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A METHOD TO DETERMINE THE VALUE OF THE GROUND PRESSURE DEVELOPED BY ONE WAY PLATE COMPACTORS

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***Abstract**: One of the most important parameters that are influencing the vibrating plate operation is represented by the ground pressure. The determination of this parameter from the design phase is an important objective for any designer. This paper came with a mathematical model, based on which the value of the ground pressure can be easily determined.*

1. INTRODUCTION

The plate compactors are equipments used to compact the soil, broken stones (non-cohesive materials), concrete or asphalt coatings, as well as residuum at the ecological ramps.

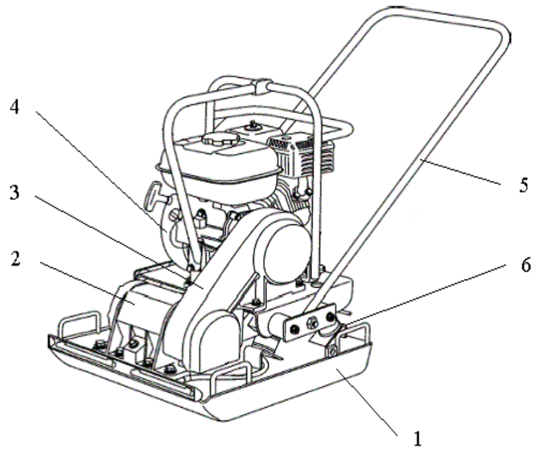

Fig.1. Plate Compactor

From constructive point of view the plate compactor (Figure 1) consists of: base plate (1), vibrations generator (2) (fitted on the base plate) driven by a combustion engine (4) by means of V-belt gearing (3), the equipment being oriented by a handle (5).

The plate compactors use a mechanism (driven by motor oil or a diesel one) which creates a descending force added to the equipment's static weight. One or two eccentrically weights turning around usually form the vibrating generator. The resulted vibrations generate the equipment's advancing movement.

In order to get compaction it is necessary to reach a certain mode of operation mainly characterized by the frequency and amplitude of vibrations due by the vibration generator.

2. MATHEMATICAL MODEL

In the Figure 2 and 3 is represented the mechanical model and the forces that act upon the components of the compacting plate.

The forces which act on the plate compactor are represented in figure 3. Starting from this, a mathematical model was developed the study proposed in this work.

In the mechanical system, subjected to study, one of the components, rather the bed plate, performs a translational motion, along the surface of the ground, on which it acts with a force, normal on the ground, denoted with ***N*** and whose value can be determined.

The supporting plate and the driving engine are involved in a plane-parallel motion, the entire system is characterized by 3 degrees of freedom (two translations along horizontal and vertical directions and a rotation around the horizontal axis).

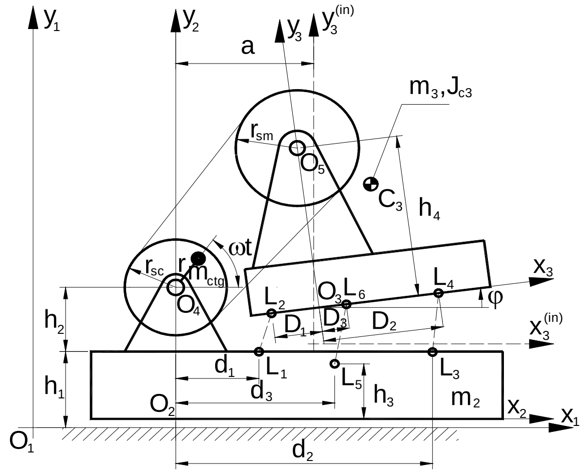

Fig. 2. Mechanical model

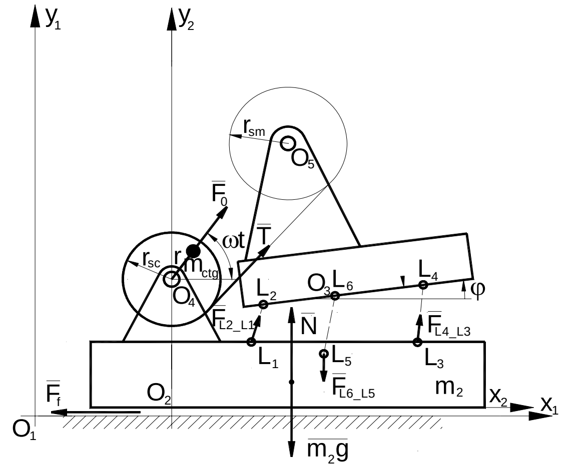

Fig. 3. Forces on the components of the plate

In order to develop the mathematical model that implies the write-in of the equations of motion, have been taken into account the following reference frames (figure 2):

$O_1x_1y_1$ – a fixed reference frame, attached to the ground. The differential equations that characterize the motion of the two analyzed bodies are written relative to this system;

$O_2x_2y_2$ – represents a mobile reference system, attached to the bed plate, which performs translation motion along the horizontal axis. To simplify the equations, it was considered that the O_2y_2 axis is passing through the center of the rotation axis of the inertial vibration generator.

$O_3x_3y_3$ – a mobile reference system, connected to the frame, involved in a plane-parallel motion.

On the basis of the mechanical model a mathematical model it was developed composed of four non-linear differential equations corresponding to the movements performed by each component part of the mechanical system, as follows:

- an equation corresponding to the plate's movement along the horizontal direction;
- three equations corresponding to the rotation of the frame around horizontal and vertical directions;

The generalized coordinates that characterize the mechanical system subjected to study, are:

- x_{O2_1} – the displacement along the horizontal direction of the origin of the system referring to the vibrating plate;
- x_{O3_1}, respectively y_{O3_1} – the displacements along the horizontal respectively the vertical direction, of the O_3 origin of the mobile system referring to the frame;
- φ, turning angle of the frame (in trigonometrically sense) due to the horizontal line;

The quantities used by the mathematical system have the following significance: m_0 – eccentric mass; m_2 – plate mass; m_3 – mass of the frame and motor; k_1 – stiffness; k_2 – axial factor of rigidity; x_{A_B}, respectively y_{A_B} – distances on the axis O_1x_1, respectively O_1y_1 between points A and B. In order to solve the mathematical model previously presented it was written a program in C language. Resolution on the system of differential equations is found with the help of Runge-Kutta fourth order method.

The study conducted on the proposed mechanical model relieved the fact that the plate is subjected to the following forces: (figure 3): the sole weight m_2g; normal reaction N; the friction force μN, which acts counterwise to the displacement of the bed plate; the rotational inertia caused by the inertial vibration generator; the stress exerted on one strand of the transmission belt, whose position (along the upper or lower strand) depends on the direction of rotation of the driving motor; the elastic forces that appear in the rubber elements;

The differential equations are established based on the momentum theorem, projected on the horizontal and vertical axes, as presented bellow:

- horizontal projection:

$$\left(m_2+m_0\right)\frac{d^2 x_{O2\_1}}{d t^2}=m_0\ r\omega^2 \cos\omega\ t-\ \mu\ N\ sign\left(\frac{d\,x_{O2\_1}}{d\,t}\right)+ \\ +(x_{L2\_1}-x_{L1\_1})\ k_1+(x_{L4\_1}-x_{L3\_1})\ k_1+T\ \cos\psi \tag{1}$$

- vertical projection:

$$0=m_0\,r\ \omega^2 \sin\omega\ t-m_2\ g+N+(y_{L2\_1}-y_{L1\_1})\ k_1+(y_{L4\_1}-y_{L3\_1})k_1+ \\ +T\ \sin\psi+\begin{cases} 0 & daca\ \ y_{L6\_1}-y_{L5\_1}>d_{\max}\ , \\ -k_2\cdot\left[d_{\max}-\left|y_{L6\_1}-y_{L5\_1}\right|\right] & daca\ \ y_{L6\_1}-y_{L5\_1}\le d_{\max}\ . \end{cases} \tag{2}$$

From equation (2) can be determined the normal reaction N, as it results from the following expression:

The expression (3) is used to determine the values for the ground pressure (force)

$$N=-\ m_0\,r\ \omega^2 \sin\omega\ t\ +m_2\ g-\ (y_{L2\_1}-y_{L1\_1})\ k_1-(y_{L4\_1}-y_{L3\_1})k_1- \\ -T\ \sin\ \psi\ +\begin{cases} 0 & daca\ \ y_{L6\_1}-y_{L5\_1}>d_{\max}\ , \\ k_2\cdot\left[d_{\max}-\left|y_{L6\_1}-y_{L5\_1}\right|\right] & daca\ \ y_{L6\_1}-y_{L5\_1}\le d_{\max}\ . \end{cases} \tag{3}$$

From equation (1) can be determined the expression for horizontal component of the acceleration that characterize the motion of the bed plate along the horizontal direction.

The equations that describe the motion of the bed plate are determined based on the theorem of the mass point motion (5).

$$\frac{d^2 x_{O2\_1}}{d t^2}=\frac{1}{m_2+m_0}\left[\begin{array}{c} m_0\,r\ \omega^2 \cos\omega\,t-\mu\ N\ sign\left(\frac{d\,x_{O2\_1}}{d\,t}\right)+\left(x_{L2\_1}-x_{L1\_1}\right)k_1\ + \\ +\left(x_{L4\_1}-x_{L3\_1}\right)k_1+T\ \cos\psi \end{array}\right] \tag{4}$$

$$m_3\,\frac{d^2 x_{C3\_1}}{d t^2}=\sum F_x,\quad m_3\,\frac{d^2 y_{C3\_1}}{d t^2}=\sum F_y \tag{5}$$

By projecting these equations on O_1x_1 and O_1y_1 directions, are obtained two differential equations of second order. Another fundamental theorem that can be applied for the same purpose is the theorem of relative angular momentum; the result is also a differential equation of second order.

The forces that are acting upon the supporting plate are graphically represented in the figure 4.

The differential equations that define the motion of the supporting plate, based on the equation of the mass point movement are represented in (5), where the right term of the equation represents the horizontal and vertical projections of all forces exerted on the chassis.

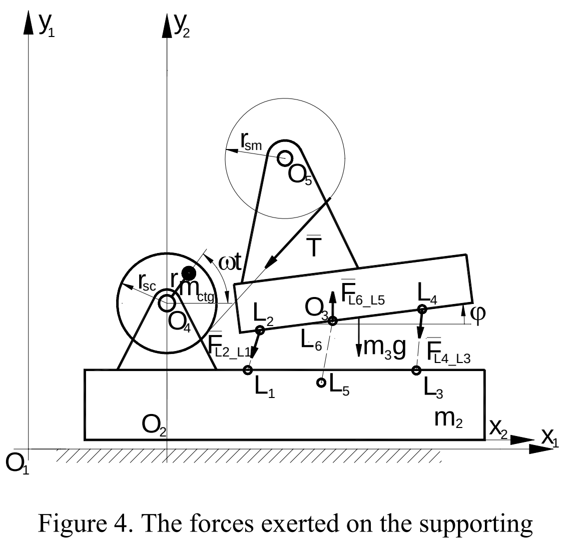

Figure 4. The forces exerted on the supporting

$$m_3\left[\frac{d^2 x_{O3\_1}}{dt^2}+\left(-x_{C3\_3}\sin\varphi-y_{C3\_3}\cos\varphi\right)\frac{d^2\varphi}{dt^2}+\left(-x_{C3\_3}\cos\varphi+y_{C3\_3}\sin\varphi\right)\left(\frac{d\varphi}{dt}\right)^2\right]= =-\left(x_{L2\_1}-x_{L1\_1}\right)k_1-\left(x_{L4\_1}-x_{L3\_1}\right)k_1-T\cos\psi\ , \tag{6}$$

$$m_3\left[\frac{d^2 y_{O3\_1}}{dt^2}+\left(x_{C3\_3}\cos\varphi-y_{C3\_3}\sin\varphi\right)\frac{d^2\varphi}{dt^2}+\left(-x_{C3\_3}\sin\varphi-y_{C3\_3}\cos\varphi\right)\left(\frac{d\varphi}{dt}\right)^2\right]= =-\left(y_{L2\_1}-y_{L1\_1}\right)k_1-\left(y_{L4\_1}-y_{L3\_1}\right)k_1-T\sin\psi-m_3\,g+ +\begin{cases}0 & daca\ \ y_{L6\_1}-y_{L5\_1}>d_{\max}\\ k_2\cdot\left[d_{\max}-\left|y_{L6\_1}-y_{L5\_1}\right|\right] & daca\ \ y_{L6\_1}-y_{L5\_1}\le d_{\max}\end{cases}, \tag{7}$$

$$J_{C3}\frac{d^2\varphi}{dt^2}=\left[-\left(x_{L2\_1}-x_{C3\_1}\right)\left(y_{L2\_1}-y_{L1\_1}\right)+\left(y_{L2\_1}-y_{C3\_1}\right)\left(x_{L2\_1}-x_{L1\_1}\right)\right]k_1+ +\left[-\left(x_{L4\_1}-x_{C3\_1}\right)\left(y_{L4\_1}-y_{L3\_1}\right)+\left(y_{L4\_1}-y_{C3\_1}\right)\left(x_{L4\_1}-x_{L3\_1}\right)\right]k_1+ +\left(x_{L6\_1}-x_{C3\_1}\right)\begin{Bmatrix}\left[d_{\max}-\left|y_{L6\_1}-y_{L5\_1}\right|\right]k_2\\ sau\\ 0\end{Bmatrix}+ +\begin{bmatrix}-\left(x_{03\_1}+r_{sm}\cdot\sin\psi\cdot\cos\varphi-\left(h_4-r_{sm}\cdot\cos\psi\right)\cdot\sin\varphi-x_{C3\_1}\right)\cdot\sin\psi+\\ +\left(y_{03\_1}+r_{sm}\cdot\sin\psi\cdot\sin\varphi+\left(h_4-r_{sm}\cdot\cos\psi\right)\cdot\cos\varphi-y_{C3\_1}\right)\cdot\cos\psi\end{bmatrix}T \tag{8}$$

The generalized coordinates in this case, are represented by: x_{O3_1} şi y_{O3_1} – the displacement along the horizontal and vertical direction of the origin O_3 of the mobile system attached to the supporting plate, respectively φ – the rotation angle of the supporting plate.

After replacing the expressions for both, the second degree derivative and bounding forces, the differential equations defined with (5) became (6) and (7).

It can be noticed that the fourth of the differential equations, above written, is obtained by applying the theorem of the angular momentum, relative to the mass center C_3 whose final expression is given by (8).

The system of four differential equations (4), (6), (7) and (8), presented above, characterizes the motion of all the elements contained by the vibrating plate.

3. NUMERICAL EXAMPLE

The study was conducted for the case of a compacting plate having the following characteristics: the engine speed: 3600 [rot/min], the vibration generator speed: 5400 [rot/min], the mass of the bed plate: m_1 = 70 [kg], the mass of the frame and driving engine: m_2 = 55 [kg], the mass of the eccentric: m_0 = 3,465 [kg], the eccentricity r = 0,0134, the constant of elasticity k_1 = 58110*2 [N/m], the constant of elasticity k_2 = 608250 [N/m], the friction coefficient between the ground and the base plate: μ = 0,35.

All these elements resulted from designing a variant of compacting plate, using the application Solid Edge, in this way, were obtained values for masses, displacements, mass centers, inertia moments, etc.

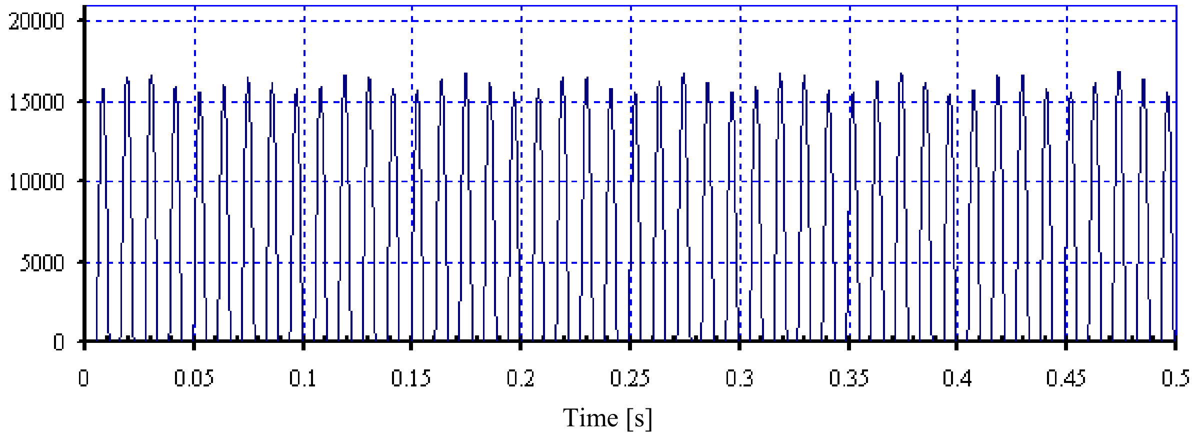

Fig. 5. The diagram of variation in time for the normal ground pressure

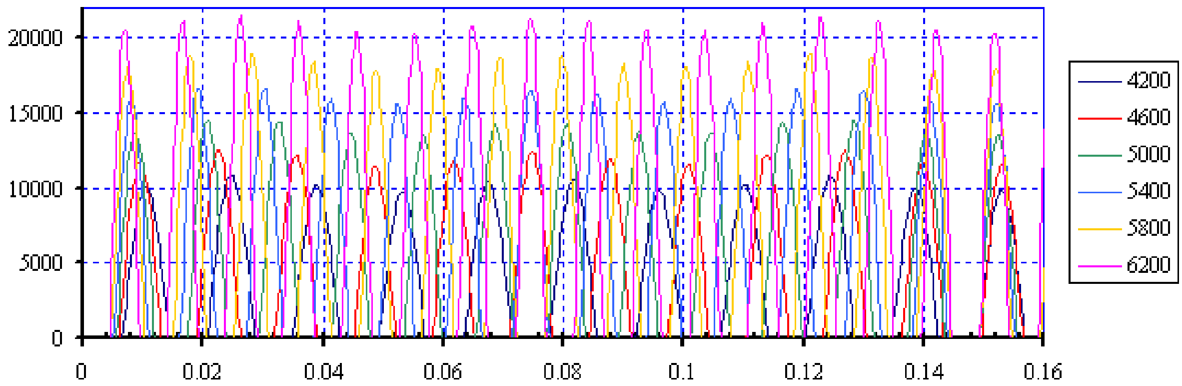

Fig. 6. Diagram of variation for the ground pressure for different values of the vibration generator speed

The solving the differential system, allows to deduce the variations in time for the plate (linear) and for the frame (linear and angular), as well as the time variation for the ground pressure (force).

In figure 5 is illustrated the time variation for the ground pressure exerted by the compacting plate, the maximum value is of 18.213 N.

This force is responsible for the process of ground compacting and this is the reason for knowing the way different process parameters are influencing its variation.

One important parameter that affects the ground pressure is represented by the speed of the vibration generator.

For the considered vibration generator, the following values were taken into study: 4200, 4600, 5000, 5400, 5800, 6200 [rot/min].

Due to the fact that the rotation speed of the driving engines take different values, in order to attain the imposed values is necessary to select the transmission ratio corresponding to the V-belt cord drive.

In the figure 6 is represented the variation in time of the ground pressure, for different values of the vibration generator speed.

It is noticed that increasing the value of the ground pressure causes an increase in the speed of the vibration generator. A possible explanation for this can be the mathematical expression used for determining the value of the ground pressure.

4. CONCLUSIONS

The increase in speed of the vibration generator has a positive effect on the ground pressure in the way of increasing, but it can also have a bad influence on other parameters (for example: the displacement speed) or even on the functioning of the compacting plate.

The value for the ground pressure depends on a lot of factors that also influence directly the well functioning of the equipment.

The mathematical model developed in this paper represents a very useful instrument in determining the values of the parameters that affect the functioning of the equipment as well as of the ground pressure exerted by the compacting plate.

Thereby, are obtained the values for the functioning parameters that correspond to a high value of the ground pressure, high speeds and reduced frame vibrations.

5. REFERENCES

[1] Buzdugan, G., Fetcu, L., Radeş, M. – Vibrations of mechanical systems (in Romanian), Academy Publishing House, Bucharest, 1975;

[2] Bratu, P. – Elastic systems for machines and machinery (in Romanian), Technical Publishing, Bucharest, 1990;

[3] Harris, C.M. - Shock and Vibration Handbook, Fourth Edition, New York, McGraw Hill, 1996;

[4] Mihăilescu, S., Vlasiu, G. – Civil engineering machines and working methods (in Romanian), Educational and Pedagogical Publishing, Bucharest, 1973;

[5] Munteanu, M. – Introduction in dynamics of vibrating machines (in Romanian), Academy Publishing House, Bucharest, 1986;

[6] Shigley, J.E., Mischkle, Ch.R. – Standard Handbook of Machine Design, Second Edition, McGraw-Hill Companies Inc., 1996, ISBN – 0-07-056958-4;

[7] Ursu-Fischer, N. – Vibrations of mechanical systems. Theory and applications (in Romanian). House of Science Book, Cluj-Napoca, 1998;

[8] Ursu-Fischer, N., Ursu, M. – Numerical methods in engineering and programming in C/C++, (in Romanian), vol. I., House of Science Book, Cluj-Napoca, 2000;

[9] Ursu-Fischer, N., Ursu, M. – Programming in C in engineering (in Romanian), House of Science Book, Cluj-Napoca, 2001;

[10] Ursu-Fischer, N., Popescu, D.I., Haiduc, N., Morariu-Gligor, R., Ursu, M. – Contributions in modelling and simulation of the vibrating plate compactor's movements. Ştiinţă şi Inginerie, vol. II, Editura AGIR Bucureşti, 2002, pg. 669-676,

[11] Ursu-Fischer, N., Popescu, D.I., Haiduc, N., Morariu-Gligor, R., Ursu, M. – Study of the operation at the plate vibrating compactor, Ştiinţă şi Inginerie, vol. III, Bucureşti, Editura AGIR, 2003, pg. 213-220.

DETERMINAREA VALORII FORȚEI DE APĂSARE NORMALE DEZVOLTATĂ DE PLĂCILE COMPACTOARE UNIDIRECȚIONALE

Rezumat: Unul din parametri cei mai importanți în funcționarea plăcilor compactoare îl reprezintă forța de apăsare asupra solului. Determinarea acesteia încă din etapa de proiectare constituie un obiectiv pentru fiecare proiectant de plăci compactoare. În lucrare este prezentat un model matematic pe baza căruia se poate determina valoarea forței de apăsare asupra solului funcție de caracteristicile constructive ale plăcii compactoare.

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