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THE GEOMETRIC EQUATIONS OF TWO MACHINE TOOL TYPE 3T3R IN COOPERATION

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Abstract: *In this paper are presented the geometric equations for the most and complex structure type 3T3R – 3T3R. The structure is compound from a positioning and orientating structure of the TOOL and the other is afferent of the PIECE. Both structures are in cooperation on a platform commune. Starting from the general cooperating model type 3T3R – 3T3R the equations can be applied for any type of kinematic structure.*
Key words: *kinematic structures, applied mechanics, cooperation, machine tool, 3T3R structure.*

1. INTRODUCTION

The cooperating and general structure type 3T3R–3T3R is compound from a positioning and orientating structure of the TOOL and the other is positioning and orientating structure of the PIECE (fig. 1).

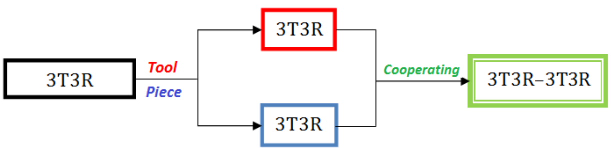

Fig. 1. The cooperation of the positioning and orientating structure of the TOOL and of the PIECE

1.1 The direct geometry equations

To achieve the equations of the cooperating general model of structure type 3T3R–3T3R are facilitated by the fact, for both structures, of the TOOL and the PIECE was determinate independently the direct and inverse geometry equations (see Acta Technica Napocenis no. 54, vol. III, 2011).

Having for both structures the equations, it can be determinate the direct geometry equations for the structures type 3T3R–3T3R. For that, are necessary the equations of the last system of the positioning and orientating structure of the TOOL and of the PIECE.

In the action of cooperating of both structures, it follows that, the workpiece, represented by system $O_P x_P y_P z_P \equiv \{P\}$ symbolized according to figure 2, to be machined by the tool represented by system $O_S x_S y_S z_S \equiv \{S\}$ after a certain technological process. By a series of homogeny transformations, as follows, the cooperating tool-piece noted with T_{SP} can be expressed with relation:

$$T_{SP} = (T_{PO})^{-1} \cdot T_{S0} \tag{1}$$

where

T_{SP} – represent the system of the tool $O_S x_S y_S z_S \equiv \{S\}$ in rapport with the system of the piece, $O_P x_P y_P z_P \equiv \{P\}$;

$(T_{PO})^{-1}$ – represent the transposed matrix of the system of the piece $O_P x_P y_P z_P \equiv \{P\}$ in rapport with the fixed system $O_0 x_0 y_0 z_0 \equiv \{0\}$;

T_{S0} – represent the system of the tool $O_S x_S y_S z_S \equiv \{S\}$ in rapport with the fixed system $O_0 x_0 y_0 z_0 \equiv \{0\}$.

The cooperation of the tool and of the piece can be expressed by the next relation:

$$T_{SP} = \begin{pmatrix} T_{SP_{00}} & T_{SP_{01}} & T_{SP_{02}} & T_{SP_{03}} \\ T_{SP_{10}} & T_{SP_{11}} & T_{SP_{12}} & T_{SP_{13}} \\ T_{SP_{20}} & T_{SP_{21}} & T_{SP_{22}} & T_{SP_{23}} \\ T_{SP_{30}} & T_{SP_{31}} & T_{SP_{32}} & T_{SP_{33}} \end{pmatrix} \tag{2}$$

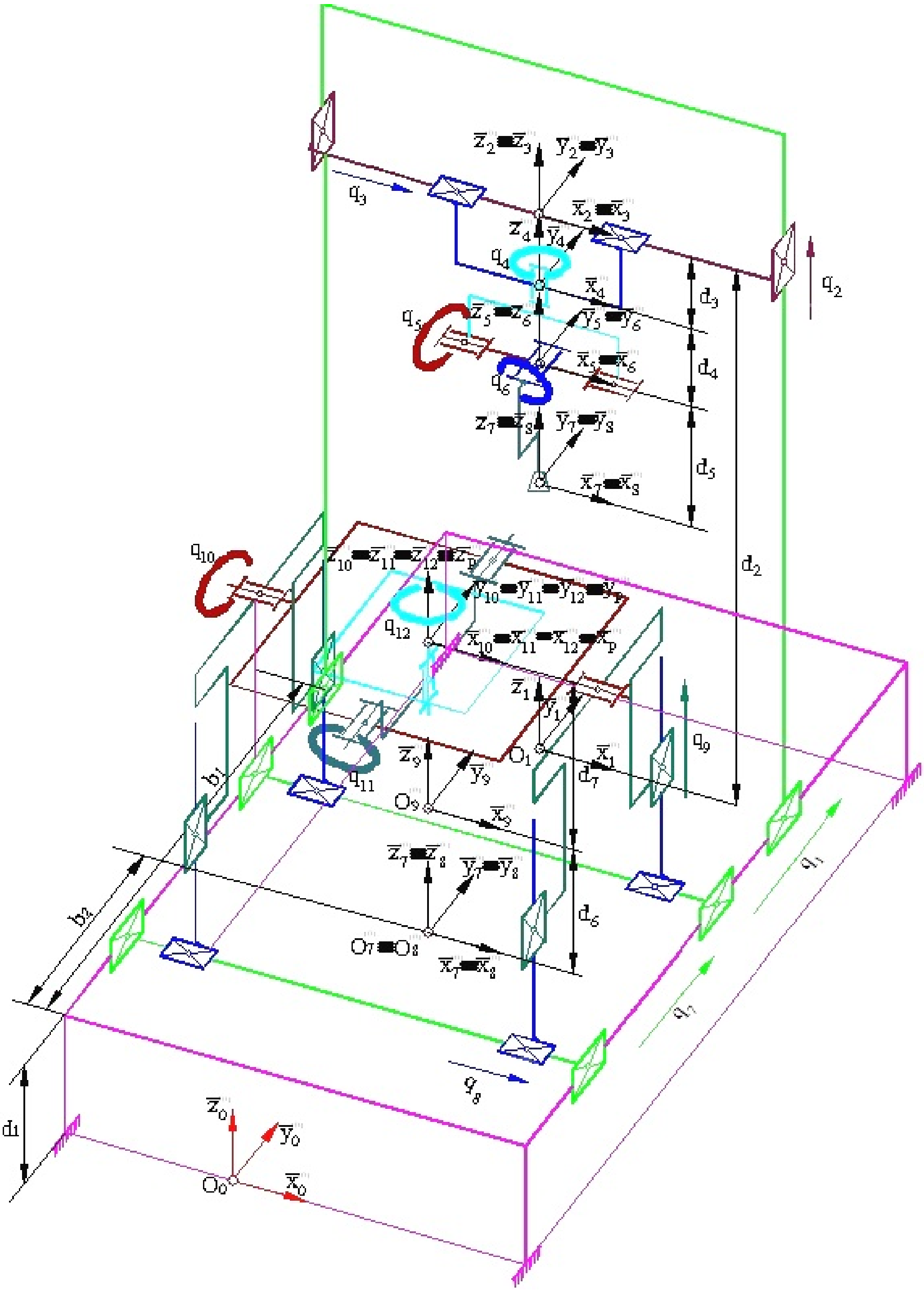

Fig. 2. The kinematical structure type 3T3R – 3T3R in the zero configuration (the cooperation of the structures 3T3R – 0 type portal and 0 – 3T3R type table)

The components of the equation (2) are obtained with the aid of orientating matrix R_{SP} which express the orientating of the tool system $\{S\}$ from system $\{P\}$. In this way it is use the relation: $R_{SP} = R\left({}^{P}\alpha_{zS};\, {}^{P}\beta_{xS};\, {}^{P}\gamma_{yS}\right)$:

$$R_{SP} \equiv R\left({}^{P}\alpha_{zS};\, {}^{P}\beta_{xS};\, {}^{P}\gamma_{yS}\right) = \\ = \begin{bmatrix} -s\alpha_{zS}\cdot s\beta_{xS}\cdot s\gamma_{yS} + c\alpha_{zS}\cdot c\gamma_{zS} & -s\alpha_{zS}\cdot c\beta_{xS} & s\alpha_{zS}\cdot s\beta_{xS}\cdot c\gamma_{yS} + c\alpha_{zS}\cdot s\gamma_{yS} \\ c\alpha_{zS}\cdot s\beta_{xS}\cdot s\gamma_{yS} + s\alpha_{zS}\cdot c\gamma_{yS} & c\alpha_{zS}\cdot c\beta_{xS} & -c\alpha_{zS}\cdot s\beta_{xS}\cdot c\gamma_{yS} + s\alpha_{zS}\cdot c\gamma_{yS} \\ -c\beta_{xS}\cdot s\gamma_{yS} & s\beta_{xS} & c\beta_{xS}\cdot c\gamma_{yS} \end{bmatrix} \quad (3)$$

From relation (5) result the column vector of the positioning and orientating parameters, define with the next calculus expression:

$$ {}^{P}\overline{X}_{SP} = \begin{pmatrix} {}^{P}\overline{p}_{SP} \\ \dots\dots \\ {}^{P}\overline{\psi}_{SP} \end{pmatrix} = \begin{bmatrix} \left({}^{P}p_{zSP} \quad {}^{P}p_{xSP} \quad {}^{P}p_{ySP} \right)^{S} \\ \dots\dots\dots\dots\dots\dots \\ \left({}^{P}\alpha_{zSP} \quad {}^{P}\beta_{xSP} \quad {}^{P}\gamma_{ySP} \right)^{S} \end{bmatrix} . \quad (4) $$

Having all parameters, the positioning vectors ${}^{P}\overline{p}_{SP}$ can be wrote as:

$$ {}^{P}p_{zSP} = \left\{ \begin{array}{c} b_1 \cdot c(q_{10}) \cdot s(q_{12}) - b_2 \cdot c(q_{10}) \cdot s(q_{12}) + d_2 \cdot s(q_{10}) \cdot s(q_{12}) - d_3 \cdot s(q_{10}) \cdot s(q_{12}) - \\ -d_4 \cdot s(q_{10}) \cdot s(q_{12}) - d_6 \cdot s(q_{10}) \cdot s(q_{12}) - d_7 \cdot s(q_{10}) \cdot s(q_{12}) + c(q_{11}) \cdot c(q_{12}) \cdot q_3 - \\ -c(q_{11}) \cdot c(q_{12}) \cdot q_8 + c(q_{10}) \cdot s(q_{12}) \cdot q_1 - c(q_{10}) \cdot s(q_{12}) \cdot q_7 + s(q_{10}) \cdot s(q_{12}) \cdot q_2 - \\ -s(q_{10}) \cdot s(q_{12}) \cdot q_9 - d_2 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) + d_3 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) + \\ +d_4 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) + d_6 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) + d_7 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) - \\ -c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) \cdot q_2 + c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) \cdot q_9 + c(q_{12}) \cdot s(q_{10}) \cdot s(q_{11}) \cdot q_1 - \\ -c(q_{12}) \cdot s(q_{10}) \cdot s(q_{11}) \cdot q_7 + b_1 \cdot c(q_{12}) \cdot s(q_{10}) \cdot s(q_{11}) - b_2 \cdot c(q_{12}) \cdot s(q_{10}) \cdot s(q_{11}) - \\ -d_5 \cdot c(q_4) \cdot c(q_{11}) \cdot c(q_{12}) \cdot s(q_6) - d_5 \cdot c(q_5) \cdot c(q_6) \cdot s(q_{10}) \cdot s(q_{12}) - \\ -d_5 \cdot c(q_{10}) \cdot s(q_4) \cdot s(q_6) \cdot s(q_{12}) - d_5 \cdot c(q_5) \cdot c(q_6) \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_{11}) - \\ -d_5 \cdot c(q_4) \cdot c(q_6) \cdot c(q_{10}) \cdot s(q_5) \cdot s(q_{12}) + d_5 \cdot c(q_6) \cdot c(q_{11}) \cdot c(q_{12}) \cdot s(q_4) \cdot s(q_5) - \\ -d_5 \cdot c(q_{12}) \cdot s(q_4) \cdot s(q_6) \cdot s(q_{10}) \cdot s(q_{11}) + d_5 \cdot c(q_4) \cdot c(q_6) \cdot c(q_{12}) \cdot s(q_5) \cdot s(q_{11}) \end{array} \right\} \quad (5) $$

$$ {}^{P}\overline{p}_{xSP} = \left\{ \begin{array}{c} b_1 \cdot c(q_{10}) \cdot c(q_{12}) - b_2 \cdot c(q_{10}) \cdot c(q_{12}) + d_2 \cdot c(q_{12}) \cdot s(q_{10}) - \\ -d_3 \cdot c(q_{12}) \cdot s(q_{10}) - d_4 \cdot c(q_{12}) \cdot s(q_{10}) - d_6 \cdot c(q_{12}) \cdot s(q_{10}) - \\ -d_7 \cdot c(q_{12}) \cdot s(q_{10}) + c(q_{10}) \cdot c(q_{12}) \cdot q_1 - c(q_{10}) \cdot c(q_{12}) \cdot q_7 + \\ +c(q_{10}) \cdot c(q_{12}) \cdot q_2 - c(q_{11}) \cdot s(q_{12}) \cdot q_3 + c(q_{11}) \cdot s(q_{12}) \cdot q_8 - \\ -c(q_{12}) \cdot s(q_{10}) \cdot q_9 + d_2 \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) - \\ -d_3 \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) + d_4 \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) - d_6 \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) - \\ -d_7 \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) + c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) \cdot q_2 - c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) \cdot q_9 - \\ -s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) \cdot q_1 + s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) \cdot q_7 - b_1 \cdot s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) + \\ +b_2 \cdot s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) - d_5 \cdot c(q_5) \cdot c(q_6) \cdot c(q_{12}) \cdot s(q_{10}) - \\ -d_5 \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_4) \cdot s(q_6) + d_5 \cdot c(q_4) \cdot c(q_{11}) \cdot s(q_6) \cdot s(q_{12}) - \\ -d_5 \cdot c(q_4) \cdot c(q_6) \cdot c(q_{10}) \cdot c(q_{12}) \cdot s(q_5) - d_5 \cdot c(q_5) \cdot c(q_6) \cdot c(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) + \\ +d_5 \cdot c(q_6) \cdot c(q_{11}) \cdot s(q_4) \cdot s(q_5) \cdot s(q_{12}) + d_5 \cdot s(q_4) \cdot s(q_6) \cdot s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) - \\ +d_5 \cdot c(q_4) \cdot c(q_6) \cdot s(q_5) \cdot s(q_{10}) \cdot s(q_{11}) \cdot s(q_{12}) \end{array} \right\} \quad (6) $$

$$ {}^{P}p_{ySP} = \left\{ \begin{array}{c} s(q_{11}) \cdot q_3 - s(q_{11}) \cdot q_8 - b_1 \cdot c(q_{11}) \cdot s(q_{10}) + b_2 \cdot c(q_{11}) \cdot s(q_{10}) + \\ +d_2 \cdot c(q_{10}) \cdot c(q_{11}) - d_3 \cdot c(q_{10}) \cdot c(q_{11}) - d_4 \cdot c(q_{10}) \cdot c(q_{11}) - d_6 \cdot c(q_{10}) \cdot c(q_{11}) - \\ -d_7 \cdot c(q_{10}) \cdot c(q_{11}) + c(q_{10}) \cdot c(q_{11}) \cdot q_2 - c(q_{10}) \cdot c(q_{11}) \cdot q_9 - c(q_{11}) \cdot s(q_{10}) \cdot q_1 + \\ +c(q_{11}) \cdot s(q_{10}) \cdot q_7 - d_5 \cdot c(q_4) \cdot s(q_6) \cdot s(q_{11}) - d_5 \cdot c(q_5) \cdot c(q_6) \cdot c(q_{10}) \cdot c(q_{11}) - \\ -d_5 \cdot c(q_6) \cdot s(q_4) \cdot s(q_5) \cdot s(q_{11}) + d_5 \cdot c(q_{11}) \cdot s(q_4) \cdot s(q_6) \cdot s(q_{10}) - \\ -d_5 \cdot c(q_4) \cdot c(q_6) \cdot c(q_{11}) \cdot s(q_5) \cdot s(q_{10}) \end{array} \right\} \quad (7) $$

1.2 The inverse geometry equations

In this case the input data are the position and the orientation of the piece in rapport with the system of the tool. Are wrote a series of equations between the systems $\{0\} \rightarrow \{S\} \rightarrow \{P\}$, system noted with T_{SP}, and meaning:

$$T_{SP}=\begin{bmatrix} R_{SP} & {}^{P}\overline{p}_{SP}^{(0)} \\ 0\,0\,0 & 1 \end{bmatrix}=\begin{bmatrix} {}^{S}\alpha_{Px} & {}^{S}\beta_{Px} & {}^{S}\gamma_{Px} & {}^{P}p_{xSP} \\ {}^{S}\alpha_{Py} & {}^{S}\beta_{Py} & {}^{S}\gamma_{Py} & {}^{P}p_{ySP} \\ {}^{S}\alpha_{Pz} & {}^{S}\beta_{Pz} & {}^{S}\gamma_{Pz} & {}^{P}p_{zSP} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The positioning vectors ${}^{P}\overline{p}_{SP}^{(0)}$ are explained with the relation:

$$ {}^{P}\overline{p}_{SP}=\begin{pmatrix} {}^{P}\overline{p}_{xSP} \\ \cdots\cdots \\ {}^{P}\overline{p}_{ySP} \\ \cdots\cdots \\ {}^{S}\overline{p}_{zSP} \end{pmatrix}=\begin{pmatrix} -p_{xP}\cdot cos(\alpha_{Px})+(b_1-p_{yP})\cdot cos(\beta_{Px})+ \\ +(d_1+d_2-d_3-d_4-d_5-p_{zP})\cdot cos(\gamma_{Px}) \\ \cdots\cdots\cdots\cdots \\ -p_{xP}\cdot cos(\alpha_{Py})+(b_1-p_{yP})\cdot cos(\beta_{Py})+ \\ +(d_1+d_2-d_3-d_4-d_5-p_{zP})\cdot cos(\gamma_{Py}) \\ \cdots\cdots\cdots\cdots \\ -p_{xP}\cdot cos(\alpha_{Pz})+(b_1-p_{yP})\cdot cos(\beta_{Pz})+ \\ +(d_1+d_2-d_3-d_4-d_5-p_{zP})\cdot cos(\gamma_{Pz}) \end{pmatrix} \quad (9)$$

The *coordinates for positioning* are:

$$\begin{cases} T_{SP_{03}}={}^{P}p_{xSP}\rightarrow \boldsymbol{q_j}; \\ T_{SP_{13}}={}^{P}p_{ySP}\rightarrow \boldsymbol{q_{j+1}}; \\ T_{SP_{23}}={}^{P}p_{zSP}\rightarrow \boldsymbol{q_{j+2}}. \end{cases} \quad (10)$$

The *coordinates for orientating* are:

$$T_{SP_{21}}=sin\,{}^{P}\beta_{xS}\rightarrow \boldsymbol{q_i};$$
$$\begin{cases} T_{SP_{20}}=-cos\,{}^{P}\beta_{xS}\cdot sin\,{}^{P}\gamma_{yS} \\ T_{PS_{22}}=cos\,{}^{P}\beta_{xS}\cdot cos\,{}^{P}\gamma_{yS} \end{cases}\rightarrow \boldsymbol{q_{i+1}}; \quad (11)$$
$$\begin{cases} T_{SP_{01}}=-sin\,{}^{P}\alpha_{zS}\cdot cos\,{}^{P}\beta_{xS} \\ T_{SP_{11}}=cos\,{}^{P}\alpha_{zS}\cdot cos\,{}^{P}\beta_{xS} \end{cases}\rightarrow \boldsymbol{q_{i+2}}.$$

Having the final results of the positioning and orientating coordinates relations (10) and (11) in the case of the tool-piece cooperating, these through particularization can be applied to any type of kinematic structure.

2. CONCLUSIONS

Mathematical modeling of the structure type 3T3R–3T3R represent an important realization, because starting from this it can be determinate the generalized coordinates for any type of kinematic structure. Also open new research directions about the modeling and simulation of machine tools by applying on that of locating matrix, usually apply on robots.

3. SELECTIVE REFERENCES

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Ecuațiile geometrice a două mașini – unelte de tip 3T3R în cooperare

Rezumat: În această lucrare sunt prezentate ecuațiile geometriei directe şi inverse pentru cea mai complexă structură de tip 3T3R – 3T3R. Aceasta este compusă din o structură de poziționare şi orienatare a SCULEI, cealaltă fiind aferentă PIESEI. Ambele structuri sunt în cooperare pe o platform comună. Pornind de la acest model general de cooperare de tip 3T3R – 3T3R ecuațiile geometriei directe i inverse pot fi aplicate pentru oricare tip de structură cinematică.

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