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# THE MODULUS OF ELASTICITY ESTIMATION BY USING FEA AND A FREQUENCY RESPONSE FUNCTION

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***Abstract:*** *In the present paper the determination of the Young's modulus by using a vibration test is under observation. The part made of the material of interest is modelled by using finite elements and a normal mode analysis is performed. The modulus of elasticity used in the material model is an assumed one. A low mode of vibration is selected, a bending mode is preferred. On the part an accelerometer is placed and a force sensor is measuring the excitation; the frequency response function is recorded. The natural frequency value of interest is picked and used as a reference for the finite element normal mode analysis. Successive runs of the model are performed while the Young modulus is tuned in order to get the same natural frequency value resulted from experiment. The process is repeated for different boundary conditions.*
***Keywords:*** *Modulus of elasticity, normal mode analysis, frequency response function.*

## 1. INTRODUCTION

An important structural characteristic of the mechanical parts for linear and elastic deformations is expressed by the modulus of elasticity E, also known as Young's modulus which expresses the axial linear deformational behavior of a large diversity of materials.

The most used experimental procedure to determine the modulus of elasticity is by employing a specialized testing machine and a test coupon or a specimen by following a corresponding testing standard [6].

The relationship between the stress and strain that a material displays during the test is known as a stress-strain curve and is unique for each material. The curve is found by recording the amount of deformation (strain) at distinct intervals of tensile or compressive loading. The stress – strain curve contains data necessary to determine the modulus of elasticity. Up to the proportional limit of the material, the stress is proportional to the strain and Hooke's Law states:

$$E = \sigma / \varepsilon \qquad (1)$$

A typical stress (σ) versus strain (ε) graphical representation for the structural steel is depicted in figure 1, where the first portion is

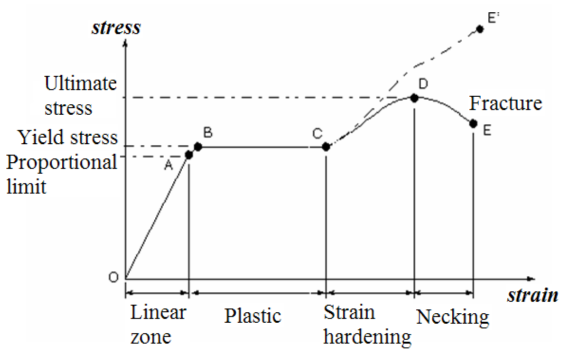


**Fig. 1.** A typical stress vs. strain curve

a linear elastic deformation characterized by the elastic modulus E, followed by a plastic zone, a strain hardening one and finally by the necking before the fracture. The width of the mentioned zones are not proper correlated, the figure just showing the sequence of the events.

Free vibrations of a structure are rapid conversions between the kinetic and potential energy of it when oscillating about the equilibrium of the structure. During small amplitude vibrations the Young modulus is the main material characteristic governing the event.

The Young modulus estimation can be performed by observing the modes of vibration and the associated natural frequencies of vibration for various boundary conditions. The simplest procedure is by observing the vibration modes when the part is free of constraints. A mini accelerometer is attached to the structure under investigation and an impact hammer is used for the structure excitation. An acquisition system is used to acquire the signals and to process them in order to get the experimental frequency response function.

## 2. EXEPERIMENTAL FREQUENCY RESPONSE FUNCTION

Exciting the structure with an impulsive load is determining the structure to vibrate. The structure will vibrate simultaneously in the first several modes of vibration which have been excited by the impulsive force. The profile of the force excitation and the response acceleration at the level of the accelerometer attaching spot are measured simultaneously.

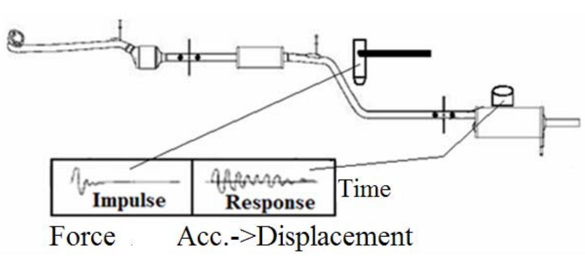


**Fig. 2.** A tipical FRF measurement

The acquisition system used is a National Instruments dynamic acquisition board with simultaneously sampling on the input channels and a Labview processing software [5]. A frequency response function (FRF) can be calculated based on the measured data from the force sensor on the first acquisition channel and the accelerometer on the second channel [4].

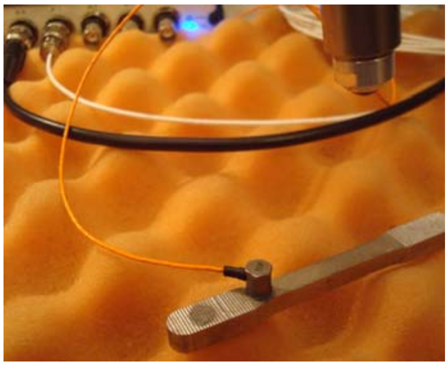

**Fig. 3.** The measurement set-up

The FRF expresses the ratio of the Fourier transformation of the acceleration, velocity or the displacement and the Fourier transformation of the excitation force for the pair of the measuring points (response, excitation) on the structure under test. More generally the FRF is the transfer function (TF) between two degrees of freedom of the structure evaluated for s=jω where s is the complex variable s=σ+jω. TF is the ratio between the Laplace transform of the response signal and the Laplace transform of the excitation signal. The response degree of freedom is associated to the acceleration sensor and the excitation degree of freedom is associated to the force sensor.

In case the acceleration is considered, the TF is of inertance type:

$$H_a(s) = \frac{s^2 X(s)}{F(s)} \tag{2}$$

where the Laplace transform is applied on the acceleration $L[\ddot{x}(t)] = s^2 X(s)$ and s is the complex variable.

In case the velocity is measured or the acceleration is integrated, the TF is of the mobility type:

$$H_v(s) = \frac{sX(s)}{F(s)} \tag{3}$$

where: $L[\dot{x}(t)] = sX(s)$.

Finally, for the displacement measurement (or double integration) the TF is of the compliance type:

$$H_d(s) = \frac{X(s)}{F(s)} \tag{4}$$

Three different graphical representations of the frequency response function are available: the coincidence-quadrature representation or the real and the imaginary parts of the FRF versus frequency, the Nyquist diagram or the imaginary versus real part representation and the Bode diagram, where the magnitude and the phase of the FRF are shown in function of the excitation frequency.

A Bode diagram is depicted in figure 4 and 5, recorded for a small analyzed part like the one in figure 3.

The first peak at about 650 Hz can be clearly seen in the modulus versus frequency graph. A shift in phase is visible in the phase versus frequency graph associated to the magnitude peak, both indicating the presence of a vibration mode of the part. The frequency value is further used to tune the first vibration mode in the finite element normal mode analysis.

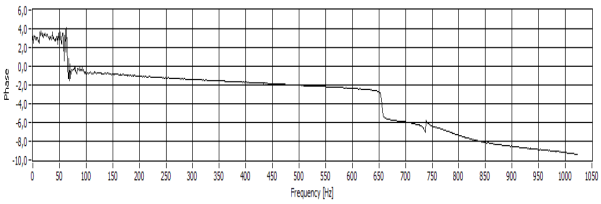

**Fig. 4.** FRF: phase versus frequency

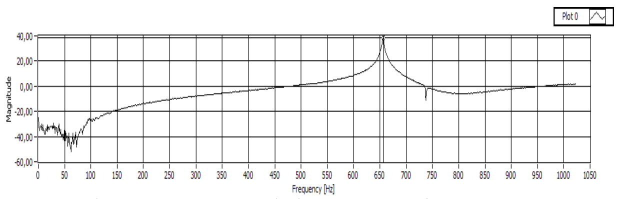

**Fig. 5.** FRF: modulus versus frequency

The experiment continues by changing the boundary conditions of the part under test, for example instead of having a free of constraints part, the part is clamped on one side in a rigid fixture. A new frequency response function is recorded by using the hammer and the accelerometer. The FRF reveals the natural frequencies and the phase changes associated to the new natural modes of vibration. The position of the accelerometer in the new measurement set-up is important as well, and has to be modeled in the right position on the finite element model.

## 3. MODAL ANALYSIS BY USING FEA

The modal analysis is run by using solvers like Ansys, Optistruct, Nastran or Code Aster. The part geometry has to be modeled in CAD, followed by the mesh of the geometry by using proper finite elements. The boundary conditions are imposed and a proper material model is selected. In the material model the Young modulus is estimated. After the first normal mode analysis the first bending mode of vibration and the associated natural frequency is considered. It is the same mode of vibration as that observed in the experimental section. The natural mode of vibration is compared and corrective values of the Young modulus are imposed. Successive runs of the model are performed while the Young modulus value is tuned to get the same natural frequency value resulted from experiment. The best value is selected and used in the sequel to validate the Young modulus by using other boundary conditions imposed to the part under test and in parallel in FEA. For example the part can be clamped in a rigid fixture and the mass of the accelerometer is added to the finite element model. In the new conditions the selected mode

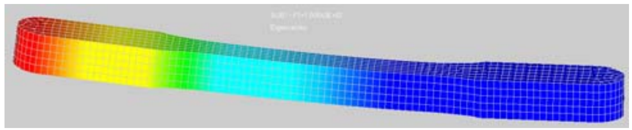

**Fig. 6.** Bending – 1st vibration mode

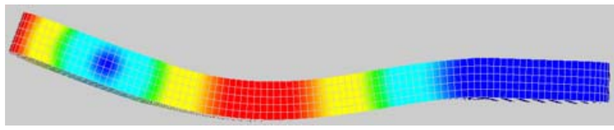

**Fig. 7.** The second vibration mode

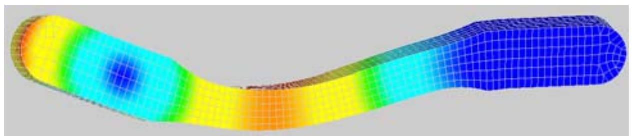

**Fig. 8.** A lateral vibration mode

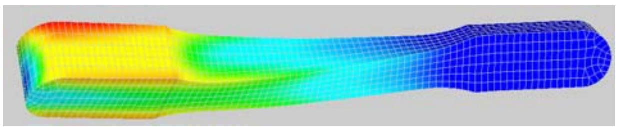

**Fig. 9.** The first torsion vibration mode

of vibration has to be similar to the associated mode of vibration resulted from experiment, in terms of the estimated mode shape and the associated natural frequency. The first mode shapes of the clamped structure are shown in figures 6, 7, 8 and 9. The first natural vibration bending mode is chosen to be compared with the first measured natural mode of the clamped structure. The mode shapes of the two modes are similar (Figure 6, bending mode) and the natural frequencies values of the modes are very close. This is a validation for the Young modulus already estimated in the first set-up. The process can continue with another set-up if is necessary.

## 4. CONCLUSIONS

In the present paper the determination of the Young's modulus by using a vibration test is under observation. The part made of the material of interest is modelled by using finite elements and a normal mode analysis is performed. The modulus of elasticity used in the material model is an assumed one. A low mode of vibration is selected, a bending mode is preferred and the torsion is avoided. In parallel, an accelerometer is attached to the structure at a spot which is not recommended to be a node of the vibration mode of interest and the frequency response function is recorded. The natural frequency value of interest is picked and used as a reference for the finite element normal mode analysis. Successive runs of the model are performed while the E value is tuned in order to get the same natural frequency value resulted from experiment. The experiment is repeated with another set-up or different boundary conditions. The first natural frequency measured on the structure (extracted from the frequency response function) has almost the same value as the natural frequency resulted from the simulation. This is a validation, indicating that the Young modulus is well estimated.

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**Estimarea modulului de elasticitate folosind FEA si functia de raspuns in frecventa**

**Rezumat**: *Articolul abordează estimarea modulului de elasticitate a unui material pe baza măsurătorilor de vibraţii asupra unor structuri confecţionate din acel material. Pentru condiţii la limită similare se măsoară prima frecvenţă naturală folosind un sistem de achiziţie de date prin calculator şi se efectuează în paralel o analiza modală folosind metoda elementelor finite. In simulare se modifică modulul de elasticitate al materialului până se obţine aceeaşi valoare în măsurare şi în experiment pentru prima frecvenţă naturală. Se repetă procedura, cu scop de validare, pentru aceeaşi piesă având alte condiţii la limită, de exemplu un capăt încastrat, caz în care se obţine din nou aceeaşi valoare pentru prima frecvenţă măsurată şi cea rezultată din analiza modală prin metoda elementelor finite.*

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