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CONTRIBUTIONS TO THE MODELING OF A MECHANICAL SYSTEM WITH TWO DEGREES OF FREEDOM CORRESPONDING TO A SEAT – DRIVER SYSTEM

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Abstract: *In literature are many mechanical models corresponding to the human body vibration, in standing or sitting position. The characteristic of these models is that the links between the discrete masses are the Kelvin-Voigt rheological systems. The study about the influence of the rheological model characteristics is presented in this paper. The relative motion between the elements of the mechanical system is studied considering different frequencies and damping's of the support displacement.*

Key words: *human body vibration, mechanical system, frequencies, damping, relative motion*

1. INTRODUCTION

The human body subject to vibratory movements from industrial environment was modeled in literature as a system of discrete masses linked with elastic and damping elements disposed in parallel, corresponding to the Kelvin-Voigt model.

There are many papers in which was studied the problem of the human body in standing or sitting position. The characteristic of these models are the use of the Kelvin-Voigt rheological model.

Using this rheological model a system of second order differential equations will be obtained. After solving this system will result the solutions.

The problem of the human body vibration was studied by many researchers in this field, over the years.

Based on these researches, numerous papers were written and are available in literature [1], [2], [3], [4], [5], [6] in which are presented different mechanical models of the human body vibrations transmitted in the whole body or only in some parts.

Different authors have developed mathematical models including experimental studies for systems with different degrees of freedom that coincide with the number of the discrete masses. The discrete masses correspond to the different parts of the human body.

Also in literature are presented studies in which were considered different rheological models containing elastic and damping elements.

Such models are considered in [1], [6] and others, for systems with two degrees of freedom or more.

2. THEORETICAL BACKGROUND

In this paper is considered that the human body is sitting on a chair. The link between the body and the chair is performed with a Kelvin-Voigt rheological system and the link between the machine carriage (which makes harmonic vibrations of known frequencies and amplitudes) and the chair is made using a Zener rheological system which contains the elastic element k_1, in parallel with the elastic element k_2 and with damping element linked in series [7], [8].

In the figure 1 is presented the corresponding mechanical model.

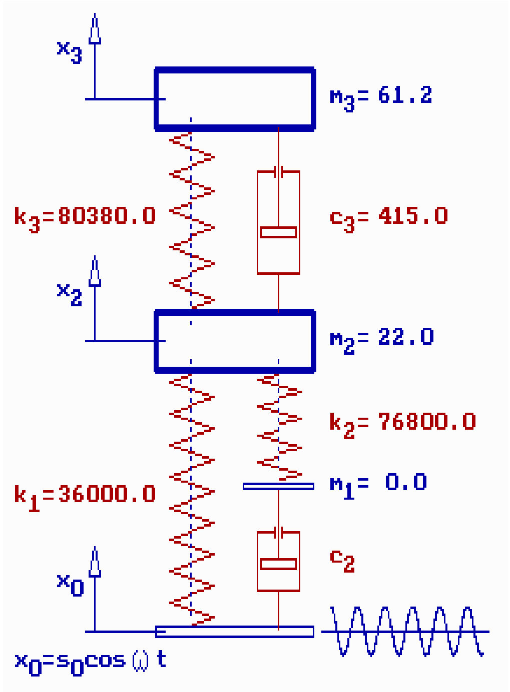

Fig.1. The mechanical model [6]

The numerical values of the masses and the elastic elements are taken from [6].

Based on second law of Newton the differential equations of the existing masses m_2, m_3 and the fictions mass m_1, which is considered between the elastic element k_2 and the damping c_2 were written:

$$
\begin{aligned}
& m_1\ddot{x}_1 = 0 = c_2(-s_0\,\omega\sin\omega t - \dot{x}_1) - k_2(x_1 - x_2) \\
& m_2\ddot{x}_2 = k_1(s_0\cos\omega t - x_2) + k_2(x_1 - x_2) - k_3(x_2 - x_3) - c_3(\dot{x}_2 - \dot{x}_3) \\
& m_3\ddot{x}_3 = k_3(x_2 - x_3) + c_3(\dot{x}_2 - \dot{x}_3)
\end{aligned} \tag{1}
$$

The three equations (1) may be written in the following form:

$$
\begin{aligned}
& \dot{x}_1 = -\frac{k_2}{c_2}x_1 + \frac{k_2}{c_2}x_2 - s_0\,\omega\sin\omega t \\
& \ddot{x}_2 = -\frac{c_3}{m_2}\dot{x}_2 + \frac{c_3}{m_2}\dot{x}_3 + \frac{k_2}{m_2}x_1 - \frac{k_1+k_2+k_3}{m_2}x_2 + \frac{k_3}{m_2}x_3 + \frac{k_1 s_0}{m_2}\cos\omega t \\
& \ddot{x}_3 = \frac{c_3}{m_3}\dot{x}_2 - \frac{c_3}{m_3}\dot{x}_3 + \frac{k_3}{m_3}x_2 - \frac{k_3}{m_3}x_3
\end{aligned} \tag{2}
$$

The first differential equation from (2) is of first order and the other two of second order.

Considering the following notations:

$$x_1 = y_1,\ x_2 = y_2,\ x_3 = y_3,\ \dot{x}_2 = y_4,\ \dot{x}_3 = y_5$$

the system (2) will be written as a system of five differential equations of first the order:

$$
\begin{aligned}
& \dot{y}_1 = -\frac{k_2}{c_2}y_1 + \frac{k_2}{c_2}y_2 - s_0\,\omega\sin\omega t \\
& \dot{y}_2 = y_4 \\
& \dot{y}_3 = y_5 \\
& \dot{y}_4 = -\frac{c_3}{m_2}y_4 + \frac{c_3}{m_2}y_5 + \frac{k_2}{m_2}y_1 - \frac{k_1+k_2+k_3}{m_2}y_2 + \frac{k_3}{m_2}y_3 + \frac{k_1 s_0}{m_2}\cos\omega t \\
& \dot{y}_5 = \frac{c_3}{m_3}y_4 - \frac{c_3}{m_3}y_5 + \frac{k_3}{m_3}y_2 - \frac{k_3}{m_3}y_3
\end{aligned} \tag{3}
$$

The matrix form on the equations (3) is:

$$
\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} =
\begin{bmatrix}
-\frac{k_2}{c_2} & \frac{k_2}{c_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\frac{k_2}{m_2} & -\frac{k_1+k_2+k_3}{m_2} & \frac{k_3}{m_2} & -\frac{c_3}{m_2} & \frac{c_3}{m_2} \\
0 & \frac{k_3}{m_3} & -\frac{k_3}{m_3} & \frac{c_3}{m_3} & -\frac{c_3}{m_3}
\end{bmatrix}
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} +
\begin{bmatrix} -s_0\,\omega\sin\omega t \\ 0 \\ 0 \\ \frac{k_1 s_0}{m_2}\cos\omega t \\ 0 \end{bmatrix} \tag{4}
$$

The zero values for the initial conditions will be considered. [9]

3. NUMERICAL RESULTS

The numerical results were obtained using the fourth order Runge – Kutta method.

The values for the parameters m_2, m_3, k_3 and c_3 were taken from the paper [6].

In order to solve the system (4) were considered five variable parameters as follows: the spring stiffness k_1 and k_2, the damping constant c_2, the oscillating support frequency and amplitude.

The support amplitude and frequency were chosen to fit with the normatives [6].

Considering the obtained numerical results the law of motions $x_2=x_2(t)$ (the motion of the chair towards a fixed reference) and $x_3=x_3(t)$ (the human body motion towards a fixed reference) were determinated.

The relative motion of the human body towards the chair presents a great interest.

In the diagrams are presented the relative displacement of the chair towards the carriage (x_S-x_2) and the human body towards the chair (x_2-x_3).

In the figures 2~4 are presented the diagrams obtained for a frequency of 3 [Hz] and amplitude of 0.0075 [m] for different values of the damping constant c_2 = 800, 1600, 2400 [N·s/m], considering k_1=36000 [N/m], k_2=76800 [N/m].

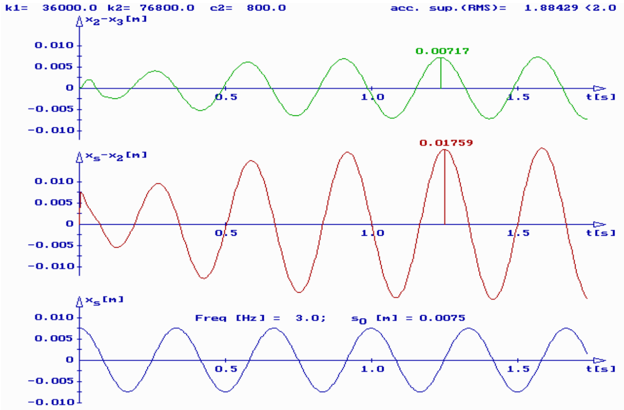

Fig.2. The relative displacement chair-carriage and human body-chair for k_1=36000, k_2=76800, c_2=800, f_0 = 3Hz and s_0=0.0075

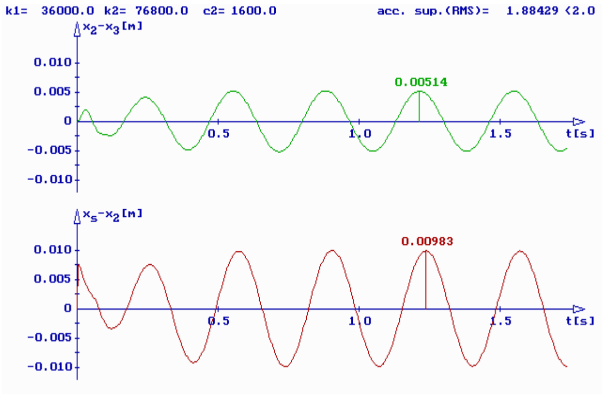

Fig.3. The relative displacement chair-carriage and human body-chair for k_1=36000, k_2=76800, c_2=1600, f_0 = 3Hz and s_0=0.0075

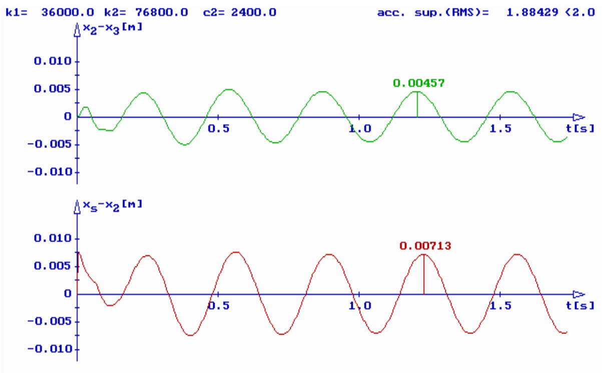

Fig.4. The relative displacement chair-carriage and human body-chair for k_1=36000, k_2=76800, c_2=2400, f_0 = 3Hz and s_0=0.0075

Observing the variation of the relative motion amplitude of the human body towards the chair the conclusion is that the values are between 4.57 ~ 7.17 mm.

Running the C program in which all five parameters, f_0, s_0, k_2, k_1 and c_2, have variable values some numerical results are presented in table 1.

Table 1.The obtained numerical results

s_0= 0.0075		f_0=3.0		
k_1	k_2	c_2	(x_s-x_2)	(x_2-x_3)
30000	70000	800	18.301	8.699
		1600	11.691	7.037
		2400	10.544	6.564
		3200	10.458	6.374
	75000	800	18.077	8.600
		1600	11.317	6.895
		2400	10.114	6.412
		3200	10.029	6.226
	80000	800	17.883	8.514
		1600	10.994	6.773
		2400	9.745	6.281
		3200	9.656	**6.095**
35000	70000	800	20.671	10.097
		1600	12.555	7.374
		2400	10.923	6.622
		3200	10.609	6.352
	75000	800	20.483	10.012
		1600	12.228	7.254
		2400	10.554	6.498
		3200	10.238	6.228
	80000	800	20.321	9.939
		1600	11.945	7.151
		2400	10.232	6.388
		3200	9.912	6.115
40000	70000	800	21.646	10.777
		1600	13.000	7.479
		2400	11.117	6.601
		3200	10.673	6.308
	75000	800	21.511	10.716
		1600	12.733	7.386
		2400	10.807	6.603
		3200	10.358	6.203
	80000	800	21.395	10.664
		1600	12.499	7.310
		2400	10.533	6.416
		3200	10.075	6.108

From table 1 results that the minimal amplitude of relative motion of human body and chair is obtained for the values for k_1=30000, k_2=80000, c_2=3200, the oscillating support performing sinusoidal movements with amplitude 7.5 mm and frequency 3 Hz.

4. CONCLUSIONS

Using the system of differential equations (4) and the C program developed by the authors, is possible to find the most suitable values for the drivers seat suspension elements.

This problem can be developed considering different values for the driver's human body and taking other values for the mass m_3 and for the elements k_3 and c_3.

We notice that was necessary to add a supplementary first order differential equation for each rheological elements that contains two elements linked in series, in our studied case we use five differential equations instead of four.

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5. REFERENCES

[1] Fritton, J. C., Rubin, C. T., Qin, Y. X., McLeod, K. J., *Whole-body vibration in the skeleton: development of a resonance-based testing device*, Annals of Biomedical Engineering, 1997, Vol. 25, pp. 831-839

[2] Garg, P.D., Ross, A.M., *Vertical mode human body vibration transmissibility*, Journal of IEEE transaction on systems, man and cybernetics Vol. SMC-6, No.2, 1976, pp.102-112

[3] Griffin J.M., *Modeling the dynamic mechanisms associated with the principal response of the seated human body*, Clinical Biomechanics16 Supplement no 1, 2011, pp. 31-44.

[4] Mansfield, N.J., Meada, S., *Comparison of the apparent mass and cross axis apparent masses of seated humans exposed to single and dual-axis whole body vibration,* Journal of Sound and Vibration, 2006, Vol. 298 (1), pp.841-853.

[5] Nigam, S.P., Malik, M.A., *Study on a vibratory model of a human body*, Transaction of ASME, Journal of Biomechanics, 1987, Vol. 109, pp. 148-153.

[6] Stein, G. J., Múčka, P., Gunston, T. P., Badura, S., *Modelling and simulation of locomotive driver's seat vertical suspension vibration isolation system*, International Journal of Industrial Ergonomics, 2008, Vol. 38, pp. 384-395

[7] Ursu-Fischer, N., Radu, I., Musca, I.A., *Contributions to the dynamic study of one degree of freedom mechanical system, actuated by a harmonic force, with different elastic and viscous elements*, Acta Technica Napocensis, Series Applied Mathematics and Mechanics, Vol. 56, Issue II, 2013, pp 283-290.

[8] Ursu-Fischer, N., Radu, I., Musca, I.A., *Contributions to the transmissibility study of one degree of freedom mechanical system, with different elastic and viscous elements*, Acta Technica Napocensis, Series Applied Mathematics and Mechanics, Vol. 56, Issue II, 2013, pp 291-298.

[9] Ursu-Fischer, N., *Vibrations of Mechanical Systems.Theory and Applications (in Romanian)*, Casa Cărții de Ştiință, Cluj-Napoca, 1998, 452 pp.

CONTRIBUȚII PRIVIND MODELAREA UNUI SISTEM MECANIC CU DOUĂ GRADE DE LIBERTATE CORESPUNZĂTOR SISTEMULUI SCAUN – CONDUCĂTOR AUTO

Rezumat: *În literature de specialitate au apărut extreme de multe modele mecanice corespunzătoare vibrațiilor coprului uman, fie în picioare, fie aşezate. O caracteristică a acestor modele este că legătura dintre masele discrete se face cu sisteme reologice tip Kelvin-Voigt. În lucrare se studiază influența amortizări în sistem în cadrul unui model reologic compus din resort cu care în paralel este un alt resort și un amortizor legat în serie. Pentru diferite frecvențe și amplitudini ale deplasării suportului se studiază mișcările relative între elementele sistemului mecanic.*

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