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THE ANALYSIS OF SINGULARITIES FOR THE 3RRS SPATIAL PARALLEL MANIPULATOR

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***Abstract**: In this article is studied the singularities of type I and type II for the 3RRS manipulator with 3 degrees of freedom. The calculation of singularities is based on the cancellation of determinants for the Jacobian matrix [A] and [B]. Also, the singularity points of type II were plotted in the manipulator's workspace.*

***Key words:** parallel manipulator, the meshing method, the Jacobian matrix, the input - output equations, the singularities, degrees of freedom.*

1.INTRODUCTION

The figure 1 shows the kinematic scheme of the 3RRS spatial parallel manipulator having three degrees of freedom and three identical kinematic chains [1], [2], [3].

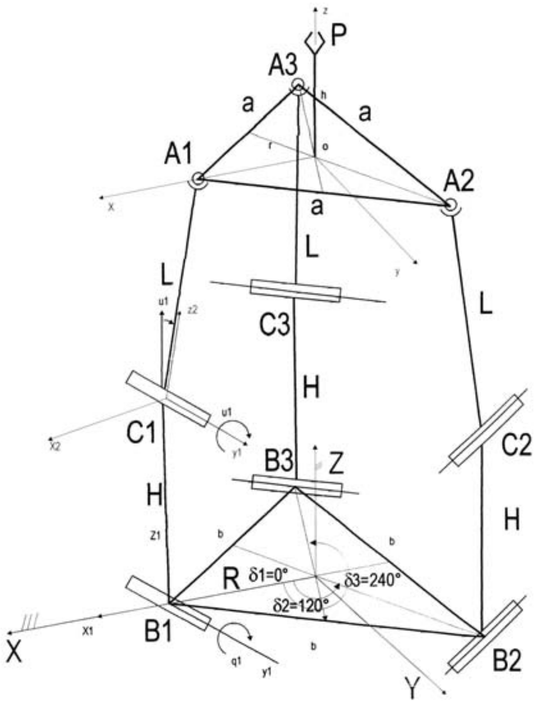

Fig.1 The kinematic scheme of the 3RRS manipulator

Only an arrangement of the kinematic chains in the three joints according to fig. 1 leads to a spatial parallel mechanism with three degrees of freedom in translation [1], [2].

Generalized coordinates of the mechanism (articular coordinates) are: qi - displacements of the driving joints, i = 1,2,3 and generalized coordinates of the mobile platform (operational coordinates) are: Z_P , Ψ, θ, that is elevation of the point P of the center of gripping device relative to to the fixed system OXYZ, precession and nutation Euler angles of the two components (mobile and fixed).

By varying the the coordinates qi, i = 1,2,3, the manipulated object can be positioned in space according the phases of manipulation operation.

2. DETERMINATION OF THE JACOBIAN MATRIX EXPRESSIONS FOR THE 3RRS MANIPULATOR.

For the 3-RRS manipulator as shown in [2], the independent variables are Ψ, θ and Z_P, the other three $\varphi = f(\Psi)$, $X_P = f(\Psi,\theta)$, $Y_P = f(\Psi,\theta)$,being functions of the independent variables. It is noted that in some point of the workspace the mobile platform will have some well-defined orientation [4], [5].

It is starts from input - output equations of the manipulator, deduced in [1] and [2]:

$$\begin{aligned}&2H\left[\left(X_{A_i}-X_{B_i}\right)\cos\delta_i+\left(Y_{A_i}-Y_{B_i}\right)\sin\delta_i\right]\sin q_i\\&-2Z_{A_i}H\cos q_i-L^2+H^2+Z_{A_i}^2+\\&\left[\left(X_{A_i}-X_{B_i}\right)\cos\delta_i+\left(Y_{A_i}-Y_{B_i}\right)\sin\delta_i\right]^2=0\\&\equiv F_i\left(\psi,\theta,Z_P\right)\end{aligned} \quad (1)$$

relations in which X_{Bi} ,Y_{Bi} are known quantities and X_{Ai}, Y_{Ai}, Z_{Ai} have the expressions established in Article [2]:

$$\begin{cases}X_{A_i}(\psi,\theta)=X_P+\alpha_1 x_{A_i}+\alpha_2 y_{A_i}-\alpha_3 h\\Y_{A_i}(\psi,\theta)=Y_P+\beta_1 x_{A_i}+\beta_2 y_{A_i}-\beta_3 h\\Z_{A_i}(Z_P,\psi,\theta)=Z_P+\gamma_1 x_{A_i}+\gamma_2 y_{A_i}-\gamma_3 h\end{cases} \quad (2)$$

It has also been established the following relations [2]:

$$\varphi=-\psi \quad (3)$$

$$\begin{aligned}Y_P(\psi,\theta)=&\frac{\sqrt{3}}{3}a\cos\psi\sin\psi(\cos\theta-1)\\&-h\cos\psi\sin\theta\end{aligned} \quad (4)$$

$$\begin{aligned}X_P(\psi,\theta)=&\frac{a\sqrt{3}}{6}\left(1-2\sin^2\psi\right)(1-\cos\theta)\\&+h\sin\theta\sin\psi\end{aligned} \quad (5)$$

The directional cosines between fixed system oxyz and mobile system OXYZ, taking into account (3) becomes:

$$\begin{cases}\alpha_1=\cos\psi\cos\varphi-\sin\psi\cos\theta\sin\varphi\\\alpha_2=-\cos\psi\sin\varphi-\sin\psi\cos\theta\cos\varphi\\\alpha_3=\sin\psi\sin\theta\end{cases}$$

$$\begin{cases}\beta_1=\sin\psi\cos\varphi+\cos\psi\cos\theta\sin\varphi\\\beta_2=-\sin\psi\sin\varphi+\cos\psi\cos\theta\cos\varphi\\\beta_3=-\cos\psi\sin\theta\end{cases} \quad (6)$$

$$\begin{cases}\gamma_1=\sin\theta\sin\varphi\\\gamma_2=\sin\theta\cos\varphi\\\gamma_3=\cos\theta\end{cases}$$

In the input-output equations of the manipulator, the variables Ψ, θ, Z_P and q_i are functions of time. By derivation of equation (1) gives three equations that can be put in a matrix form (7).

$$[A]\dot{q}=[B]\dot{q}_P \quad (7)$$

In (7), $\dot{q}$ is the column matrix of articular velocities and $\dot{q}_P$ is the column matrix of operational velocities having expressions given by equation (8). The matrices [A] and [B] is called Jacobian matrix, their expressions being given by the relations (9).

$$\dot{q}=\begin{bmatrix}\dot{q}_1\\\dot{q}_2\\\dot{q}_3\end{bmatrix}\quad \dot{q}_P=\begin{bmatrix}\dot{\psi}\\\dot{\theta}\\\dot{Z}_P\end{bmatrix} \quad (8)$$

$$[A]=\begin{bmatrix}\frac{\partial F_1}{\partial q_1}&0&0\\0&\frac{\partial F_2}{\partial q_2}&0\\0&0&\frac{\partial F_3}{\partial q_3}\end{bmatrix}$$

$$[B]=\begin{bmatrix}\frac{\partial F_1}{\partial\psi}&\frac{\partial F_1}{\partial\theta}&\frac{\partial F_1}{\partial Z_P}\\\frac{\partial F_2}{\partial\psi}&\frac{\partial F_2}{\partial\theta}&\frac{\partial F_2}{\partial Z_P}\\\frac{\partial F_3}{\partial\psi}&\frac{\partial F_3}{\partial\theta}&\frac{\partial F_3}{\partial Z_P}\end{bmatrix} \quad (9)$$

The partial derivatives that appear as elements of the matrix [A] is obtained immediately:

$$\frac{\partial F_i}{\partial q_i}=2Z_{Ai}H\sin q_i+2H\begin{bmatrix}\left(X_{Ai}-X_{Bi}\right)\cos\delta_i+\\\left(Y_{Ai}-Y_{Bi}\right)\sin\delta_i\end{bmatrix}\cos q_i \quad (10)$$

where X_{Ai} , Y_{Ai}, Z_{Ai} is calculated by (2); sin q_i and cos qi is calculated with the relations established in article [2].

To calculate elements of matrix [B] is necessary to calculate some partial differential whose expressions are shown below:

$$\frac{\partial \alpha_1}{\partial \psi} = 2\sin\psi\cos\psi(\cos\theta - 1)$$

$$\frac{\partial \alpha_2}{\partial \psi} = (\cos^2\psi - \sin^2\psi)(1 - \cos\theta)$$

$$\frac{\partial \alpha_3}{\partial \psi} = \sin\theta\cos\psi$$

$$\frac{\partial \beta_1}{\partial \psi} = (1 - \cos\theta)(\cos^2\psi - \sin^2\psi)$$

$$\frac{\partial \beta_2}{\partial \psi} = 2\sin\psi\cos\psi(1 - \cos\theta)$$

$$\frac{\partial \beta_3}{\partial \psi} = \sin\psi\sin\theta$$

$$\frac{\partial \gamma_1}{\partial \psi} = -\sin\theta\cos\psi, \quad \frac{\partial \gamma_2}{\partial \psi} = -\sin\theta\sin\psi$$

$$\frac{\partial \gamma_3}{\partial \psi} = 0, \quad \frac{\partial \alpha_1}{\partial \theta} = -\sin^2\psi\sin\theta$$

$$\frac{\partial \alpha_2}{\partial \theta} = \sin\psi\cos\psi\sin\theta, \quad \frac{\partial \alpha_3}{\partial \theta} = \sin\psi\cos\theta$$

$$\frac{\partial \beta_1}{\partial \theta} = \sin\psi\cos\psi\sin\theta, \quad \frac{\partial \beta_2}{\partial \theta} = -\cos^2\psi\sin\theta$$

$$\frac{\partial \beta_3}{\partial \theta} = -\cos\psi\cos\theta, \quad \frac{\partial \gamma_1}{\partial \theta} = -\sin\psi\cos\theta$$

$$\frac{\partial \gamma_2}{\partial \theta} = \cos\theta\cos\psi, \quad \frac{\partial \gamma_3}{\partial \theta} = -\sin\theta$$

$$\frac{\partial X_P}{\partial \psi} = \frac{2\sqrt{3}}{3}a\sin\psi\cos\psi(\cos\theta - 1) + h\sin\theta\cos\psi$$

$$\frac{\partial X_P}{\partial \theta} = \frac{\sqrt{3}}{6}a(1 - 2\sin^2\psi)\sin\theta + h\sin\psi\cos\theta$$

$$\frac{\partial Y_P}{\partial \psi} = \frac{\sqrt{3}}{3}a(\cos\theta - 1)(\cos^2\psi - \sin^2\psi) + h\sin\theta\sin\psi$$

$$\frac{\partial Y_P}{\partial \theta} = \frac{\sqrt{3}}{3}a\cos\psi\sin\psi\sin\theta - h\cos\psi\cos\theta$$

$$\frac{\partial Z_P}{\partial \psi} = 0, \quad \frac{\partial Z_P}{\partial \theta} = 0$$

$$\frac{\partial X_{Ai}}{\partial \psi} = (1 - \cos\theta)\begin{bmatrix} \sin\psi\cos\psi\left(-\frac{2\sqrt{3}}{3}a - 2x_{Ai}\right) + \\ y_{Ai}(\cos^2\psi - \sin^2\psi) \end{bmatrix}$$

$$\frac{\partial X_{Ai}}{\partial \theta} = \sin\theta\begin{bmatrix} \frac{\sqrt{3}}{6}a(1 - 2\sin^2\psi) - x_{Ai}\sin^2\psi \\ + y_{Ai}\sin\psi\cos\psi \end{bmatrix}$$

$$\frac{\partial Y_{Ai}}{\partial \psi} = (1 - \cos\theta)\begin{bmatrix} (\sin^2\psi - \cos^2\psi)\left(\frac{\sqrt{3}}{3}a - x_{Ai}\right) \\ + 2y_{Ai}\sin\psi\cos\psi \end{bmatrix}$$

$$\frac{\partial Y_{Ai}}{\partial \theta} = \sin\theta\begin{bmatrix} \cos\psi\sin\psi\left(x_{Ai} - \frac{\sqrt{3}}{3}a\right) \\ - y_{Ai}\cos^2\psi \end{bmatrix}$$

$$\frac{\partial Z_{Ai}}{\partial \psi} = -\sin\theta(x_{Ai}\cos\psi + y_{Ai}\sin\psi)$$

$$\frac{\partial Z_{Ai}}{\partial \theta} = \cos\theta(-x_{Ai}\sin\psi + y_{Ai}\cos\psi) + h\sin\theta$$

Using all of these partial derivatives in determining expressions of the matrix elements [B] is obtained:

$$\frac{\partial F_i}{\partial \psi} = 2\left\{ \begin{array}{l} \left(\frac{\partial X_{Ai}}{\partial \psi}\cos\delta_i + \frac{\partial Y_{Ai}}{\partial \psi}\sin\delta_i\right) \\ \left[\begin{array}{l} -H\sin q_i + \\ \begin{pmatrix} X_P + \alpha_1 x_{Ai} + \alpha_2 y_{Ai} \\ -\alpha_3 h - X_{Bi} \end{pmatrix} + \\ \begin{pmatrix} \cos\delta_i Y_P + \beta_1 x_{Ai} + \beta_2 y_{Ai} \\ -\beta_3 h - Y_{Bi} \end{pmatrix}\sin\delta_i \end{array} \right] + \\ \frac{\partial Z_{Ai}}{\partial \psi}\begin{pmatrix} -H\cos q_i + Z_P + \gamma_1 x_{Ai} \\ + \gamma_2 y_{Ai} - \gamma_3 h \end{pmatrix} \end{array} \right\} \tag{11}$$

$$\frac{\partial F_i}{\partial Z_P} = 2\begin{pmatrix} -H\cos q_i + Z_P + \gamma_1 x_{Ai} \\ + \gamma_2 y_{Ai} - \gamma_3 h \end{pmatrix} \tag{12}$$

$$\frac{\partial F_i}{\partial \theta} = 2\left\{ \begin{array}{l} \left(\frac{\partial X_{Ai}}{\partial \theta}\cos\delta_i + \frac{\partial Y_{Ai}}{\partial \theta}\sin\delta_i \right) \\ \left[\begin{array}{l} H\sin q_i + \\ \left(\begin{array}{l} X_P + \alpha_1 x_{Ai} + \alpha_2 y_{Ai} \\ -\alpha_3 h - X_{Bi} \end{array} \right) + \\ \left(\begin{array}{l} \cos\delta_i Y_P + \beta_1 x_{Ai} + \beta_2 y_{Ai} \\ -\beta_3 h - Y_{Bi} \end{array} \right)\sin\delta_i \end{array} \right] + \\ \frac{\partial Z_{Ai}}{\partial \theta}\left(\begin{array}{l} -H\cos q_i + Z_P + \gamma_1 x_{Ai} \\ +\gamma_2 y_{Ai} - \gamma_3 h \end{array} \right) \end{array} \right\} \quad (13)$$

Thus the relations (11), (12), (13) provide the expressions of the matrix elements [B]. Having matrix elements expressions [A] and [B] and implicitly the determinants, can be analyze singularities of the mechanism.

The singularities of type I appear when detA=0. In this case the manipulator loses degrees of freedom, and the mobile platform remain fixed when the engines is running.

With a special program was checked in which of the workspace point the determination of the matrix [A] is zero (or has very low value).

Covering workspace with the smallest step on all three axes, it is observed that the values of the determinant of the matrix [A] not in the range (-10^{-2}; 10^{-2}), as shown in Table 1, therefore there is no question the emergence of this type of singularity.

It also observes that if an element of the main diagonal of the determinant [A] has little value (i.e. close to 0,5), the other elements on the main diagonal are very highfinally obtaining very high values for det [A].

Table 1 Some values of det [A] at different points of the workspacefor the 3RRS manipulator.

Date de intrare					**Date de ieşire**	
X_P [mm]	**Y_P [mm]**	**Z_P [mm]**	**ψ [^{0}]**	**θ [^{0}]**	**a_{11} a_{22} a_{33}**	**Val. detA**
-6	-5	72	-46	10	-1065,37 -2,61 9763,71	27141830
-6	5	72	46	-10	-2,61 -1065,37 9763,71	27141830
-1	-8	72	-14	10	-2,61 9763,71 -1065,37	27141830
-1	8	72	14	-10	-1065,37 9763,71 -2,61	27141830
1	-12	78	-4	14	-7697,60 4,20 18979,18	-613149347
1	12	78	4	-14	4,20 -7697,60 18979,18	-613149347
4	8	60	-26	-10	4,20 18979,18 -7697,60	-613149347
4	-8	60	26	10	-7697,60 18979,18 4,20	-613149347
5	8	60	-34	-10	12114,75 9434,57 -3,74	-427460712
5	-8	60	34	10	9434,57 12114,75 -3,74	-427460712
7	16	76	-16	-18	9434,57 -3,74 12114,75	-427460712
7	-16	76	16	18	12114,75 -3,74 9434,57	-427460712
11	14	76	-44	-18	2,72 6651,77 17889,87	324162934
11	-14	76	44	18	6651,77 2,72 17889,87	324162934
22	39	68	-24	-40	6651,77 17889,87 2,72	324162934
22	-39	68	24	40	2,72 17889,87 6651,77	324162934
23	39	68	-36	-40	10864,88 16089,76 -1,00	-174045235

23	-39	68	36	40	10864,88 -1,00 16089,76	-174045235

3. GRAPHICAL REPRESENTATIONS OF SINGULARITIES OF TYPE II IN THE WORKSPACE OF THE 3-RRS MANIPULATOR

The singularities of type II appear when detB=0. In this unwanted situation the mobile platform earn degrees of freedom and can not take the forces or moments in one direction – when the engines are blocked.

A special program was checked in which of the workspace point the determination of the matrix [B] is zero, or has very low value (-10^{-2}; 10^{-2}).

These points were retained, they even determining workspace's OZ axis, colored as illustrated in Fig. 2. These singularity points were obtained by following up workspace with the lowest possible step on the three axes. If they work one step greater the possibility arises loss of these points, they can be skipped.

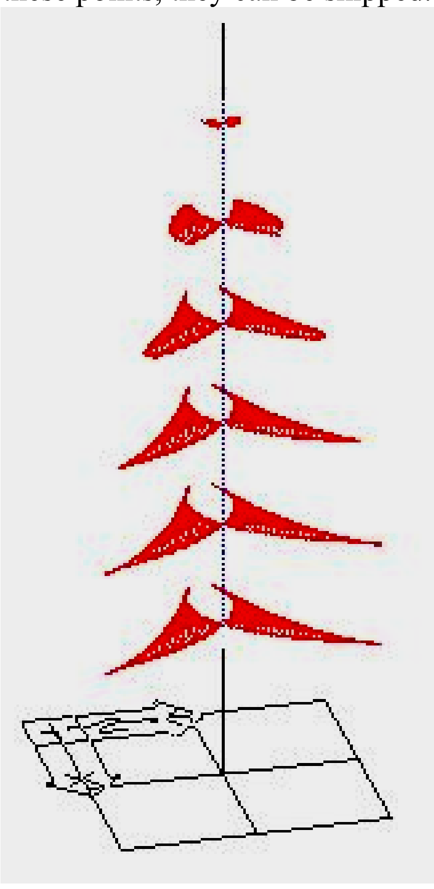

Fig. 2 An indication of singularity in the workspace of the 3RRS manipulator

4. CONCLUSIONS

The conclusions that detach from the study of parallel manipulator with 3 degrees of freedom 3RRS can be summed up as follows:

- Was devised a program for graphical representation of points of singularity (type II) in the 3D workspace, thus obtaining a clear view of workspace areas to be avoided.
- The expressions of Jacobian matrices elements [A] and especially [B] are very complicated and must established multiple computing intermediary relations.
- The singularities of type I does not arise because det [A] has very high values in every workspace's points (Table 1).
- The singularities of type II even arise along the Z axis, i.e. narrow zone connecting the two symmetrical parts of the manipulator workspace. Therefore, the manipulator will work either in one or the other of the plane Y = 0 (taking into account how has been chosen the fixed reference system OXYZ), which is a great disadvantage (fig. 2).

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Analiza singularităților manipulatorului paralel spațial 3RRS

Rezumat: *În acest articol se studiază singularitățile de tipul I şi II ale manipulatorului 3RRS care are 3 grade de libertate. Calculul singularităților se bazează pe anularea determinanților matricelor Jacobiene [A] respectiv [B]. De asemenea, punctele de singularitate de tipul II au fost reprezentate grafic în spațiul de lucru al manipulatorului.*
Cuvinte cheie: *manipulator paralel, matrice Jacobiană, singularități, grade de libertate.*

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