TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 57, Issue IV, November, 2014

# CONSIDERATIONS ON AN ELEMENTARY FLUID-STRUCTURE COUPLING SYSTEM

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***Abstract:*** *In the present paper a one dimensional fluid-structure coupling system is under observation. The system is composed of two coupled subsystems. The first one is a structural mass - spring - damper and the second one is a tube filled with air with a piston which represents the mass of the first subsystem. The solutions of a derived transcendental equation stand for the natural frequencies of the structure−air coupled system. The natural frequencies of both uncoupled subsystems are observed graphically in parallel to the natural frequencies of the coupled system. The coupled system's natural frequencies are discussed in function of the value of the unique structural natural frequency relative to the natural frequencies of the air filling the tube.*
***Keywords:*** *fluid-structure coupling system, natural frequency, transcendental equation.*

## 1. INTRODUCTION

The reduction of the interior vibration and noise in automobile passenger compartment is of continuous interest. Noise and vibrations are generated from sources like the engine, transmission or driveline, wind, the exhaust system and from the tires-road contact. These perturbations are transmitted through the body structure (structure born) and the air (air born) to the comfort points on the passenger compartment and degrading the acoustic comfort in the habitacle. The acoustics of the compartment is determined as well by the structural modal characteristics and the acoustic modal characteristics of the cabin and on the other side by the nature of the coupling between the structural and the acoustic subsystems. For the coupled air-structure analysis an elementary air-structure system has been considered and solved in order to enrich the perception on the coupling effect [2], [5], [10], [15]. This system is similar to the coupling between a body in white panel and the air of the passenger compartment.

Vibrations are reduced by using isolators and absorbers. Noise is controlled through insulation (used to separate the noise source from the target) and through absorption materials and structures. Semi-active and active vibration and noise control systems are also available [3].

## 2. ONE DIMENSIONAL COUPLING SOLUTION

In order to analytically observe the coupling or interaction between a volume of air and the elastic enclosure, an elementary system is considered (Figure 1). The straight tube is assumed elastic at one end, acoustically rigid at the opposite end and with constant section. The structure elasticity is introduced and modeled by a single degree of freedom mass-spring-damper system. The elastic end of the tube can be seen like a flat piston of section area A (A diameter being less than the sound wavelength) and mass $m_s$, which displacement *u(t)* is small about the static position. The system is working along a single direction, which is the tube axis.

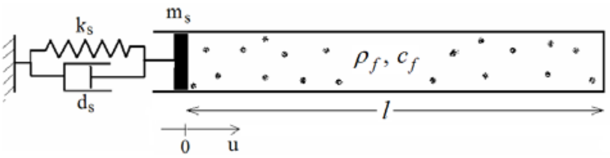


**Fig. 1.** A one d.o.f. structure coupled with the air of a tube

This simplified model is similar to the coupling between a sheet metal panel of the body in white and the air of the car cabin. In case one of the natural frequencies of the enclosure equals or is close to a natural frequency of the air in the cabin, the excitation at the resonance will happen.

In case we are taking independently the two subsystems of the simplified model we would have one degree of freedom for the structural component and n degrees of freedom for the fluid component of the system. The natural frequency $f_{s0}$ associated to the structure subsystem and natural frequencies $f_{f,n}$ , *n=1,2,...* associated to the fluid subsystem, would be:

$$f_{s0} = \frac{1}{2\pi}\sqrt{\frac{k_s}{m_s}} \quad (1)$$

and [8] $$f_{f,n} = \frac{c_f}{2l}n, \quad n = 1,2,3,... \quad (2)$$

where $k_s$ and $m_s$ are the spring stiffness and respectively the piston mass, $c_f$ is the sound velocity in air, *l* is the length of the air column filling the tube and *n* is the acoustic mode index.

In order to determine the natural frequencies of the coupled system the differential equation of each subsystem influenced by the other one, has to be written. The differential equation associated to the mass-spring-damper system is [1], [11]:

$$m_s\ddot{u} + d_s\dot{u} + k_s u = F_s + F_p \quad (3)$$

where *u(t)* registers the piston or structure position along the tube axis with respect to the static position, $\dot{u}$ and $\ddot{u}$ are the piston velocity and acceleration, $d_s$ is the viscous damping, $F_s(t)$ is the external active force acting on the piston (not coming from the air), $F_p$ is the external force, acting on the piston lateral surface:

$$F_p = -A \cdot p(0) \quad (4)$$

caused by the air pressure *p* at the left side (*u=0*) of the tube and A is the area of the tube or the piston cross section.

The general acoustic wave partial differential equation for longitudinal pressure perturbations in 3D homogeneous enclosed space (cavity) and time, is [1], [3]:

$$\nabla^2 p = \frac{1}{c_f^2}\frac{\partial^2 p}{\partial t^2} \quad (5)$$

in which the sound pressure or the deviation from equilibrium or ambient pressure ($p_0$) is dependent on position in rectangular Cartesian space and time $p = p(x,y,z,t)$ and $\nabla^2$ *or* $\Delta$ is the Laplacian operator.

The speed of sound on the adiabatic propagation case (the temperature variations do not have enough time to leave the area):

$$c_f = \sqrt{\frac{B}{\rho_0}} \quad (6)$$

is the square root of the adiabatic bulk modulus (*B*) of the gas and the gas density at rest ($\rho_0$) ratio. The gas adiabatic bulk modulus [15]:

$$B = \gamma p_0 \quad (7)$$

is the ratio $\gamma$ of the specific heats for the gas multiplied by the gas pressure at rest $p_0$.

On the boundary surface of the air a momentum balance [12] is considered:

$$\frac{\partial p}{\partial n} = -\rho_f \ddot{u} \quad (8)$$

where $\ddot{u}$ is the normal acceleration component of the bounding of the vibrating surface, $\rho_f$ the mean ambient air density and *n* stands for the normal direction to the bounding surface. In case of acoustically rigid bounding surface results:

$$\partial p / \partial n = 0 .$$

For an opening on the bounding surface the pressure at the volume limit equals the ambient pressure: $p = 0$.

Assuming a time harmonic solution for the cavity pressure field (at resonance for instance), which allows the separation of time from the spatial variables and preserving the notation for pressure:

$$p = p(x, y, z)e^{j\omega t} \tag{9}$$

one can obtain by substitution in (5) the three dimensional second order Helmholtz differential equation:

$$\nabla^2 p + \frac{\omega^2}{c_f^2} p = 0 \tag{10}$$

where *p=p(x,y,z)*. Considering the acoustic wave number $k = \frac{\omega}{c_f}$, where $\omega = 2\pi f$ is the angular velocity, results:

$$\nabla^2 p + k^2 p = 0 \tag{11}$$

which is an eigenvalue problem. With the appropriate boundary conditions will result a set of eigenvalues and the associate eigenmodes or acoustical modes or spatial pressure distribution associated to each eigenvalue. From the eigenvalues the natural frequencies of the enclosed space will result. For the majority of the enclosures the solution of the relation (11) has to be found numerically, specially by using the finite element method. The resulted matrix eigenvalue problem is of the form [3], [4]:

$$(K - \omega^2 M)p = 0 \tag{12}$$

where *p* is the vector of pressure amplitudes at the fluid elemental nodes inside the enclosure and K, M are respectively the stiffness and mass of the acoustic field.

Taking the differential equation valid for the fluid pressure along x direction only, in the acoustic tube, results:

$$\frac{\partial^2 p}{\partial x^2} + k^2 p = 0 \tag{13}$$

In order to solve the above partial differential equation, the acoustic boundary conditions have to be considered. For a harmonic solution and small vibration amplitude movement $u_0$ of the bounding surface

$$u(t) = u_0 e^{j\omega t} \tag{14}$$

one have:

$$\frac{\partial p}{\partial n} = \rho_f \omega^2 u_0 \tag{15}$$

or for velocity amplitude $v_0$, one have:

$$\frac{\partial p}{\partial n} = -j\rho_f \omega v_0 \tag{15b}$$

Relations (15) and (15b) link the normal boundary surface displacement or velocity to the normal derivative of the pressure. These relations are useful for the treatment of coupling between the fluid and the surrounding structure.

For a bounding surface covered with a sound absorbing material of specific acoustic impedance $z_s$, the following relation is valid:

$$\frac{\partial p}{\partial n} = -j\rho_f \omega \frac{p}{z_s}$$

## 3. NATURAL FREQUENCIES OF THE COUPLED AIR-STRUCTURE SYSTEM

In order to simplify the system, the external forces on the piston (excepting the gas) and the damping in structure are neglected: $F_s$=0, $D_s$=0. Let us assume a harmonic solution for the pressure distribution of the air along the x axis of the tube:

$$p(x, k) = a_1 \cos kx + a_2 \sin kx \tag{16}$$

where $k = \omega / c_f$. The integral constants $a_1$ and $a_2$ have to be determined from the boundary conditions imposed at both tube ends. Replacing the solution (16) in the following relation which is valid for the elastic wall:

$$\left. \frac{\partial p(x,k)}{\partial x} \right|_{x=0} = \rho_f \omega^2 u \tag{17}$$

the $a_2$ constant is determined:

$$a_2 = \rho_f \omega^2 u / k,$$

where *u* is the displacement of the piston in the coupled system. From the rigid tube end, where the variation of the pressure normal to the rigid wall is null:

$$\left.\frac{\partial p(x,k)}{\partial x}\right|_{x=l}=0 \qquad (18)$$

the constant $a_1$ results:

$$a_1=\rho_f\omega^2 u/k\cdot(\tan kl)^{-1}$$

Substituting the constants in the proposed solution, we get:

$$p(x,k)=\frac{\rho_f\omega^2 u}{k}\cdot (\tan kl)^{-1}\cos kx+\frac{\rho_f\omega^2 u}{k}\cdot\sin kx \qquad (19)$$

By considering the fluid impedance $Z_0=\rho_f\cdot c_f$, after some calculations yields:

$$p(x,\omega)=Z_0 u\omega\cdot(\tan\frac{\omega l}{c_f})^{-1} \{(\tan\frac{\omega l}{c_f})\cdot(\sin\frac{\omega}{c_f}x)+\cos\frac{\omega}{c_f}x)\} \qquad (20)$$

Now, the expression of the air pressure variation at the left tube end (x=0), becomes:

$$p(0,\omega)=\omega Z_0 u(\tan\frac{\omega l}{c_f})^{-1} \qquad (21)$$

Let us observe the structure subsystem behavior when the harmonic excitation is present. The assumed solution:

$$u(t)=u_0 e^{j\omega t}$$

is substituted in the differential equation (3) of the structure subsystem, resulting:

$$(-\omega^2 m_s+j\omega d_s+k_s)u=F_s+F_p \qquad (22)$$

Being a normal modal analysis in which the damping is neglected and the only external force is the one from the air counterpart, results:

$$(-\omega^2 m_s+k_s)u=-Ap(0,\omega) \qquad (23)$$

Substituting (21) which stands for the air pressure at *x=0*, results:

$$-\omega^2 m_s+k_s=-A\omega Z_0(\tan\frac{l\omega}{c_f})^{-1}$$

or:

$$\tan\frac{l\omega}{c_f}=\frac{AZ_0\omega}{\omega^2 m_s-k_s} \qquad (24)$$

Introducing the structure subsystem natural circular frequency $\omega_{s0}=\sqrt{m_s/k_s}$, as a reference, we get:

$$\tan(\frac{l}{c_f}\omega)=\frac{AZ_0\omega}{m_s(\omega^2-\omega_{s0}^2)} \qquad (25)$$

or by using the frequency *f*, instead of the circular frequency $\omega$, results:

$$\tan(\frac{2\pi l}{c_f}f)=\frac{AZ_0 f}{2\pi m_s(f^2-f_{s0}^2)} \qquad (26)$$

The left member of (26) equals zero for $\pi n$ radians *(n=1,2,3...)* of the argument of the tangent function:

$$\frac{2lf}{c_f}=n,$$

resulting the known (2) relation:

$$f_{f,n}=\frac{nc_f}{2l} \qquad n=1,2,3...$$

that gives the eigenvalues $f_{f,n}$ of the fluid subsystem with rigid ends.

The right member of the equation (26) has a null denominator for $f=f_{s0}$. This member increases asymptotically to +∞ for the argument $f$ decreasing to $f_{s0}$ and tends asymptotically to -∞ for $f$ increasing to $f_{s0}$.

The solutions $f_i$, $i=1,2,3,...$ of the transcendental equation (26) are the natural frequencies of the structure-air coupled system.

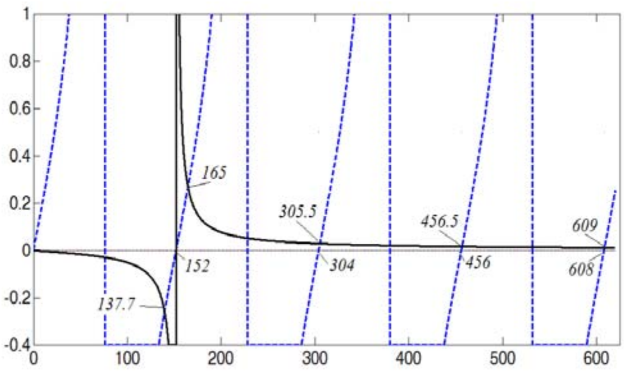


**Fig. 2.** $M_s$=0.01 $k_s$=9122

Let us consider a spring-mass-tube system with the same value of 152 Hz for the first natural frequency of each of the two uncoupled subsystems. This is realized for the following system characteristics [SI]:

$M_s = 0.01$, $k_s = 9122$, l=1.13152, A=0.001

The natural frequencies 137.7Hz, 165Hz, 305.5Hz etc. of the coupled system are obtained graphically by using Matlab and can be observed in Figure 2. The solutions are placed at the multiple intersections between the two curves, one associated to the left and the second associated to the right member of the equation (26). The first natural frequency $f_i = 137.7$ Hz of the coupled system will be different from the unique natural frequency $f_s = 152$ Hz of the mass-spring subsystem that equals the first natural frequency of the air in the tube $f_{f,1} = 152$ Hz. All subsequent frequencies originating from the air in the tube will be a little different from the associated acoustic mode order $i = 2,3,...$, of the tube with rigid ends, as one can observe in row no. 1 of Table 1. The eigenvalue of the spring-mass subsystem

Table #1

| L=1.13152; A=0.001; $\rho_f$=1.205; $c_f$=344 | | | | |
|---|---|---|---|---|
| | $M_s$ $k_s$ | Mass-spring: $f_s$ [Hz] | Tube with air, rigid ends [Hz] $f_{f,n}, n=1,2,3,4$ | Whole coupled system [Hz] $f_i$, $i=1,2,3,...$ |
| 1 | 0.01, 9122 | 152. | 152, 304 456, 608 | 137.7, 165, 305.5, 456.5, 609 |
| 2 | 0.0035, 9122 | 256.9 | 152 304 456 608 | 148.9 251.8, 312.6 459.6 610.3 |

can be close to a higher eigenvalues of the air in tube or between to adjacent eigenvalues.

Hence, for a smaller piston mass the natural frequency of the mass-spring subsystem

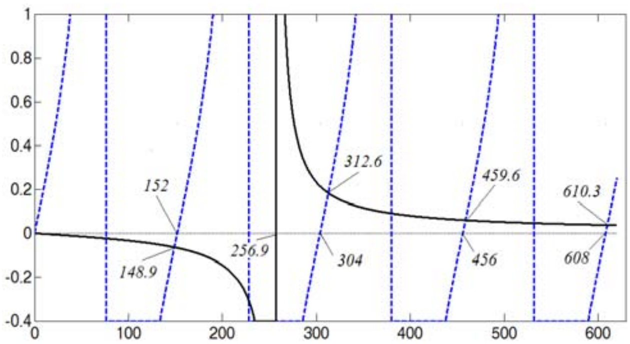


**Fig. 3.** $M_s$=0.0035 $k_s$=9122

increases and the associated curve is shifting to the right with regards to the right member graph, in the graph. The abscissa of the intersection points are changing like for example in Figure 3 and the associated row no. 2 of Table 1, where the intersection point is at the natural frequency $f_2 = 251.8$ Hz and y=-1.815 is not visible in the figure. As far as the natural frequency of the spring-mass subsystem is next to a natural frequency of the tube with rigid walls subsystem the tube natural frequency is replaced with two different natural frequencies of the coupled system.

## 4. CONCLUSIONS

A one dimensional fluid−structure coupling system has been observed. The tube filled with air have a rigid end and a mobile one represented by a mobile piston, part of a single degree of freedom mass–spring–damper system. Hence, the two subsystems are coupled. The solutions of a derived transcendental equation stand for the natural frequencies of the structure−air coupled system. The natural frequencies of both uncoupled subsystems are observed graphically in parallel to the natural frequencies of the coupled system. The piston and the air will move together for each eigenmode of the coupled system. The pressure in a tube with rigid ends are maximal at the walls for each acoustic mode, resulting a harmonic excitation on the walls. In the coupled system if one imagine the air pressure harmonically exciting the mass (piston) of the structural system, the piston is put into harmonic motion and vice versa. In case the frequencies of excitation $f_{f,n}$ are far from the unique structural natural frequency $f_s$, the natural frequencies $f_i$ for the entire coupled system are very close to $f_{f,n}$ and slightly increased. For a $f_{f,n}$ value close to $f_s$ the associated $f_i$ differs more from the natural frequencies of the individual subsystems. In case the structural system's natural frequency is altered by changing the mass or the stiffness, the curve is translated along the abscissa while the curve associated to the tube is fixed, hence the intersection points are changing.

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**Considerații asupra unui sistem elementar de cuplare fluid – structură**


**Rezumat**: *Articolul abordează un sistem cuplat format dintr-o componentă structurală de tipul masă – arc – amortizor şi un subsistem reprezentat de un tub umplut cu aer, la un capăt tubul fiind rigid iar la celălalt având un piston mobil. Pistonul reprezintă masa subsistemului structural. Este dedusă o ecuație transcendentală a cărei soluții reprezintă frecvențele naturale ale sistemului cuplat. Frecvențele naturale ale celor două sisteme luate independent sunt urmărite în mod grafic, în paralel cu frecvențele naturale ale sistemului cuplat. Este analizată dependența frecvențelor naturale ale întregului sistem în funcție de poziționarea unicei frecvențe naturale aparținând sistemului masă-arc relativ la frecvențele naturale ale coloanei de aer din tub.*



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