p} = \frac{c_{pbrut}}{k_v}$

$c_{pbrut} = 0{,}43$ - for simplified construction

$k_v = [\, 0{,}9 \div 1]$ - for rapid turbines

Thus: $k_{\Delta p} = \frac{0{,}43}{0{,}9} = 0{,}467$

From where: $\Delta p_{global} = 0{,}2802 \cdot v^2$ [N/m²] - for rapid turbines.

On the basis of the Prandtl and Betz's data, the following correlation has been accepted:

$$\frac{\Delta p_{global\infty}}{\Delta p_{global}} = \left(1 + \frac{1{,}678}{0{,}8 \cdot \frac{R}{l_m}} \right), \qquad (16)$$

$$\Delta p_{global\infty} = \Delta p_{global} \cdot \left(1 + \frac{1{,}678}{0{,}8 \cdot \frac{R}{l_m}} \right). \qquad (17)$$

For the sections from the hub of the rotor we have y_{02} □ [0,8 ÷ 1,8] whit a 0,2 step and we can calculate the values:

$$\Delta p_{loc.02} = y_{02} \cdot \Delta p_{global\infty} \ [\text{N/m}^2] \qquad (18)$$

By using the fundamental equation of turbomachines, we have at the hub of the rotor:

$$\Delta v_{u02} = \frac{\Delta p_{loc.02}}{\rho \cdot u_{02}} \ [\text{ m/s }]. \qquad (19)$$

From where: $u_{02} = \frac{\pi \cdot n}{30} \cdot 0{,}2 \cdot R$ (20)

At the hub of the rotor we also have the angle:

$$\beta_{02} = \text{arctg} \left(\frac{v_{ax}}{u_{02} - \frac{\Delta v_{u02}}{2}} \right) \ [\,^\circ\,] \qquad (21)$$

and w_∞ at hub part of blade:

$$w_{\infty 02} = \frac{v_{ax}}{\sin \beta_{\infty 02}} \quad (22)$$

The formula that can be used for verification:

$$w_{\infty} = \sqrt{v_{ax}^2 + \left(u + \frac{\Delta v_u}{2}\right)^2} \quad [\text{m/s}] \quad (23)$$

In order to determine the stress of the blade at the hub:

$$\left(C_y \cdot \frac{l}{t}\right)_{02} = \frac{2 \cdot \Delta p_{loc02}}{w_{\infty 02} \cdot u_{02} \cdot \rho} = \frac{2 \cdot \Delta v_{u02}}{w_{\infty 02}}. \quad (24)$$

C_y - coefficient of the lift; l - profil chord

$t = \frac{2\pi r}{z}$ is the step of the network profiles

z – the number of blades

The number of blades of a SWT established by the designer by taking into account the turbine rotation, its solidity (materials and technology used) and the conditions through which its dynamic stability can be ensured without complications. Lately the most used version has been the one where the rotor has three blades, as the dynamic balancing is much easier to ensure in this case and the influence of the network functioning of the blades is negligible.

After computing the values for $y_{02}=\{0,8 \div 1,8\}$ with a pace of 0.2, the value of y_{02} for which the bearing coefficient C_{y02} is most favorable is chosen. Using the above model, value calculations are made for the other sections $x \in [0,2 \div 1]$ of the blade, resulting y_k cu $k \in \{2, 4, 6, 8, 10\}$.

Thus the polynomial curves $y_k = f(x = r/R)$ can be drawn, out of which the most favorable version is chosen. The number of calculation sections is chosen depending on the accuracy desired in the design of the blade.

Substituting this calculation for each section of radius r, the aerodynamic load that will be obtained:

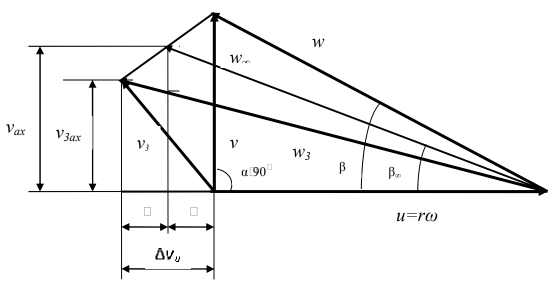

Fig. 1. Speed triangle

$$C_y \cdot l = \frac{4\pi}{z} \cdot \frac{\Delta v_u}{w_{\infty}} \cdot r \quad (25)$$

We thus determine $C_y = f(r)$ for each individual section.

In order to have a low inertia at startup and high working speed (most SWT do not have a gearbox in order not to increase costs) slender blades are preferred, made of light materials and which get narrower and thinner from the hub to the blade tip.

In order to obtain the values of the speeds and angles made by the air current when entering and exiting the blade profile, we have at the entrance triangle [3]:

$u = \omega \cdot r$; $v_{ax} = v_1$ and:

$\text{tg}\ \beta_1 = \frac{v_{ax}}{u}$, from where:

$\beta_1 = \text{arctg}\left(\frac{v_{ax}}{u}\right)$.

Also from the entrance triangle:

$\sin \beta_1 = \frac{v_{ax}}{w_1}$, from where: $w_1 = \frac{v_{ax}}{\sin \beta_1}$.

The variation of the local pressure on the profile: $\Delta p_{tloc} = y \cdot \Delta p_{global}$ (26)

$\frac{\Delta p_{tloc}}{\rho} = u \cdot \Delta v_u$, thus: $\Delta v_u = \frac{\Delta p_{tloc}}{\rho \cdot u}$

For the computation of β_3 we have:

$\text{tg}\ \beta_3 = \frac{v_{ax}}{u + \Delta v_u}$, from where:

$$\beta_3 = \text{arctg}\left(\frac{v_{ax}}{u + \Delta v_u}\right). \quad (27)$$

Knowing β_1 and β_3 we have: $\Delta\beta = \beta_1 - \beta_3$

And from:

$\sin \beta_3 = \frac{v_{ax}}{w_3}$ we have: $w_3 = \frac{v_{ax}}{\sin \beta_3}$

In order to determine the angle α_3 we have:

$\text{tg}\,(180° - \alpha_3) = \frac{v_{ax}}{\Delta v_u}$ from where:

$$\alpha_3 = 180° - \text{arctg}\ \frac{v_{ax}}{\Delta v_u} \quad (28),$$

and $\Delta\alpha = \alpha_3 - \alpha_1 = \alpha_3 - 90°$. (29)

$\sin\,(180° - \alpha_3) = \frac{v_{ax}}{v_3}$, from where:

$$v_3 = \frac{v_{ax}}{\sin(180° - \alpha_3)} \quad (30)$$

β_∞ is obtained from:

$$\text{tg } \beta_\infty = \frac{v_{ax}}{u + \frac{\Delta v_u}{2}}, \quad \text{from where:}$$

$$\beta_\infty = \text{arctg}\left(\frac{v_{ax}}{u + \frac{\Delta v_u}{2}}\right). \tag{31}$$

Also: $\sin \beta_\infty = \frac{v_{ax}}{w_\infty}$, from where:

$$w_\infty = \frac{v_{ax}}{\sin \beta_\infty}. \tag{32}$$

From $\text{tg}(180° - \alpha_\infty) = \frac{v_{ax}}{\frac{\Delta v_u}{2}}$

we have:

$$\alpha_\infty = 180° - \text{arctg}\left(\frac{2 \cdot v_{ax}}{\Delta v_u}\right) \tag{33}$$

v_∞ is also determined from:

$$\sin(180° - \alpha_\infty) = \frac{v_{ax}}{v_\infty} \quad \text{and:}$$

$$v_\infty = \frac{v_{ax}}{\sin(180° - \alpha_\infty)}. \tag{34}$$

Knowing these, the computation of the stress of the blade, the blading's pace, the bearing coefficient C_y and the incidence of the profile can be made, meaning:

For the computation of the stress of the blade:

$$(C_y \cdot l/t) = \frac{2 \cdot \Delta p_{loc}}{w_\infty \cdot u \cdot \rho} = \frac{2 \cdot \Delta v_{u02}}{w_\infty} \tag{35}$$

The step $t = \frac{2 \cdot \pi \cdot r}{z}$ (36)

where z is the blade number.

$$C_y \cdot l = (C_y \cdot l/t) \cdot t = \frac{2 \cdot \Delta v_{u02}}{w_\infty} \cdot t \tag{37}$$

The angle of incidence is:

$$i_{0c}[°] = -85 \cdot f/l - \frac{16 \cdot f/l}{1 + 0{,}05 \cdot (d/l \cdot 100)}, \tag{38}$$

where: f/l = 0,04 – relative arrow

d/l – the relative thickness of the profile

d/l = 0,35 - at the hub and d/l = 0,15 at the periphery. There is a linear correlation between the width at the hub and the one from the periphery.

The incidence angle i_c is obtained from:

$$C_a = k \cdot (i_c - i_{0\,calc}),$$

$$i_c = \frac{C_a + k_{i0calc}}{k}, \quad \text{where } k = 0{,}1$$

$i_{0\,calc}$ – the computed null bearing angle. The curve $i_c = f(r/R)$ can be drawn, which can be adapted in order to have a continuous variation of the profile's incidence angle: i[°].

Having calculated the incidence angle i = f(r/R), the profile's installation angle can be determined: $\beta_i = \beta_\infty - i$

In order to determine the coordinates of the profiles, the Oy axis is considered perpendicular on the chord in the board of attack the Ox axis placed on the profile's chord. Firstly the curve of the case $y_s(x)$ and the width function $y_d(x)$ is determined, using [5] and the ratios of determination of the NACA 4 digit series profiles:

$$\frac{y_d(x)}{l} = \frac{d}{l}\left[\begin{array}{l} 1{,}4845\sqrt{\frac{x}{l}} - 0{,}63 \cdot \frac{x}{l} - 1{,}758 \cdot \left(\frac{x}{l}\right)^2 \\ +1{,}4215\left(\frac{x}{l}\right)^3 - 0{,}5075\left(\frac{x}{l}\right)^4 \end{array}\right] \tag{39}$$

The curve radiuses for the board of attack and output board have:

$$\frac{r_A}{l} = 1{,}1019 \cdot \left(\frac{d}{l}\right)^2 \tag{40}$$

and:

$$\frac{r_F}{l} = 0{,}105\left(\frac{d}{l}\right)^2 \tag{41}$$

In order to fully know the profile's coordinates, the case function is also needed:

$$\frac{y_s}{l} = \frac{\frac{f}{l}}{\left(\frac{x_s}{l}\right)^2} \cdot \left[2 \cdot \frac{x_s}{l} \cdot \frac{x}{l} - \left(\frac{x}{l}\right)^2\right] \tag{42}$$

$$0 \le \frac{x}{l} \le \frac{x_s}{l}$$

$$\frac{y_s}{l} = \frac{\frac{f}{l}}{\left(1 - \frac{x_s}{l}\right)^2} \cdot \left[\left(1 - 2\frac{x_s}{l}\right) + 2 \cdot \frac{x_s}{l} \cdot \frac{x}{l} - \left(\frac{x}{l}\right)^2\right] \tag{43}$$

$$\frac{x_s}{l} \le \frac{x}{l} \le 1$$

The airfoil section is formed in this case through the junction of two parabola arcs in point $\frac{x_s}{l}$.

For the computation of the ordinates of extrados and intrados we thus have:

$$y_{ex} = y_s + y_d \quad (44)$$

$$y_{in} = y_s - y_d \quad (45)$$

4. CONCLUSION

This is how the computation of the blade profile can be made, depending on the section made for the radius of the rotor and using the calculus steps exposed above. Consequently, after a careful economic calculation it can be proceded to the selection of a supplier from the profile catalogs and of the turbine rotor or even the small power wind turbine which is suitable for our site of demand.

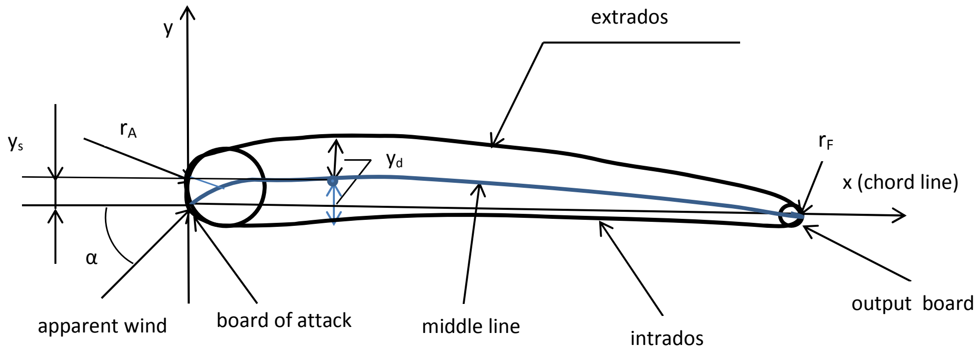

Fig. 2. Blade profile

11. REFERENCES

1. Desire de Gourieres, Énergie eoliene – Theorie, conceptions et calcul practique des instalations. Erolles Editeur, Paris 1980.
2. Gyulai Francisc. Hydraulic machinery and equipment for specific technologies, UP Timisoara, 1985.
3. Gyulai F., Consequences on the optimization of building wind turbines with horizontal axis, IPTV Timisoara, 1988.
4. Gyulai F., □te□co L., Optimizer studies of wind turbine blades load . IPTV Timisoara.
5. Zidaru Gh., Potential and hydrodynamic motions network profiles, EDP, Bucureşti 1981.
6. Bej Adrian, Wind turbines. Editura Politehnică Timişoara 2003

MODEL DE CALCUL PENTRU PROIECTAREA PALETAJULUI ROTORULUI UNEI TURBINE EOLIENE DE MICĂ PUTERE CU AX ORIZONTAL

Scopul prezentului studiu este de a realiza un model simplu de calcul pentru paletajul unei turbine de vânt de mici dimensiuni. Având datele inţiale despre viteza vântului la înălţimea de instalare a turbinei, puterea la arborelui turbinei corelată cu necesarul de consum calculat, se considera o distributie a caderii de presiune sub formă de arce de parabolă. Prin calcule trigonometrice aplicate asupra triunghiului de viteze se ajunge la determinarea functiei de schelet si apoi a coordonatelor profilului paletei.

Nichifor TODOR, Engineer, Associate Professor UT Cluj Napoca, Research Manager, SC Toni Green Energy SRL Alba Iulia, E-mail tonigreenenergy@gmail.com, Office Phone 0744771605, Alba Iulia, Emil Racovita Street, 64, Phone/fax 0358103961 .