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# MODAL ANALYSIS OF AN INFLATED TIRE BY SIMULATION

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***Abstract:*** *The automakers are continuously interested in NVH solutions for high quality vehicles. The dynamics of a tire, which is excited mainly from the road, is under observation. In this paper the modal parameters of an inflated tire are determined by using finite element analysis. The natural frequencies and the modal vectors of the prestressed structure for a couple of pressure values are presented. The pressure influence on the natural frequencies can be assessed. Further studies will observe the experimental modal analysis and the frequency response analysis of the structure.*
***Keywords:*** *inflated tire, modal analysis, modal parameters.*

## 1. INTRODUCTION

Two important issues that appear when engineers are confronting with automotive design are improving the vehicle's dynamics and NVH (Noise, Vibration and Harshness) refining. The wheels are one important target when dealing with these problems because they have the role of sustaining the vehicle and responding the maneuvers of the driver on the road [9]. As a consequence they are, along with the engine, the main sources of vibrations when are dynamically excited by the unevenness of the road, or by braking, steering and accelerating the vehicle [4].

Modal analysis is a good method to formulate a mathematical model for the dynamic behavior of the wheel. It consists on the process of determining its inherent dynamic characteristics like natural frequencies, damping factors and mode shapes in order to build the modal model.

There are theoretical models based on the physics of the tire construction, and empirical models which are based on experimental data.

One simple, yet reliable theoretical model, was conceived by Zagelaar [11] and it consists of a rigid ring that represents the thread-band, which is suspended on translational and rotational springs and dampers representing the tire sidewalls and the pressurized air inside, which are linked to a smaller rigid ring representing the rim. This model can only simulate the low frequency in-plane rigid modes of the tire, which means that the thread-band keeps its circular shape and the movements consist in rotational (Fig. 1), a horizontal and a vertical displacement of the ring (Fig. 2). Such a simple model is mostly used for studying the ride and comfort of vehicles in the low and medium frequency range (0-100Hz)

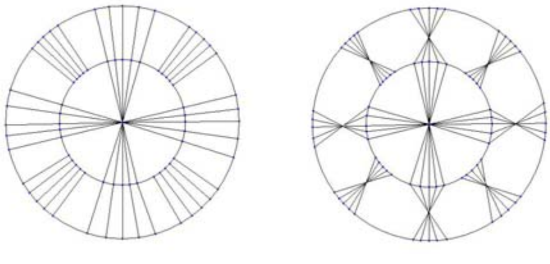

**Fig. 1**. Rotational modes (in plane)

However, when studying the dynamic behavior of the tire at higher frequencies, a model that simulates a more complex deformation is needed. As a consequence, in general a finite element model is being introduced for performing modal analysis.

Brinkmeier et al. describe [1] this kind of approach for determining the radiating noise produced by a rolling tire. To predict the dynamic forces exerted on the tire axis, an

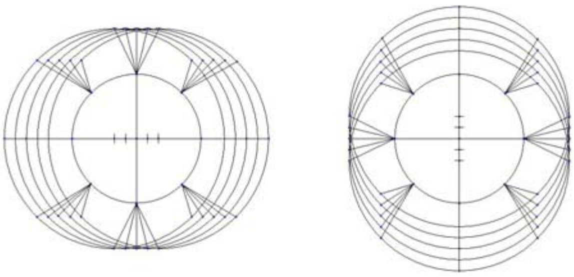

**Fig. 2**. Translational modes (in plane)

analysis of a 3-D tire model with finite elements passing over an obstacle is studied by Cho et al. in their work [2]. The effects of the inflating pressure and rolling speed are taken into account during these investigations.

Finite elements analysis is also being used for examining the contact pressures between the tire and the road for different thread patterns [3], or to validate a FEM model of a tire by simulating several test procedures in order to be used in future vehicle dynamics analysis [7].

## 2. THE FINITE ELEMENT MODEL

Sheet metal wheels are mounted on most of the cars because they are economic, they have a high stress limit and can be easily serviced. They are constructed from two distinguished parts, the rim and the wheel disc (attachment face) and then welded together.

The material from which they are fabricated is a cold-formable sheet metal or band steel, with a high elongation and thicknesses from 1.8 to 4.0 mm for the rim and 3.0 to 6.5 mm for the disc, depending on the wheel loads.

The metal plate from which the rim is fabricated comes in a rectangular form, and then is bent to produce a cylindrical sleeve with two free edges which will be welded afterwards. To give the sleeve the corresponding thickness profile it follows at least one cylindrical flow spinning operation followed by calibration.

The steel disc is obtained by stamping a metal plate. It has to have appropriate holes for fitting the center hub and the bolts which will fasten the wheel to the hub. The disc can be perforated also for purposes like better brake cooling or just reducing weight. Also many strength requirements are placed on the wheel disc because it has to absorb vertical, lateral and longitudinal forces coming from the road and to transfer them to the wheel through the fixing bolts.

Starting from the surfaces of the geometry the mesh, mainly based on shell elements, has been done. The tire size is 185/65/R15 (tire width [mm]/ratio of the tire height to width [%]/ rim diameter [inch]. The height of the sidewall of the tire is 65% of the width (120.25 mm). The cylindrical rim structure is supported by a central disc made by stamping. This part has appropriate holes for the center hub and lug nuts. The outer surface of the wheel disc has a cylindrical portion to support the connection to the rim. Both, the rim and the central disc are modeled by using shells

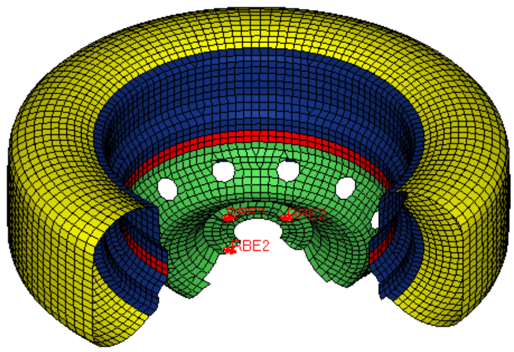

**Fig. 3.** The wheel mesh layout

and the material assigned is steel: Young modulus: 2.1e5MPa, Poisson ratio of 0.3, the density 7.9e-9tons/mm3. The rim and the disc thickness values are 2.3mm and 4.2 mm respectively. The rubber material for this study is considered as isotropic E=200MPa, Poisson ratio ν=0.49, density ρ=2.1e-9 and thickness=6.24mm

## 3. NORMAL MODE ANALYSIS OF THE PRELOADED SYSTEM

### 3.1. The prestressed eigenvalue analysis

The dynamical equations of some mechanical systems [5], [6] can be written in a matrix form (1), as follows:

$$M\ddot{Q} + C\dot{Q} + KQ = F \tag{1}$$

where Q is the vector of the system generalized coordinates (system degrees of freedom), M, C, K are the mass (inertia), viscous damping and stiffness matrices and F is the vector of generalized forces. The matrices M, C and K are considered symmetrical, with n x n elements.

When doing the normal mode analysis of the system, the damping is neglected and the external forces are not acting, resulting (2):

$$M\ddot{Q} + KQ = 0 \qquad (2)$$

Assuming a harmonic and synchronous motion in the structure, when all coordinates perform in time the motion in phase or out of phase, the following solution is proposed:

$$Q(t) = u \cdot \cos(\omega t - \varphi) \qquad (3)$$

where ω is considered the natural frequency of the whole system and $u$ is a constant n–vector of amplitudes.

By replacing the proposed solution in the system of differential equations, results:

$$Ku - \omega^2 Mu = 0 \;\; or \;\; (K - \omega^2 M) \cdot u = 0 \qquad (4)$$

The set of the homogeneous algebraic equations (4) has the unknown vector $u$. Considering $\lambda = \omega^2$ as a parameter, one gets:

$$Ku = \lambda Mu \qquad (5)$$

known as the eigenvalue problem when trying to determine $\lambda$ values for which the system (4) has nontrivial solutions. By solving for λ and nontrivial solution (u≠0), the following characteristic equation (6) is obtained:

$$\det(K - \lambda M) = 0 \qquad (6)$$

where $\lambda_r$ ($r=1,2...,n$) values are the eigenvalues (or characteristic values) of the system. The natural frequencies of the system are derived:

$$f_r = \sqrt{\lambda_r} / 2\pi \quad r = 1,2,...n \qquad (7)$$

For each determined eigenvalue $\lambda_r$, a vector $u_r$ ($r=1,2...,n$) named eigenvector, defining the associated mode shape of vibration is calculated. $u_r$ is satisfying the following equation:

$$Ku_r = \lambda_r Mu_r \quad r = 1,2,...,n \qquad (8)$$

In case of a preloaded structure like the one under observation the stiffness matrix becomes:

$$K = K_0 - K_{PL} \qquad (9)$$

where the preloading is defined or included in the matrix $K_{PL}$. The relation (5) becomes:

$$(K_0 - K_{PL})u = \lambda M u \qquad (10)$$

or:

$$(K_0 - K_{PL} - \lambda M)u = 0$$

The matrix $K_{PL}$ has to be determined in an initial static analysis case of the type:

$$(K_0 - K_{PL})U_0 = F \qquad (11)$$

where $U_0$ and $F$ are the static deformation of the structure and the external preloading force respectively.

### 3.2. Natural frequencies of the system

The normal modal analysis, by using Optistruct [12], has been applied to identify the structure natural frequencies and the associated mode shapes. The tire is constrained by using four bolts at the central area. The recommended tire pressures are 1.9 and 2.1 bars for the front and the rear tires respectively. For the structure critical locations the strain energy distribution has been observed. The natural frequencies for a pressure of 0.2MPa are listed in Table 1.

*Table 1*

**Natural frequencies of the tire**

| Mode # | Frequency (Hz) | Mode # | Frequency (Hz) |
|---|---|---|---|
| 1 | 18.6 | 16 | 186.8 |
| 2 | 51.5 | 17 | 186.8 |
| 3 | 70.1 | 18 | 191.3 |
| 4 | 70.1 | 19 | 191.4 |
| 5 | 128.9 | 20 | 206.7 |
| 6 | 129.4 | 21 | 206.8 |
| 7 | 142.2 | 22 | 207.4 |
| 8 | 142.2 | 23 | 207.4 |
| 9 | 151.8 | 24 | 216.0 |
| 10 | 151.8 | 25 | 229.1 |
| 11 | 167.3 | 26 | 229.1 |
| 12 | 167.3 | 27 | 240.8 |
| 13 | 173.6 | 28 | 240.9 |
| 14 | 180.2 | 29 | 250.3 |
| 15 | 181.6 | 30 | 250.3 |

### 3.3. Mode shapes

The first mode shape, depicted in figure 4, is

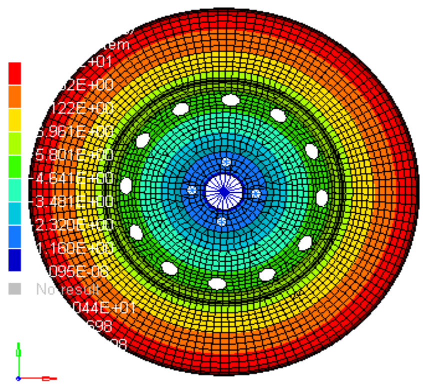

**Fig. 4**. Mode shape #1

characterized by the rotation of the outer zone of the tire (red area) in the wheel plane, around the wheel axis. All figures presented in the sequel are showing the displacement associated to the mode shapes. The steel is much more rigid in comparison to the rubber, consequently the displacement is reduced, the central area being in

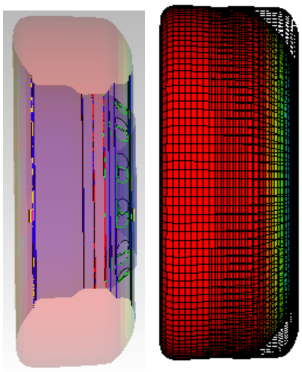

**Fig. 5**. Mode shape #2

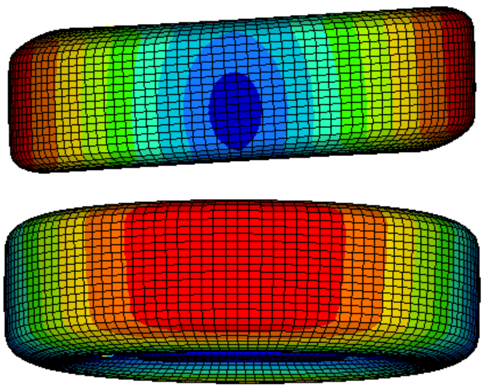

**Fig. 6**. Modes #3 and #4

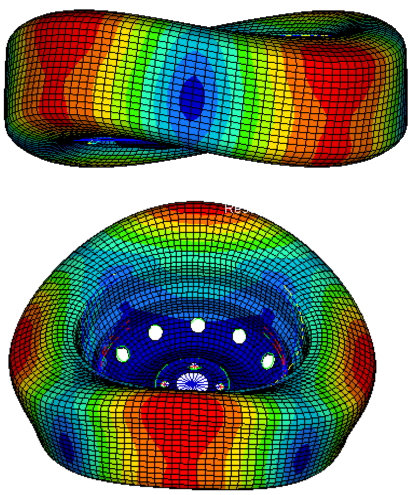

**Fig. 7**. Mode shape #5

dark blue colour.

In the second mode of vibration the tire is moving out of the wheel plane and along the wheel axis (Fig. 5). Both, the deformed (amplitude) and the undeformed shape of the system for this mode can be observed.

In the third and the fourth modes the tire is

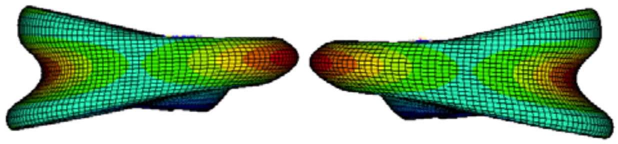

**Fig. 8**. Mode shape #7,#8

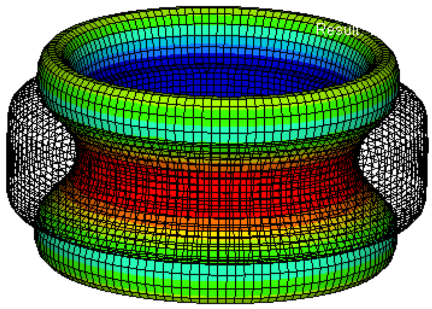

**Fig. 9**. Mode #13

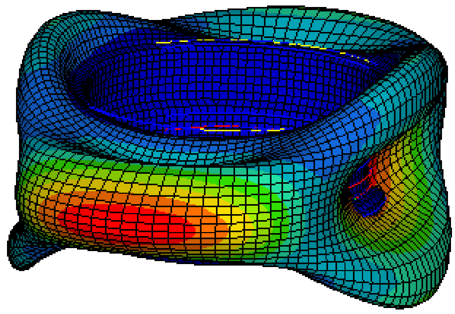

**Fig. 10**. Radial lobes

moving out of the wheel plane, rotating around an axis found in the wheel plane (Fig. 6).

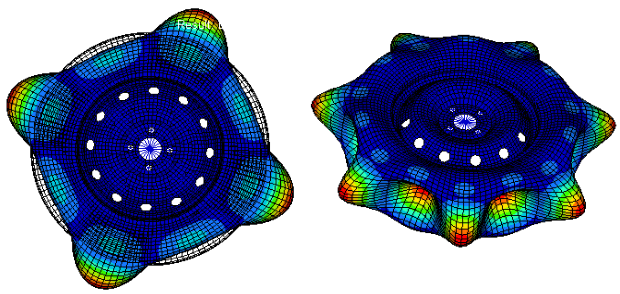

**Fig. 11**. Multiple radial lobes

Figure 7 is presenting the fifth mode from two viewpoints, a side one and a top view. The movement is mainly axial. Two diametric opposite zones are moving out of phase to the other perpendicular and diametric opposite zones.

The modes number seven and eight are similar and are depicted in figure 8. Mode #13 is depicted in figure 9. Both the deformed and wire frame undeformed structures are visualized.

As long as the mode and the associated frequency are higher, the associated mode shape is more complicated deformed presenting an increasing number of lobes like the modes depicted in figures 10 and 11.

### 3.4. The pressure variation influence on the natural frequencies

The natural frequencies are influenced by the degree of the structure pretension as we can see, as well, from the relations (10) and (11).

The finite element analysis has been applied multiple times by considering each time a different tire pressure and hence a different prestressed tire structure. The tire pressure list of values is 0.18, 0.2, 0.22, 0.24 [MPa]. The results are centralized in Table #2.

*Table 2*

**Natural frequencies variation with pressure**

| | Natural Frequencies [Hz] | | | |
|---|---|---|---|---|
| **Mode #** | 0.18 MPa | 0.2 MPa | 0.22 MPA | 0.24 MPA |
| 1 | 17.7 | 18.6 | 19.3 | 20.1 |
| 2 | 49.8 | 51.5 | 53.1 | 54.6 |
| 3 | 68.9 | 70.1 | 71.2 | 72.4 |
| 4 | 68.9 | 70.1 | 71.3 | 72.4 |
| 5 | 127.9 | 128.9 | 129.9 | 130.9 |
| 6 | 128.3 | 129.4 | 130.4 | 131.4 |
| 7 | 140.9 | 142.2 | 143.4 | 144.6 |
| 8 | 141.0 | 142.2 | 143.5 | 144.7 |
| 9 | 149.9 | 151.8 | 153.6 | 155.4 |
| 10 | 149.9 | 151.8 | 153.7 | 155.4 |
| 11 | 164.6 | 167.3 | 169.9 | 172.3 |
| 12 | 164.7 | 167.3 | 169.9 | 172.4 |
| 13 | 171.7 | 173.6 | 175.3 | 177.0 |
| 14 | 178.7 | 180.2 | 181.8 | 183.3 |
| 15 | 180.1 | 181.6 | 183.1 | 184.6 |

As we can see from the results gathered in Table 2, the variation of the frequency for the same frequency mode is within 2 to 5 Hz for a pressure variation of 0.6 MPa. An experimental modal analysis has to be done for the simulation validation.

## 4. CONCLUSION

The modal analysis of an inflated tire is important for the tire dynamics, revealing the resonant frequencies and the associated eigenmodes complexity of the structure. For the tire size of 185/65/R15, the variation of these frequencies as a consequence of the tire pressure variation has been observed. This task has been accomplished by using the finite element simulation.

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**Analiză modală aplicată unei anvelope folosind metoda elementelor finite**

**Rezumat:** În lucrare sunt determinaţi parametrii modali ai unei roţi de automobil prin analiză cu elemente finite. Deoarece diferenţa de rigiditate dintre anvelopă şi jantă este mare, modurile de frecvenţă mai joasă vor presupune în primul rând deplasarea anvelopei. La început este determinată starea de tensiuni a roţii ca urmare a presiunii aerului din anvelopă. In a doua etapă sunt calculate frecvenţele naturale şi modurile de vibraţie ale structurii pretensionate. In continuare este observată variaţia frecvenţelor naturale în raport cu variaţia presiunii din pneuri.

**PRE-STRESSES TYRE MODAL ANALYSIS**

```
SUBCASE      1
  SPC =       3
  LOAD =       2
SUBCASE      2
  SPC =       3
  METHOD(STRUCTURE) =       1
STATSUB(PRELOAD)  =  1
$
BEGIN BULK
GRID    6600040       2724.98639.17379215.7784
GRID    6608089       2903.16496.76384149.8047
$$
$ RBE2 Elements - Multiple dependent nodes
$
RBE2       8122 6608084  123456 6600126 6600127 6600199 6600291 6600292
+       6600293 6600337 6600341 6600342 6604529 6605606 6605608 6605610
+       6605614 6605932 6605934 6605935 6605936 6607930 6607933
. . . .
RBE2       8126 6608087  123456 6600413 6606180 6606183 6606839 6607928
+       6607929
CTRIA3       66       3 6607943 6607945 6607944
. . . .
CTRIA3     5206       6 6606141 6602188 6606139
$
CQUAD4       59       2 6607963 6607960 6607961 6607962
CQUAD4     8100       1 6603540 6600043 6603534 6604916
CQUAD4     8101       1 6603539 6603535 6603534 6600042
$
$HMNAME PROP                1"PNEU_D" 4
PSHELL       1  6600126.24     660012        660012        0.0
$HMNAME PROP                2"ZONE_RECOUVREMENT_D" 4
PSHELL       2  6600116.5      660011        660011        0.0
$HMNAME PROP                3"DISQUE_DE_ROUE_D" 4
PSHELL       3  6600114.2      660011        660011        0.0
$HMNAME PROP                4"JANTE_DE_ROUE_D" 4
PSHELL       4  6600112.3      660011        660011        0.0
$HMNAME PROP                5"EVENTS_D" 4
PSHELL       5  6600126.24     660012        660012        0.0
$HMNAME PROP                6"EVENTS_D_2" 4
PSHELL       6  6600126.24     660012        660012        0.0
MAT1     660011210000.0      0.3     7.9-9
MAT1     660012200.0         0.49    2.1-9
EIGRL         11.0    300.0                          MASS
SPC          3 6608087  3     0.0    SPC          3 6608086  3     0.0    SPC          3 6608084  1245  0.0    SPC          3 6608088  3     0.0    SPC
3 6608089  3     0.0
PLOAD2       20.2        3037 PLOAD2       20.2        623  PLOAD2       20.2        6350
$$
ENDDATA
```

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