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THE ANALYSIS OF MAXIMAL WORKSPACE OF THE 3-RRS SPATIAL PARALLEL MANIPULATOR

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Abstract*: In this article is studied the graphical representation using the meshing method (based on input-output equations and on a program designed in AutoCAD) for the maximal workspace of the 3RRS manipulator with 3 degrees of freedom. Is calculated the areas of the different plane sections (Zp=constant), and the workspace volume.*
Key words: *parallel manipulator, the meshing method, degrees of freedom, Euler angles.*

1. INTRODUCTION

The figure 1 shows the kinematic scheme of the 3RRS spatial parallel manipulator having three degrees of freedom and three identical kinematic chains [1], [2], [3].

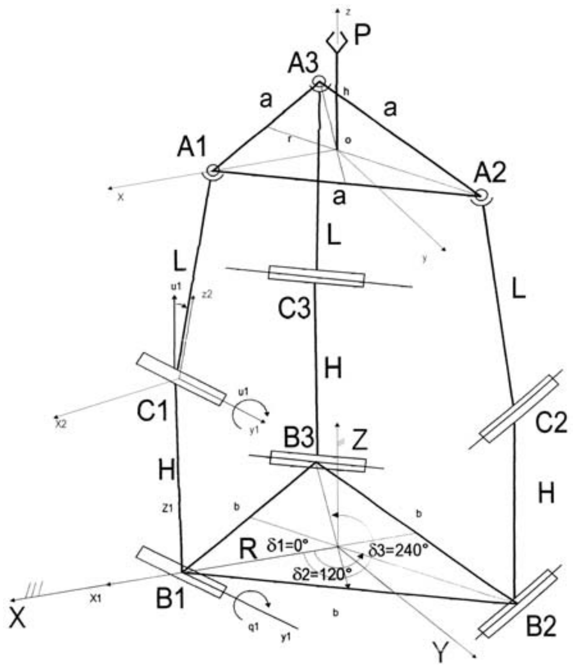

Fig.1 The kinematic scheme of the 3RRS manipulator

Only an arrangement of the kinematic chains in the three joints according to fig. 1 leads to a spatial parallel mechanism with three degrees of freedom in translation [1], [2].

Generalized coordinates of the mechanism (articular coordinates) are: qi - displacements of the driving joints, i = 1,2,3 and generalized coordinates of the mobile platform (operational coordinates) are: Z_P , Ψ, θ, that is elevation of the point P of the center of gripping device relative to to the fixed system OXYZ, precession and nutation Euler angles of the two components (mobile and fixed).

By varying the the coordinates qi, i = 1,2,3, the manipulated object can be positioned in space according the phases of manipulation operation.

2. THE STUDY AND THE GRAPHIC REPRESENTATIONS FOR THE WORKSPACE OF THE 3-RRS MANIPULATOR USING THE MESHING METHOD

For the 3-RRS manipulator as shown in [2], the independent variables are Ψ, θ and Z_P, the other three $\varphi = f(\Psi)$, $X_P = f(\Psi,\theta)$, $Y_P = f(\Psi,\theta)$,being functions of the independent variables. It is noted that in some point of the workspace the mobile platform will have some well-defined orientation [4], [5].

In conclusion will be studied the maximal workspace, i.e. that area that can be touched by the point P at least an orientation of the mobile platform, according to the algorithm described below.

Is required Z_P coordinate of the characteristic point P, and the Ψ and θ parameters varies with the specific step (fixed) between the minimum and maximum required values.

For each pair (Ψ, θ) from the plane Z_P =const, Y_P and X_P is calculated with the formulas (1) or (2) established in article [2].

$$Y_P = \frac{\sqrt{3}}{3} a \cos\psi \sin\psi (\cos\theta - 1) - h\cos\psi \sin\theta \qquad (1)$$

$$X_P = \frac{a\sqrt{3}}{6}\left(1 - 2\sin^2\psi\right)(1 - \cos\theta) + h\sin\theta\sin\psi \qquad (2)$$

For each point (X_P, Y_P) from the plan Z_P = const (where the mobile platform has a specific orientation) is calculated using the inverse geometric model q_i. If:

$$q_{i\min} \le q_i \le q_{i\max} \quad i = 1, 2, 3 \qquad (3)$$

then the point (X_P, Y_P) from the plan Z_P = const belongs of the workspace and be retained as such. The points satisfy the conditions (3) determines a section of the workspace discreet in the form.

By changing the value of Z_P shall be determined a sequence of sections for which borders are the isohypses of the maximal workspace.

To achieve the 3D workspace and its various sections, was developed a program whose menu can be seen in fig. 2.

Can be accessed the following options:

- the 3D representation of the workspace with its rotation and the possibility of choosing a desired viewing angle.
- the graphical representation of the sectional planes: Z = ct , Ψ = ct, θ = ct;
- the graphical representation of a plane containing the Z axis and which makes an angle α (between 0^0 şi 180^0) to the OX axis.

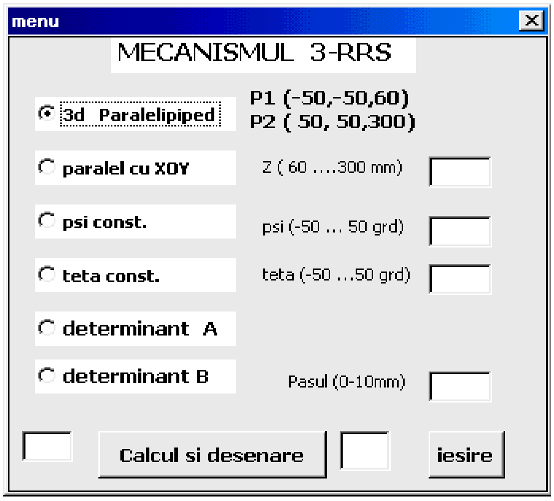

Fig. 2. The interface computer program for calculating workspace of the manipulator 3-RRS

For the 3RRS manipulator having the following constructive data:

$$\begin{cases} a = 200mm, b = 400mm, H = 100mm, \\ L = 150mm, \quad h = 50mm, \\ \delta_1 = \delta_1' = 0^0, \delta_2 = \delta_2' = 120^0, \delta_3 = \delta_3' = 240^0 \\ q_{i,\min} = -80^0, \; q_{i,\max} = 80^0, \end{cases}$$

$$\begin{cases} pas\,\psi = 1^0, \quad \psi_{\min} = -50^0, \quad \psi_{\max} = 50^0 \\ pas\,\theta = 1^0, \quad \theta_{\min} = -50^0, \quad \theta_{\max} = 50^0 \end{cases}$$

have been obtained in the figures 3 – 10, the 3D maximal workspace which is comprised of plane sections Zp = ct. (the red ones), 40mm in step, between the quotas Z = 60 mm, Z = 300 mm. The difference is: in the figure 3 the plane θ = 0^0 is seen (the black section) which in fixed OXYZ system appears as a curved surface that unites "the wing tips" of sections planes Z = ct with the OZ axis, regarded everything of the point (x = 0 mm, y = 0,015 mm Z = 1 mm).

In the figure 4 is shown the projection of the image from figure 3, in the XOY plan.

In the figures 5, 6 and 7 are seen from the point (X=0,05 mm; Y=0,05 mm; Z=1 mm), the planes θ=0^0, θ=-50^0 and θ=50^0 respectively.

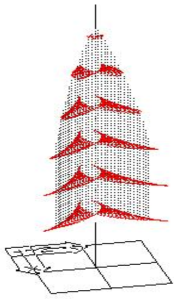

Fig. 3. 3D Workspace for 3-RRS [Zp=ct, θ = 0^0, (0;0,015;1)]

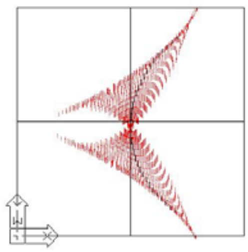

Fig. 4. 3D Workspace for 3-RRS [Zp=ct,θ = 0^0 ,(0;0;1)]

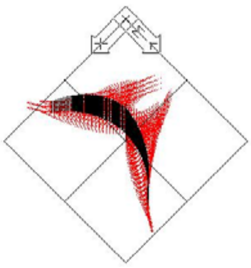

Fig. 5. 3D Workspace for 3-RRS [Zp=ct, θ = 0^0,(0,05; 0,05; 1)]

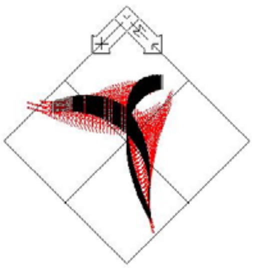

Fig. 6. 3D Workspace for 3-RRS [Zp=ct, θ= 0^0,θ = -50^0, (0,05; 0,05; 1)]

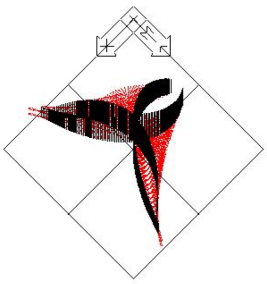

Fig. 7. 3D Workspace for 3-RRS[Zp=ct,θ=0^0,θ=-50^0 ,θ = 50^0 , (0,05; 0,05; 1)]

Fig. 8. 3D Workspace for 3-RRS[Zp = ct, Ψ = -30^0 , (3; - 0,125; 2,68)]

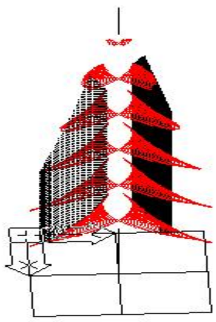

Fig. 9. 3D Workspace for 3-RRS[Zp = ct, Ψ = -30^0 Ψ= 45^0, (3; -0,125; 2,68)]

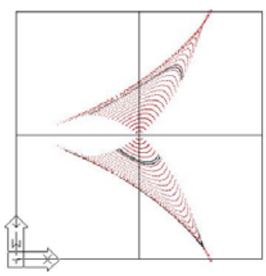

Fig. 10. 3D Workspace for 3-RRS[Zp = ct, Ψ = -30^0, Ψ= 45^0, Ψ = 12^0, (3; -0,125; 2,68)]

In the figures 8 and 9 it can be seen from the point (X=3 mm; Y=-0,125 mm; Z=2,68 mm), in the 3D workspace, added successively, the planes Ψ= -30^0 and Ψ=45^0 respectively (ones the black).

In the figure 10 is shown the projection of the image from figure 9, in the XOY plan to which was added the plane Ψ=12^0.

Was devised a program that calculates the volume of workspace for the 3-RRS mechanism. For structural data shown above, the volume of the workspace is:

$$\text{Vol 3-RRS} = 1384250 \text{ mm}^3$$

The table 1 shows the values of the different planar sectional areas Zp = ct. (between Z = 60 mm and Z = 300 mm), and in figure 11 are plotted these areas.

Table 1.

The values of the different planar sectional areas Z_P

Zp	Aria supraf.	Zp	Aria supraf.
(mm)	**(mm^2)**	**(mm)**	**(mm^2)**
60	10075	180	6550
70	10075	190	5450
80	10075	200	4700
90	10075	210	3750
100	10025	220	2950
110	9975	230	2100
120	9925	240	1450
130	9675	250	800
140	9150	260	350
150	8550	270	0
160	8200	280	0
170	7200	290	0

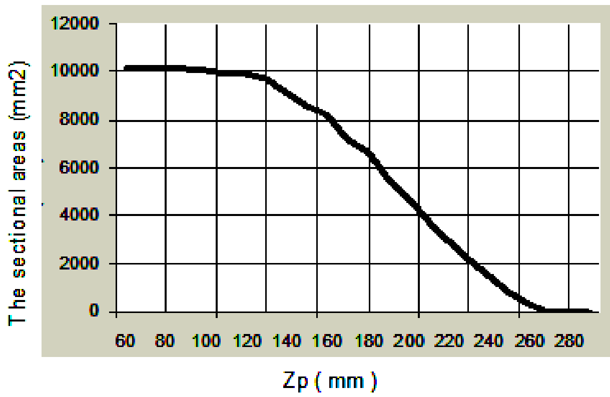

Fig. 10. The areas of the planar sections for different Z_p coordinates

3. CONCLUSIONS

The conclusions stemming from this study relating to the workspace of the manipulator 3RRS are:

• It was determined the maximal workspace in 3D and its different sections (Zp = const, Ψ = const, θ = const), using the meshing method.

• Mathematical models have been proposed and corresponding algorithms of computer-aided solutions to the problem of the workspace and singularities of the mechanism.

• Determination of the workspace through the meshing method, using existing programs (under AUTOCAD) in the initial phase of research, by plotting, brought important clarifications on the nature and extent thereof. The method can also be used with educational purposes.

• The program is conversational and can be easily adapted to solve of the workspace and of other 3-RRS manipulators who have other constructive parameters.

The conclusions drawn from the above-mentioned program are:

• Horizontal sections of the workspace have the form of open wings of a butterfly symmetrical to the axis OX and joined to a very small portion even around the the OZ axis. Such an arrangement is less convenient.

• Planar sectional areas of the workspace (for the constructive dimensions shown), have the maximum value approximately 10.000 mm^2 over a interval of Z 60 - 130 mm, and then begin to slowly decrease with the increase rate Z (Fig. 11).

• The workspace is made up of two identical parts, symmetrically in relation to the axis OX and even joined along the axis OZ on a very narrow area, which is a disadvantage.

• The workspace volume is about 1.384.250 mm^3, which does not mean much for the constructive data.

4. REFERENCES

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Analiza spaţiului de lucru maximal al manipulatorului paralel spaţial 3-RRS

Rezumat: *În acest articol se studiază reprezentarea grafică prin metoda discretizării (pe baza ecuaţiilor de intrare-ieşire si a unui program conceput în AutoCad) a spaţiului de lucru maximal pentru manipulatorul 3RRS care are 3 grade de libertate. Se calculează ariile diferitelor secţiuni plane* Z_P *= cst. şi volumul spaţiului de lucru.*

Cuvinte cheie: *manipulator paralel, metoda discretizării, grade de libertate, unghiurile lui Euler.*

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