TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 54, Issue IV, 2011

ACOUSTIC NORMAL MODES SIMULATION OF A CLASSROOM

Iulian LUPEA, Roxana TIBREA

Abstract: *The acoustic of enclosed spaces continues to be an important issue in building acoustics, high level of comfort in car habitacles of quality vehicles, industrial facilities and others. The management of acoustic modal parameters plays an important roll in reaching the above targets. In this article the finite element method is used to perform the acoustic normal modes analysis of a classroom, finding out the resonant frequencies and the acoustic modes or the pressure distribution for each mode. The results are compared to a set of values determined by a close related formula. The acoustic modes simulation is an important step for the frequency response analysis and the acoustic optimization.*
Keywords: *room acoustic, finite elements, acoustic normal modes.*

1. INTRODUCTION

Many practical problems involve the acoustic of enclosed spaces like rooms, auditoria, concert halls, acoustic couplers for calibrating microphones, car habitacles, tanks and others [1], [5]. In such enclosures one can highlight sound resonances at specific frequencies and associated sound pressure fields or acoustic mode shapes. A sound resonance and the associated mode shape are determining an acoustic normal mode of the enclosure. In this situation we have to solve the three dimensional wave propagation equation [1], [2], [3], [4]:

$$\frac{\partial^2 p}{\partial t^2} = c_0^2 \nabla^2 p \qquad (1)$$

where *p(x,y,z,t)* is the deviation from the ambient pressure, ∇^2 is the Laplace operator, $c_0 = \sqrt{\frac{K}{\rho}}$ [m/s] is the sound speed or the longitudinal wave speed in air, K [Pa] is the bulk modulus or the modulus of bulk elasticity for gas mediums, measuring the air's resistance to uniform compression and ρ is the air density.

For solving equation (1) we need to use the proper boundary conditions imposed by the walls of the classroom under study. The wall surface acoustical properties are of great interest in room acoustic. These properties are completely characterized by the reflection factor for all incidence angles and for the frequency band of interest. The behavior of the walls is well characterized by the wall impedance, too. Wall impedance is based on the particle normal to the wall velocity caused by the sound wave pressure. The reflection factor and the wall impedance are in general complex values.

The Helmholtz equation with the boundary conditions of the rigid walls becomes an eigenvalue problem, resulting eigenvalues or natural frequencies and eigenvectors.

For simple regular shapes, the natural frequencies and the associated pressure fields can be determined analytically. For more complicated enclosures only numerical methods implemented in finite element or boundary element solvers can help in solving the eigenvalue problem.

2. NORMAL MODES IN A RECTANGULAR ROOM

For a rectangular enclosure (l_x, l_y, l_z) the wave equation becomes:

$$\frac{\partial^2 p}{\partial t^2} = c_0^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) \tag{2}$$

and the solution of the differential equation, assuming separation of variables, takes the form:

$$p(x,y,z,t) = p_x(x) p_y(y) p_z(z) \cdot T(t) \tag{3}$$

The boundary conditions of the walls are influenced by the acoustical impedance $z_w = p/u$, where p is the complex acoustic pressure at the wall surface and u is the complex particle velocity at the wall level. In general, the acoustical impedance is complex, having a real or resistive component collecting the energy loss mechanisms and an imaginary or reactive component which expresses the potential energy stored in the air:

$$z_w = z_R + jz_I \tag{4}$$

For rigid walls the impedance is infinite: $z_w = \infty$, resulting zero the normal to the wall (n) sound pressure gradient:

$$\frac{\partial p}{\partial n} = 0 \tag{5}$$

or:

$$\frac{\partial p}{\partial x} = 0, \quad for \quad x = 0,\ x = l_x; \tag{6a}$$

$$\frac{\partial p}{\partial y} = 0, \quad for \quad y = 0,\ y = l_y. \tag{6b}$$

$$\frac{\partial p}{\partial z} = 0, \quad for \quad z = 0,\ z = l_z. \tag{6c}$$

The air particles velocity is zero at the rigid walls level. By considering this situation on the boundary surfaces we are in an undamped case at which by increasing the impedance real part we are adding damping.

One assumes a complex exponential for the time dependent factor of the solution (3):

$$p(x,y,z,t) = p_x(x) p_y(y) p_z(z) \cdot e^{j\omega t} \tag{7}$$

where $j = \sqrt{-1}$ and the circular frequencies ω have to be determined.

By inserting the proposed solution (7) into the equation (2), one get:

$$\frac{-\omega^2}{c_0^2} p_x(x) p_y(y) p_z(z) = \frac{d^2 p_x(x) \cdot p_y(y) p_z(z)}{dx^2} + \frac{d^2 p_y(y) p_x(x) p_z(z)}{dy^2} + \frac{d^2 p_z(z) p_x(x) p_y(y)}{dz^2}$$

Dividing by *p(x,y,z)*, results:

$$\frac{-\omega^2}{c_0^2} = \frac{d^2 p_x(x) \cdot}{dx^2} \frac{1}{p_x(x)} + \frac{d^2 p_y(y)}{dy^2} \frac{1}{p_y(y)} + \frac{d^2 p_z(z)}{dz^2} \frac{1}{p_z(z)} \tag{8}$$

and for: $\omega^2 / c_0^2 = k^2$, (9)

one get:

$$\frac{d^2 p_x(x) \cdot}{dx^2} \frac{1}{p_x(x)} + \frac{d^2 p_y(y)}{dy^2} \frac{1}{p_y(y)} + \frac{d^2 p_z(z)}{dz^2} \frac{1}{p_z(z)} + k^2 = 0 \tag{10}$$

The terms of the previous equation are independent each to the other. Each term can be equaled with a constant, in order from the list $-k_x^2, -k_y^2, -k_z^2$, resulting:

$$\frac{d^2 p_x(x)}{dx^2} + k_x^2 p_x(x) = 0 \tag{11a}$$

$$\frac{d^2 p_y(y)}{dy^2} + k_y^2 p_y(y) = 0 \tag{11b}$$

$$\frac{d^2 p_z(z)}{dz^2} + k_z^2 p_z(z) = 0 \tag{11c}$$

and the sum of the three constants equals k^2:

$$k_x^2 + k_y^2 + k_z^2 = k^2 . \tag{12}$$

The solutions of the equations (11 a,b,c) are:

$$p_x(x) = C_{1x} e^{jk_x x} + C_{2x} e^{-jk_x x} \tag{13a}$$

$$p_y(y) = C_{1y} e^{jk_y y} + C_{2y} e^{-jk_y y} \tag{13a}$$

$$p_z(z) = C_{1z} e^{jk_z z} + C_{2z} e^{-jk_z z} \tag{13a}$$

The solution (7) of the equation, becomes:

$$p = (C_{1x} e^{jk_x x} + C_{2x} e^{-jk_x x}) \cdot \cdot (C_{1y} e^{jk_y y} + C_{2y} e^{-jk_y y}) \cdot \cdot (C_{1z} e^{jk_z z} + C_{2z} e^{-jk_z z}) \cdot e^{j\omega t} \tag{14}$$

From the boundary conditions at x=0 indicating a null pressure gradient at the rigid wall, results:

$$\left.\frac{\partial p_x}{\partial x}\right|_{x=0} = 0 \quad => C_{1x} = C_{2x} \qquad (15a)$$

$$\left.\frac{\partial p_y}{\partial y}\right|_{y=0} = 0 \quad => C_{1y} = C_{2y} \qquad (15a)$$

$$\left.\frac{\partial p_z}{\partial z}\right|_{z=0} = 0 \quad => C_{1z} = C_{2z} \qquad (15a)$$

Replacing these constant relations in the solutions (13), results:

$$p_x(x) = C_{1x}(e^{jk_x x} + e^{-jk_x x}) = 2C_{1x}\cos(k_x x) \quad (16a)$$

$$p_y(y) = 2C_{1y}\cos(k_y y) \qquad (16b)$$

$$p_z(z) = 2C_{1z}\cos(k_z z) \qquad (16c)$$

For the boundary condition at the opposite wall, direction OX, results:

$$\left.\frac{\partial p_x}{\partial x}\right|_{x=l_x} = -2C_{1x}k_x \sin(k_x l_x) = 0 \qquad (17)$$

From this, and for all three directions, results:

$$k_x l_x = \pi\, n_x, \quad n_x = 0,1,2,... \qquad (18a)$$

$$k_y l_y = \pi\, n_y, \quad n_y = 0,1,2,... \qquad (18b)$$

$$k_z l_z = \pi\, n_z, \quad n_z = 0,1,2,... \qquad (18c)$$

The sound pressure expression becomes a sum of terms of the following type:

$$p(x,y,z) = C_{n_x,n_y,n_z}\cos(\frac{\pi\, n_x}{l_x}x)\cos(\frac{\pi\, n_y}{l_y}y)\cos(\frac{\pi\, n_z}{l_z}z) \qquad (19)$$

where C_{n_x,n_y,n_z} is the modal amplitude of the acoustic mode, identified by the integer numbers n_x, n_y, n_z and x, y, z variables are confined inside the rectangular room:

$$0 \le x \le l_x\,, \quad 0 \le y \le l_y\,, \quad 0 \le z \le l_z$$

Finally, the pressure expression at the position (*x,y,z*) and moment *t*, can be written as a summation of acoustic modes:

$$p(x,y,z,t) = \sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty}\sum_{n_z=0}^{\infty} C_{n_x,n_y,n_z}\cdot \cos(\frac{\pi\, n_x}{l_x}x)\cos(\frac{\pi\, n_y}{l_y}y)\cos(\frac{\pi\, n_z}{l_z}z)e^{j\omega t} \qquad (20)$$

From relations (9) and (12) where the eigenvalue k^2 ($k = \omega / c_0$) is defined and relation (18) results:

$$\omega_{n_x,n_y,n_z} = c_0[(\frac{\pi\, n_x}{l_x})^2 + (\frac{\pi\, n_y}{l_y})^2 + (\frac{\pi\, n_z}{l_z})^2]^{1/2}$$

The modal frequencies of the cavity in function of the wave numbers are:

$$f_{n_x,n_y,n_z} = \frac{c_0}{2}[(\frac{n_x}{l_x})^2 + (\frac{n_y}{l_y})^2 + (\frac{n_z}{l_z})^2]^{1/2} \qquad (21)$$

The acoustic modal analysis of the room generates two parameter types: the mode frequencies and the mode shapes.

Modes may be interpreted as sums of interfering traveling waves.

Analytical solutions like those derived for rectangular volumes can be obtained for cylindrical and spherical volumes, as well.

In case of an irregular enclosure the nodal planes become curved nodal surfaces.

For getting the acoustical normal modes of the enclosure, all six walls have been considered rigid. In this situation the particle velocity is zero at the walls surfaces and the pressure variation normal to the wall is null, as well. At the walls level the waves are reflected and standing pressure waves are generated. These standing waves are considered the modes shapes of the air in the cavity. The dissipation of the modes energy, in case of the damping presence, makes the modes amplitudes to decrease and the same for the sound pressure in the cavity. At the resonant frequency determination one can starts with perfectly rigid walls and the air without damping. Later the modal damping is added like in the case of structural modal analysis, frequency response or transient response [7].

3. NORMAL MODE SIMULATION BY USING FEA

3.1. The model set-up

The geometry of the enclosure has been designed by using CAD. Based on the surfaces the mesh has been generated [8]. In order to model the system under observation the CAD model of the air envelope has been created first. The classroom interior side of the walls and the windows are meshed by using triangular shell elements. These are structural nodes, each one with six degrees of freedom, three translational and three rotational. The interior volume of the classroom occupied by the air is meshed by using tetrahedral volume elements, a fixed mesh proper for the fluids [8]. Each node of air has only one degree of freedom, expressing the pressure at that location. The boundary conditions for the air are imposed by the structural nodes of the surrounding structure. The maximum number of the acoustic modes to be modeled is dictated by the mesh size. At least six elements per wavelength are recommended. The mesh density of the acoustic model should be able to predict modes up to the upper bound of the frequency of interest.

The material model MAT10 has been used to model the air. The following parameters are requested in the general case: the bulk modulus, the mass density, the speed of sound, the fluid element damping coefficient and the normalized admittance coefficient for porous material.

3.2. Modal frequencies of the cavity

The normal modal analysis, by using Optistruct [8], has been run to identify the structure natural frequencies and the associated acoustic mode shapes. A first set of natural frequencies resulted, are listed in Table 1.

Table 1

Comparison rectangular vs. room (FEA)

Mode#	n_x	n_y	n_z	Rectangular	FEA	Diff.
1	1	0	0	11,41	11,3	1,05%
2	0	1	0	22,67	22,3	1,55%
3	2	0	0	22,82	22,4	1,83%
4	1	1	0	25,38	24,9	1,95%
5	2	1	0	32,16	31,5	2,14%
6	3	0	0	34,23	34,8	-1,59%
7	3	1	0	41,05	41,5	-0,96%
8	0	2	0	45,33	44,1	2,87%
9	4	0	0	45,64	44,5	2,49%
10	0	0	1	45,95	45,8	0,36%
11	1	2	0	46,75	46,9	-0,39%
12	1	0	1	47,34	48,8	-3,01%
13	2	2	0	50,75	49,3	2,88%
14	4	1	0	50,96	50	1,91%
15	0	1	1	51,23	51,6	-0,79%
16	2	0	1	51,30	52,5	-2,29%
17	1	1	1	52,49	52,9	-0,82%
18	2	1	1	56,09	55,2	1,60%
19	3	2	0	56,80	56,3	0,93%
20	5	0	0	57,05	56,8	0,45%
21	3	0	1	57,29	59,5	-3,64%
22	5	1	0	61,39	59,6	3,05%
23	3	1	1	61,62	62,7	-1,65%
24	4	2	0	64,33	63	2,19%
25	0	2	1	64,55	63,6	1,49%
26	4	0	1	64,76	64,4	0,50%
27	1	2	1	65,55	66	-0,75%
28	0	3	0	68,00	66,9	1,58%
29	6	0	0	68,46	68,2	0,42%
30	2	2	1	68,46	68,5	-0,03%
31	4	1	1	68,61	69,1	-0,68%
32	1	3	0	68,95	69,4	-0,58%
33	2	3	0	71,73	70,3	1,99%
34	6	1	0	72,11	71	1,56%
35	5	2	0	72,87	71,8	1,50%
36	3	2	1	73,06	72,9	0,26%
37	5	0	1	73,25	73,6	-0,49%
38	3	3	0	76,13	73,9	3,02%
39	5	1	1	76,68	76,7	0,02%

The eigenvalues distribution is depicted in Fig. 1. The values are evenly distributed indicating a relatively good room design.

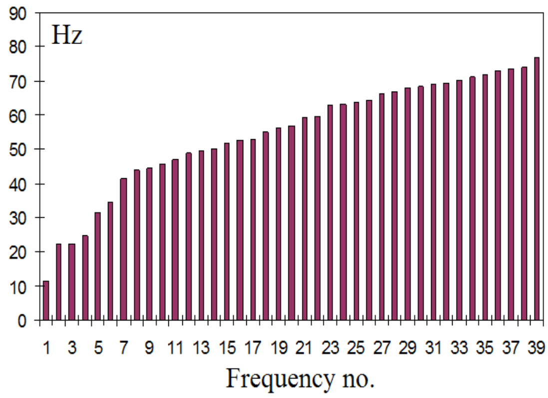

Fig. 1. Modal frequencies distribution

One can observe the axial modes of the types (nx, 0, 0), (0, ny, 0) or (0, 0, nz), the tangential modes of the types (nx, ny, 0), (nx, 0, nz) or (0, ny, nz) and the oblique or tree-dimensional modes (nx, ny, nz) in which all of the indices nx, ny and nz are non-zero.

Often it is of interest to find out the density of the eigenfrequencies in a specific frequency band like an octave or an octave fraction.

3.3. The modal pressure distribution

The first acoustic mode (an axial mode) [100] is along OX axis, obtained at 11.3 Hz (Fig. 2).

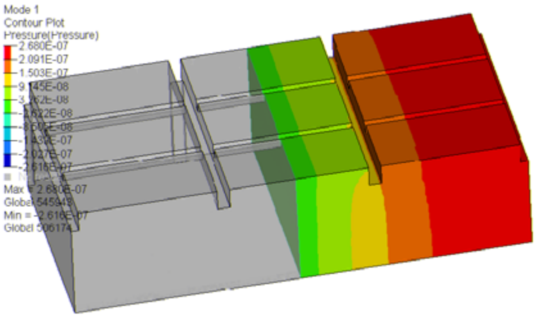

Fig. 2. Mode 1: 11,3 Hz [1 0 0]

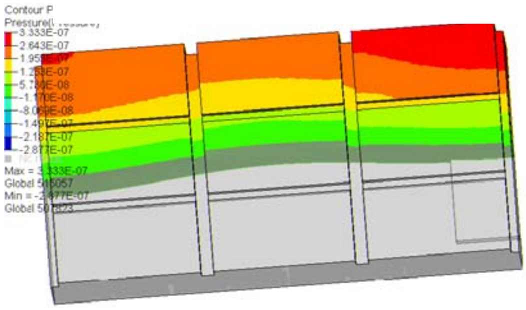

Fig. 3. Mode 2: 22,3Hz [0 1 0]

This is normal considering that $l_x > l_y$ and $l_x > l_z$. Zero pressure surface is about at the middle section of the length l_x of the classroom.

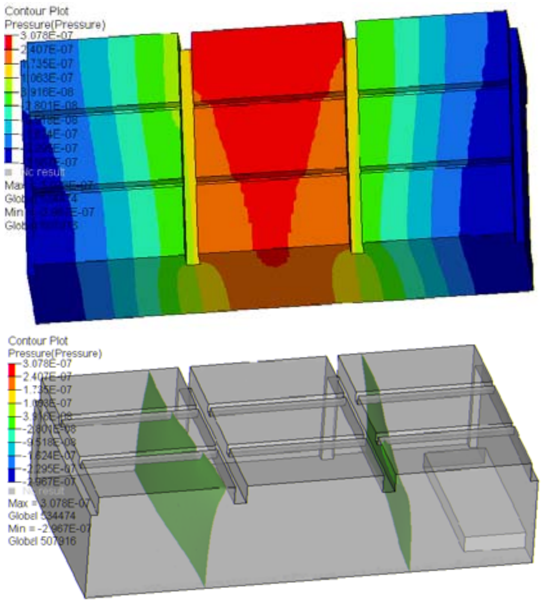

Fig. 4. Mode 3: 22.4Hz [2 0 0]

The frequency of 22.4 Hz associated to the second axial mode along OX axis [200] shown in figure 4, is very close to the frequency of 22.3 Hz associated to the first axial mode (along OY axis [010] (Fig. 3). For the mode 3 [200], the pressure distribution and the two zero pressure surfaces, can be seen in figure 4. These two

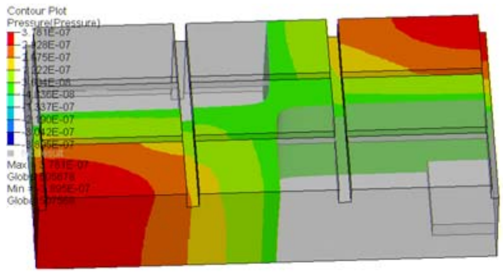

Fig. 5. Mode #4 [1 1 0]

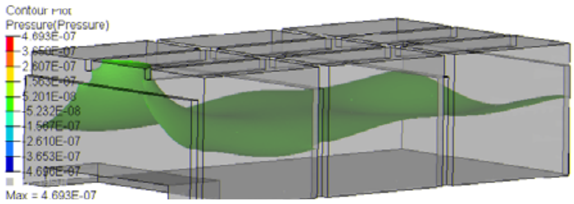

Fig. 6. Mode 10: 45,8Hz [0 0 1]

surfaces are deformed because of the details of the classroom. For a parallelepiped enclosure these two surfaces would be planar. The maximum pressure is located at the two opposite walls, following the relation:

$$\frac{p(x)}{C_{2,0,0}} = \cos(\frac{2\pi}{l_x} x)$$

The first tangential mode [110] (Fig. 5) was obtained at 24.9 Hz. The maximal pressure is located at the two opposite corners.

The first axial mode [001] along the OZ axis or the height of the classroom is depicted in figure 6, where one can see the zero pressure surface. The associated frequency was obtained at 45.8Hz.

Another tangential acoustic mode has two zero pressure surfaces along OX direction and one null pressure along OY direction [210] (Fig. 7). A much higher oblique acoustic mode [331] is depicted in figure 8. For such a mode the particle velocity has components on all three coordinate axes. The reflections cover all six walls. The higher the mode the more complicated the pressure distribution is.

4. CONCLUSIONS

The acoustic modes of a classroom has been observed from analytical and simulation points of view. The values of the fundamental resonant frequencies calculated by using the relation (21) and the values resulted from simulation are in good agreement. The

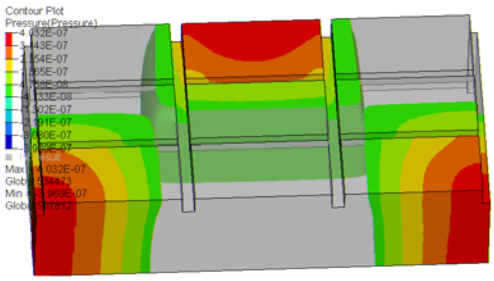

Fig. 7. Mode #5, [2 1 0]: 31.5Hz

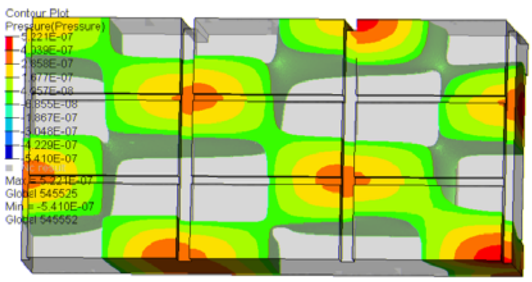

Fig. 8. Mode [3 3 1]

free oscillations problem or the homogeneous problem has been observed in the paper. In this case we had initial conditions to start oscillations, the boundary conditions were for perfectly rigid walls and no damping in the air and at the walls.

Often in the room we have sound sources. In this case forced oscillations are generated. When the source is a pure tone at a specific frequency only that frequency can be observed in the room. An acoustic mode can be excited when the frequency of the source fits a natural frequency of the classroom or is close to it. The excitation is depending, as well, on the position of the source in relation to the nodes or the antinodes of the pressure distribution.

The acoustic modal analysis by using finite elements is a good and confident approach in observing acoustic systems with applications. Further studies will observe the frequency response and the pressure distribution as a response to the human voice.

5. REFERENCES

[1] Bădărău, E., Grumăzescu, M., *Bazele acusticii moderne*, Ed. Academiei RPR, 1961.

[2] Everest F., Pohlmann, K., *Master handbook of acoustics*, McGraw Hill, 2009.

[3] Fahy, F., *Foundation of engineering acoustics*, Elsevier Academic Press, 2005.

[4] Lawrence, E., Austin, R., Coppens, A., *Fundamentals of acoustics*, John Wiley & Sons, Inc. NewYork, 2000.

[5] Heylen, W., Lammens, S., Sas P., *Modal Analysis Theory and Testing*, Leuven: Katholieke Universteit Leuven, Department Werktuigkunde, 2002.

[6] Lupea, I., *Vibration and Noise measurement by using Labview programming*, Casa Cărţii de Ştiinţă Publisher, Cluj-Napoca, 2005.

[7] Lupea, I., Szathmari, R., *Vibroacoustic frequency response on a passenger compartment*, J. of Vibroengineering, Kaunas, Lithuania vol. 12 (4), Dec. 2010.

[8] ** HyperWorks - Optistruct, Altair Eng., Troy, MI, USA.

Analiza modală acustică a unei săli de cursuri folosind elemente finite

Rezumat: În lucrare sunt determinaţi prin simulare parametrii modali ai unei săli de cursuri. Se consideră modelarea geometrică a sălii, discretizarea pereţilor şi a volumului de aer. Folosind solverul Optistruct, se extrag frecvenţele naturale şi modul de distribuţie a presiunii pentru fiecare mod acustic. Valorile provenite din simulare sunt comparate cu cele calculate folosind o relaţie obţinută analitic, pentru o sală asimilată cu un paralelipiped. Sunt făcute aprecieri şi formulate concluzii asupra fenomenului studiat.

Iulian Lupea, Professor Ph.D., Technical University of Cluj-Napoca, The Department of Applied Mechanics and Computer Programming, iulian.lupea@mep.utcluj.ro, 0040 264401691.

Roxana Tibrea, Assistant professor, Technical University of Cluj-Napoca, The Department of Civil Engineering.