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USING THE SPHERE AND THE REVOLVING CONE AS GEOMETRIC LOCI FOR GRAPHICAL CONSTRUCTIONS

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Abstract: *The graphical solving of some problems in spatial geometry is facilitated by the use of some auxiliary surfaces, as geometric loci, which comply with the requirements of the problem. The paper proposes some ways of utilizing sphere and revolving cone, as auxiliary surfaces, for graphically determining the positions occupied in space by some geometrical elements, materialized by parts in machine building, of distances between them or of angles they form or have to form with other elements, within the requirements imposed by some concrete design problems.*

Key words: *geometrical loci, sphere, revolving cone*

1. INTRODUCTION

The use of some geometric loci for graphical solving of some problems in spatial geometry, such as problems of distances and angles, most often leads to the simplification and shortening the time of designing work [1]. Among these, the frequently used in graphical constructions are:

a) the sphere as locus of the points equally distanced from a given point;

b) the circular straight cylinder, as a locus of the points equally distanced from a given line;

c) the revolving cone as locus of the lines intersecting in the same point and which meet a plane at a given angle;

d) the bisector plane of the dihedral angle formed by the two given planes.

Knowing that, the locus of the points equidistant from a plane, a sphere, a cylinder or a revolving cone is, as the case, a system of two parallel planes to the first, two concentric spheres with that considered, two cylinders and two cone having the same axis as the considered surface, it results that the problems may have one or more solutions [2].

In the present paper there are analyzed some modalities of using the sphere and revolving cone, as geometric loci, for graphical determination, on the basis of some initially known data, of the positions in space of some geometric elements, materialized by parts in machine building, as well as of some distances or angles needed in designing process.

The geometric elements are assimilated with vertices, edges and plane faces, respectively, of some machine parts, exemplified within the paper.

2. RESULTS AND DISCUSSIONS

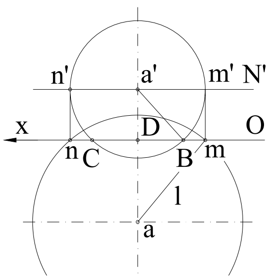
2.1. The auxiliary surface is the spherical surface

A. Determining the vertex coordinates of some prismatic parts, lying on axis *Ox*, at a given distance from the known point

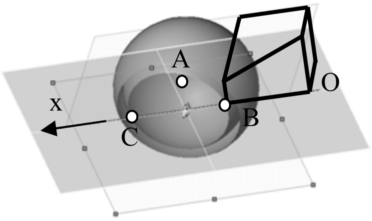
The vertex of part from Figure 1, looked for, belongs to axis *Ox* and to a sphere described by a given point *A*(*a*,*a'*) considered as center, with radius equal to the given distance.

The level plane [*N*] drawn through *a'* determines one of the big circles of the sphere whose projection is obtained describing the circle from point *a* as center, with radius *l*.

The circle, thus drawn, cuts axis *Ox* in *M* and *N*. *MN* is the diameter of the section in the sphere effected with the vertical plane [*V*]. For its determination it is necessary to draw the circle with *MA* as radius. Points *B* and *C* are the solutions of the problem.



a)



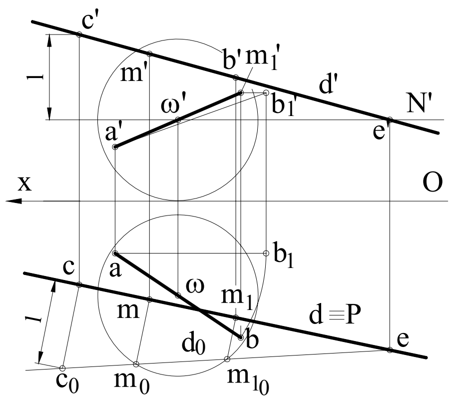
b)

Fig. 1. Points lying on *Ox*, at distance *l* to *A*(*a*,*a'*).

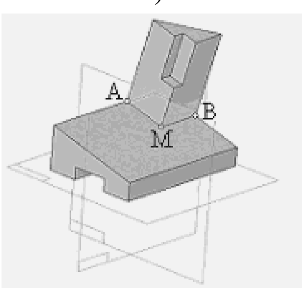
The actual size of segment *AB*(*ab*,*a'b'*) results by transformation of *AB* → *AM*|| [*H*], [3].

B. <u>Determining the vertex coordinates of a prismatic part with base a right angled triangle</u>

There are known the vertexes *A*(*a*,*a'*), *B*(*b*,*b'*) of the base with the right angle in point *M* looked for (Fig.2).



a)



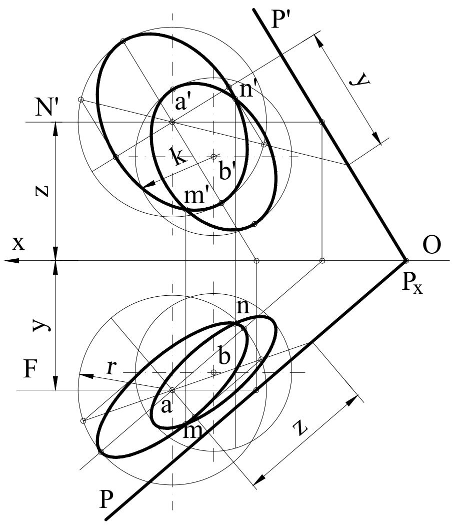
b)

Fig. 2. Determining the vertex of a triangular prism.

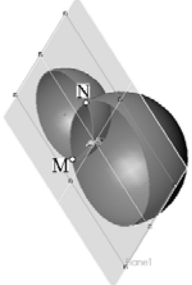
One determines the actual size of segment *AB* and draws the sphere with diameter *2R= AB* and with center in the middle of this segment. The straight line *D*(*d*,*d'*) meets the sphere in points *M*(*m*,*m'*) and M_1(m_1,m_1*'*) which are the solutions of the problem, as each of them, together with points *A* and *B,* determines a big circle in the sphere. In order to intersect line *D* with the sphere, the auxiliary plane [*P*], drawn through line *D*, is used and a rotation of this plane and line is effected to the level plane [*N*] which passes through the center of the sphere. By this rotation, m_0 and m_{10} are obtained which then, by elevation of rotation, become *M*(*m*,*m'*) and M_1(m_1,m_1*'*) respectively.

C. <u>Determining a vertex lying in a given plane, at known distances with respect to two points</u>

The point looked for will be placed, in the same time, on a sphere with center *A(a,a')* and radius *r*, on a sphere with center *B(b,b'*) and radius *k* and on the given plane [*P*] (Fig. 3). In each sphere one determines the section effected by plane [*P*], and points *M*(*m*,*m'*) and *N*(*n*,*n'*) common to the two circles thus obtained, are the solutions of the problem.



a)



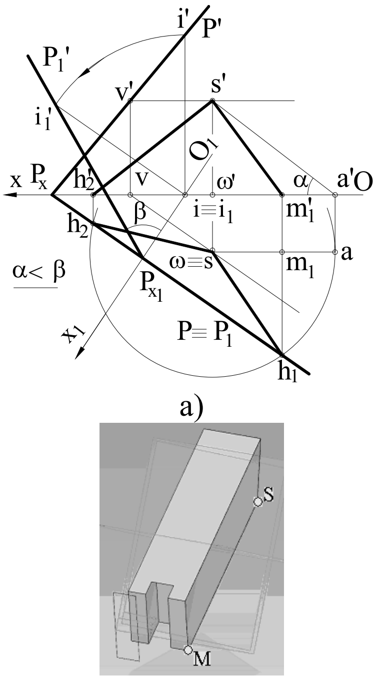
b)

Fig. 3. Point lying in a given plane, at known distances with respect to two points.

2.2. The auxiliary surface is the conical surface

A. Determining a straight line which passes through a point *S* contained in plane [*P*] forming an angle α with plane [*H*]

The auxiliary surface employed is a revolving cone with vertex in point *S*(*s*,*s'*) whose generatrices form angle α with plane [*H*] (Fig. 4). *SA*(*sa*,*s'a'*) is the frontal generatrix of the cone. Plane [*P*] cuts the cone through lines SM_1 or SM_2. The condition that the problem be possible is that $sa \geq sm_1$ or $\omega' a' \geq \omega' m_1'$, respectively, $ctg\ \alpha \geq ctg\ \beta$ or $\alpha \leq \beta$. Thus, it follows that: a) the problem has two solutions for $\alpha < \beta$; b) a single solution for $\alpha = \beta$ and c) is impossible for $\alpha > \beta$.



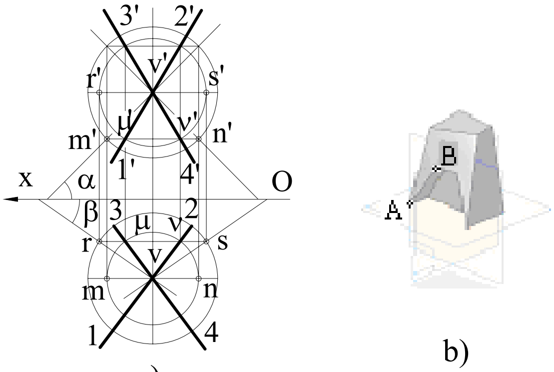
a)

b)

Fig. 4. Line forming a given angle with plane [*H*].

B. Determining a straight line passing through a given point forming known angles with planes [*H*] and [*V*]

The line looked for is the common generatrix of two revolving cones with the common vertex in the given point, whose generatrices form angles α and β with the two projection planes (Fig. 5). In order to intersect the two cones, a sphere with variable radius with center in the given point *V*(*v*,*v'*) is used. The sphere intersects the cone which has the end axis through the frontal circle with diameter *rs* = *r's'* and the vertical cone through the horizontal circle with diameter *m'n'* = *mn*. The two circles have points (μ,μ') and (ν,ν') common, which together with point *V*(*v*,*v'*) determine the required generatrices.



a)

b)

Fig. 5. The line forming known angles with planes [*H*] and [*V*].

The problem admits for $\alpha + \beta < \pi/2$, four solutions, for $\alpha + \beta = \pi/2$, two solutions, and no solution for $\alpha + \beta > \pi/2$.

C. Determining a plane containing the given point, inclined with respect to [*H*] and [*V*]

a) A line *D*(*d*,*d'*) is drawn, in accordance with the method previously presented, which passes through the known point *A*(*a*,*a'*) and which forms, with the projection planes [*H*] and [*V*], the complementary angles ($\pi/2-\alpha$) and ($\pi/2-\beta$) (Fig. 6-a). The plane drawn through point *A*(*a*,*a'*), perpendicular to line *D*(*d*,*d'*), [*P*] ⊥ *D*, is the required plane. The plane [*P*] is defined by the horizontal line *G*(*g*,*g'*) and frontal line *F*(*f*,*f'*) and forms angles α and β with planes [*H*] and [*V*].

b) A sphere of arbitrary radius and the center in point *A*(*a*,*a'*) (Fig. 6-b) is used. The required planes are parallel to the tangent plane to the sphere forming with planes [*H*] and [*V*] the dihedral angles α and β respectively. Two cones with vertical axis are circumscribed to the sphere, respectively the straight end axis and vertexes *V*(*v*,*v'*), *S*(*s*,*s'*) respectively. The generatrices of the two cones form with the projection planes the angles ($\pi/2-\alpha$) and ($\pi/2-\beta$) respectively. The cone *V*(*v*,*v'*) meets the sphere through the horizontal circle *bc*, and cone *S*(*s*,*s'*) through the frontal circle *de*. The contact points between the sphere and the cone are the contact points between the sphere and the tangent planes looked for, i.e. (1,1')

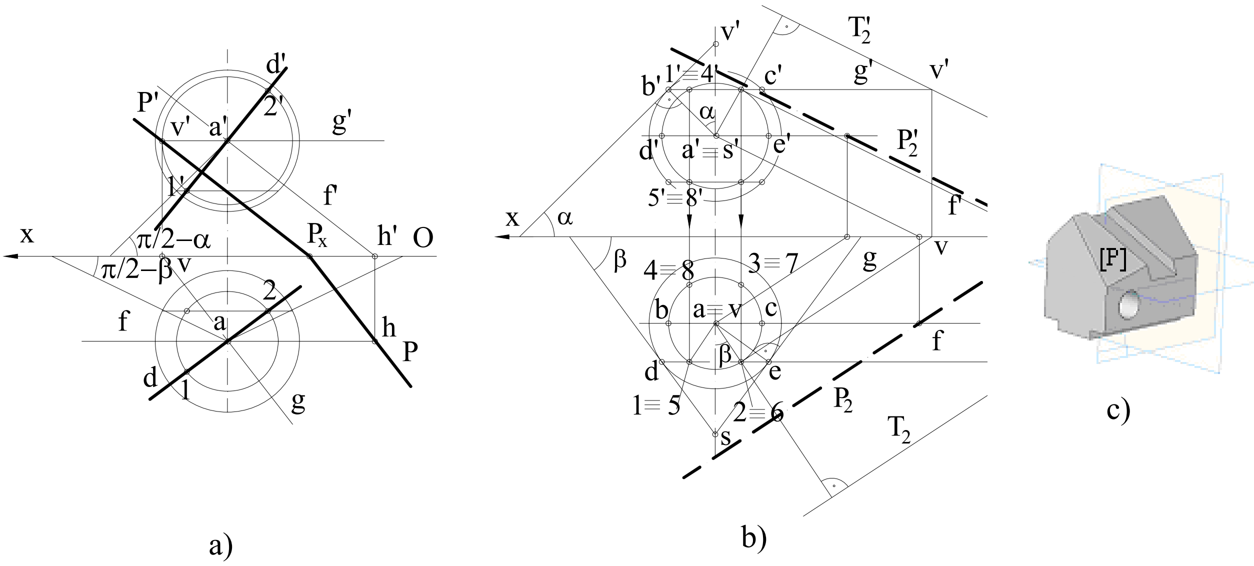


Fig. 6. Inclined plane with respect to projection planes.

As other two cones, symmetrical to the former ones can be also circumscribed to the sphere, with respect to the center of the sphere, it results eight tangent planes, two by two parallel. In order that the problem be possible, it is necessary that $ak \leq ab$. The problem admits: four solutions if $\alpha + \beta > \pi / 2$ (Fig. 6-a,b), two solutions if $\alpha + \beta = \pi / 2$ and no solutions if $\alpha + \beta < \pi / 2$.

3. CONCLUSIONS

The paper analyzes and presents concrete cases of establishing positions, in space, of some geometric elements using the sphere or revolving cone as geometric loci containing the respective elements. The constructions developed are relatively simple, sufficiently accurate and, in some cases, can eliminate certain analytical calculations, reducing on this way, the time for designing.

4. REFERENCES

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Utilizarea sferei şi conului de rotaţie ca suprafeţe auxiliare în rezolvări grafice

Rezumat: Rezolvarea grafică a unor probleme din geometria spaţială este facilitată de utilizarea unor suprafeţe auxiliare, ca locuri geometrice, ce răspund cerinţelor problemei. Lucrarea prezintă câteva modalităţi de utilizare a sferei şi conului de rotaţie, ca suprafeţe auxiliare, pentru determinarea grafică a poziţiilor pe care le ocupă în spaţiu unele elemente geometrice, materializate pe piese din construcţia de maşini, a distanţelor dintre acestea sau a unghiurilor pe care acestea le formează sau trebuie să le formeze cu alte elemente, în condiţiile impuse în cadrul unor probleme concrete de proiectare. Rezultatele, suficient de precise, obţinute exclusiv prin metode grafice simple şi rapide, elimină în cele mai multe cazuri, necesitatea efectuării, în prealabil, a unor calcule analitice.

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