TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 58, Issue I, March, 2015

CORRECTIVE ACTION TO ELIMINATE THE DYNAMIC FAULTS OF ROTATING MACHINERY

Marinel CÎMPAN, Mariana ARGHIR

***Abstract**: This paper contains the theoretical substantiation to eliminate the dynamic behaviour of machinery and equipment, with parts in rotation motion, for the purpose of cataloging backup failures. They analyze the dynamic rotational motion, for the elastic rotor, non symmetricaly, without damping. The main defects have a different meaning as they are determined by this study, and they can be removed by static and dynamic balancing, which apply directly to predictive maintenance, for obtaining the theoretical performance given by this study.*

***Key words:** elastic rotor, non symmetry bearing, without damping.*

1. GENERAL CONSIDERATIONS FOR PREDICTIVE MAINTENANCE

Corrective action to eliminate the main defects that occur in dynamic rotating machines are one of the most important components of the implementation of predictive maintenance and these are designed to bring the machines in a dynamic mode compatible with the system and it is necessary to work at a *Good grade* or *Utilizable*. The main defects that occur in dynamic rotating machines are imbalance and misalignment.

The rotor is a subset of these machines, consisting of a shaft which is one or more discs and executing a movement of rotation around its own axis. As a form, they can be simple or complex, but, regardless of type, being a moving element rotation determines the dynamic properties specific to rotor machines, which do not occur in the other types of machinery or structures.

From great machine rotor class belong the following subclasses of machines: motors, generators, turbines, compressors, pumps and blowers.

2. DYNAMIC ROTARY MACHINES

In machine operation, the rotor is subjected to vibration of bending and twisting. These vibrations are dependent on the geometry of the rotor and bearing type, and the excitatoare forces. The rotor, the precession, he turns his own Foundation. Complexity of dynamic phenomena is increased if it takes into account the fact that the hydro and aerodynamic forces can act upon the rotor, with variable gradient of temperature and pressure fields, electromagnetic fields etc. [Arg 02].

The main features of dynamics of rotor machines, compared with those without rotor systems are:

- All dynamic phenomena that occur during the operation of the rotor machines, are related to the movement of rotation of the rotor, with an energy transfer from the direction of rotation by the movement of precession.
- While, in the case of passive structures, a mode of vibration is characterized by its shape, the active structures, vibration of the rotor movement

is defined by the precession mode. Therefore, the movement of vibration of the rotor comprises two lateral parts, inseparable, it has been agreed to be referred to as vertical-horizontal component of precession.

- The machine rotor dynamics, generally due to the existence of some small differences, without symmetry, system characteristics on both vertical and horizontal directions, precession modes appear in pairs-for example: the first horizontally and the first vertically.
- Another unique feature of the rotor is the fact that they have their own disruptive force, which occurs as a result of the existence of unbalanced masses in rotary motion. This is due to the fact that they correspond to the modes of the rotor itself, and the fact that they are generally poorly written off. As a result, in the study of machines with rotor dynamic interest is granted with priority to the first own modes [Urs 98].

Types of bearings used in rotor machines are [Don 02]: bearings with ball bearings, bearings, sliding bearings, bearing with gas.

The machines with large power, the most commonly encountered are the berings with sliding, without friction, because of their special features: high capacity, high durability, small depreciation, which is the study of this paper.

3. ELASTIC ROTOR, NONE SYMMETRICALLY PLACED BETWEEN BEARINGS, WITHOUT DUMPING

The disc, being placed symmetrically between bearings, vibrates only in his own plan. But when he is placed nearer to the one of bearings, or when prepare at one end of shaft located in the console, it will no longer vibrate only in his own plan. In fact, the two systems are shown in figure 1. No longer have the same critical speed, in spite of the fact that they show the same spring constant shaft, in whose end is same table. It is due that centrifugal force acting on the disc, forces that are no longer found in a single plan, form a torque tending to redress the shaft.

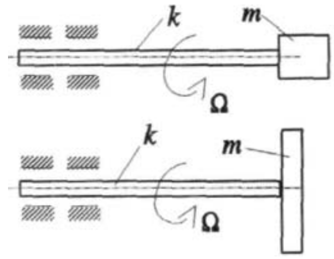


Fig. 1. Discs in Console

Either two coordinate systems, attached point G of the disc (Fig. 1): a system G whose axes remain always parallel to the axes fixed of co-ordinates system, OXYZ, and a system Gxyz whose axes Gx and Gy remain always in the median of the disc, and Gz axis coincides with the axis of rotation of the disc when Ω=0 and the shaft is not deformed, the two systems of reference coincide.

They can note with J_T and J_P the mechanical moments of inertia of the disc in relation to two main axes 1 and 2. For a disc of diameter D and thickness b, there are relations:

$$J_T = \frac{mD^2}{16} + \frac{mb^2}{12}; \quad J_P = \frac{mD^2}{8} \tag{1}$$

For a thin disk, there is the relationship $J_P = 2J_T$. For the determine the moments which acts on the disc can be used the momentum theorem in relation to point G (Fig. 1), as:

$$K_X = J_T \dot{\alpha}_G; \quad K_Y = J_T \dot{\beta}_G; \quad K_Z = J_P \Omega; \tag{2}$$

For the rotated axes (Figure 2) the relations are:

$$\begin{aligned} K_{x'} &= K_X \cos\beta_G + K_Z \sin\beta_G \\ K_{y'} &= K_Y \cos\alpha_G + K_Z \sin\alpha_G \end{aligned} \tag{3}$$

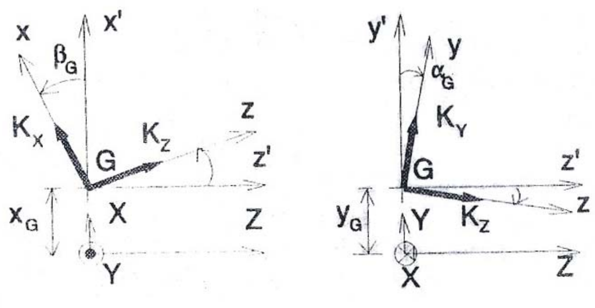


Fig. 2. Moments Acting over the Disc

Taking into account the small deformations, they can note: $\cos\alpha_G \approx \cos\beta_G \approx 1$ and $\sin\alpha_G \approx \alpha_G$, $\sin\beta_G \approx \beta_G$ so, the relations (3) with the help of (2) become:

$$\begin{cases} K\,x' = J_T\,\dot{\alpha}_G + J_P\,\Omega\beta_G \\ K\,y' = J_T\,\dot{\beta}_G - J_P\,\Omega\alpha_G \end{cases} \quad (4)$$

Appling the momentum theorem, they can obtain the moments that act over the disc. They are:

$$\begin{cases} M_G^X = \dot{K}_X = J_T\,\ddot{\alpha}_G + J_P\,\Omega\dot{\beta}_G \\ M_G^Y = \dot{K}_Y = J_T\,\ddot{\beta}_G - J_P\,\Omega\dot{\alpha}_G \end{cases} \quad (5)$$

Over the shaft the moments are:

$$M_C^X = -M_G^X;\; M_C^Y = -M_G^Y \quad (6)$$

But, about the shaft act and the inertia forces and they are:

$$F_C^X = -m\,\ddot{X}_G;\; F_C^Y = -m\,\ddot{Y}_G \quad (7)$$

Those representations are given in the figure 3.

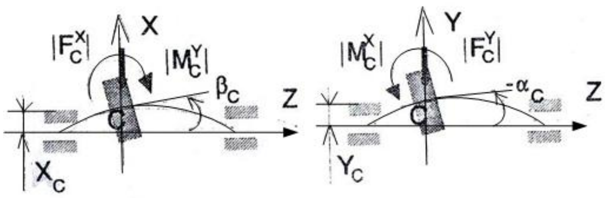


Fig. 3. The Inertia Forces that Act over the Shaft

The motion equations using the influence coefficients have the expressions for the mass centre of the system:

$$\begin{aligned} X_C &= -\left|F_C^X\right|\delta_{11} - \left|M_C^Y\right|\delta_{12} \\ Y_C &= -\left|F_C^Y\right|\delta_{11} + \left|M_C^X\right|\delta_{12} \\ -\alpha_C &= -\left|F_C^Y\right|\delta_{21} + \left|M_C^X\right|\delta_{22} \\ \beta_C &= -\left|F_C^X\right|\delta_{21} - \left|M_C^Y\right|\delta_{22} \end{aligned} \quad (8)$$

The forces, due to the rotor motion, can be presented as in figure 4, where the two points are C – for the mass centre, and G – for the gravity force.

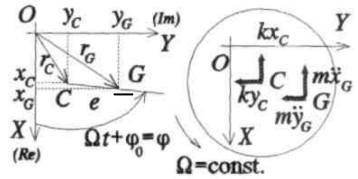


Fig. 4. The Acting Forces over the Elastic Rotor

But, between the two point C and G, with D'Alembert principal, taking into account the acting forces over the elastic rotor, there are the differential equations:

$$\begin{cases} m\,\ddot{x}_G + kx_C = 0 \\ m\,\ddot{y}_G + ky_C = 0 \end{cases} \quad (9)$$

There are the linkage relations for the two points C and G (with e – eccentricity), as:

$$\begin{cases} x_G = x_C + \bar{e}\cos\varphi \\ y_G = y_C + \bar{e}\sin\varphi \end{cases} \Bigg| \times i \quad (10)$$

Additional symbol is $i=\sqrt{-1}$. If it is multiplied the second relation by "i", and adding the second relation (10) with the firts, the result is:

$$r_G = r_C + \bar{e}\,e^{i\varphi} \quad (11)$$

where

$$\begin{cases} r_G = x_G + iy_G \\ r_C = x_C + iy_C \\ e^{i\varphi} = \cos\varphi + i\sin\varphi \end{cases} \quad (12)$$

Looking in the figure 4, the angular velocity Ω is a constant in the rotor motion, and $\varphi = \Omega t + \varphi_0$, hence the (11) relation becomes:

$$r_G = r_C + \bar{e}\,e^{i(\Omega t + \varphi_0)} \quad (13)$$

With (10) and (12) relations, in the (13) vectorial relation and using the (8) system, the result becomes:

$$\begin{aligned} &m\,\delta_{11}\ddot{X}_G + J_T\delta_{12}\ddot{\beta}_G - J_P\Omega\delta_{12}\dot{\alpha}_G + X_C = 0 \\ &m\,\delta_{11}\ddot{Y}_G - J_T\delta_{12}\ddot{\alpha}_G - J_P\Omega\delta_{12}\dot{\beta}_G + Y_C = 0 \;\Big|\; \times i \\ &m\,\delta_{21}\ddot{X}_G + J_T\delta_{22}\beta_G - J_P\Omega\delta_{22}\dot{\alpha}_G + \beta_C = 0 \\ &m\,\delta_{21}\ddot{Y}_G - J_T\delta_{22}\ddot{\alpha}_G - J_P\Omega\delta_{22}\dot{\beta}_G + \alpha_C = 0 \;\Big|\; \times i \end{aligned} \quad (14)$$

In the system need to multiply by "i" the second relation, its addition with first forms an equation, and need to multiply by "i" the fourth relation, its addition with third forms an equation. The result can be obtained if they used the system (12), and both form the system with influence coefficients, for the small angles.

$$\begin{cases} m\ \delta_{11}\ddot{r}_G + J_T\delta_{12}\ddot{\varphi}_G - iJ_P\Omega\delta_{12}\dot{\varphi}_G + r_C = 0 \\ m\ \delta_{21}\ddot{r}_G + J_T\delta_{22}\ddot{\varphi}_G - iJ_P\Omega\delta_{22}\dot{\varphi}_G + \varphi_C = 0 \end{cases} \quad (15)$$

The system (15) can be written in matricial form. It is:

$$\begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}\begin{pmatrix} m & 0 \\ 0 & J_T \end{pmatrix}\begin{Bmatrix} \ddot{r}_C \\ \ddot{\varphi}_C \end{Bmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \quad (16)$$
$$\begin{pmatrix} 0 & 0 \\ 0 & -iJ_P\Omega \end{pmatrix}\begin{pmatrix} \dot{r}_C \\ \dot{\varphi}_C \end{pmatrix} + \begin{pmatrix} r_C \\ \varphi_C \end{pmatrix} = 0$$

3.1. Particular Cases

1. If the disc has an eccentricity. In this situation: $\bar{e} \neq 0$, and the mass centre position vector has a time variation law given by:

$$r_G = r_C + \bar{e}e^{i(\Omega t + \varphi_0)} \quad (17)$$

2. If the disc is mounted in an oblique position on the shaft at an angle of low value, can be written as:

$$\varphi_G = \varphi_C + \psi e^{i(\Omega t + \varphi_0)} \quad (18)$$

In this situation, the relation (16) has a new form, depending of the oblique angle.

$$\begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}\begin{pmatrix} m & 0 \\ 0 & J_T \end{pmatrix}\begin{Bmatrix} \ddot{r}_C \\ \ddot{\varphi}_C \end{Bmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & -iJ_P\Omega \end{pmatrix}$$
$$\begin{pmatrix} \dot{r}_C \\ \dot{\varphi}_C \end{pmatrix} + \begin{pmatrix} r_C \\ \varphi_C \end{pmatrix} = \Omega^2\begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}\begin{Bmatrix} m\bar{e}e^{i\varphi_0} \\ (J_T - J_P)\psi e^{i\psi_0} \end{Bmatrix} \quad (19)$$

For the elastic rotor, the influence coefficients can be expressed using the elasticity's coefficients of the mechanical system. The matrix is:

$$\begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}^{-1} = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \quad (20)$$

Using (20) the (19) becomes:

$$\begin{pmatrix} m & 0 \\ 0 & J_T \end{pmatrix}\begin{Bmatrix} \ddot{r}_C \\ \ddot{\varphi}_C \end{Bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -iJ_P\Omega \end{pmatrix}\begin{pmatrix} \dot{r}_C \\ \dot{\varphi}_C \end{pmatrix} + \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}\begin{pmatrix} r_C \\ \varphi_C \end{pmatrix} =$$
$$= \Omega^2\begin{Bmatrix} m\bar{e}e^{i\varphi_0} \\ (J_T - J_P)\psi e^{i\psi_0} \end{Bmatrix} \quad (21)$$

This is the matrix differential equation for the elastic rotor, subjected to connections without dumping, placed none symmetrically between the bearings. The disc is mounted in an oblique position on the shaft with an angle of low value.

4. CRITICAL SPEED

For the purpose of calculating critical speeds will be considered the second member of (21) equation equal with zero. The expression will be:

$$\begin{pmatrix} m & 0 \\ 0 & J_T \end{pmatrix}\begin{Bmatrix} \ddot{r}_C \\ \ddot{\varphi}_C \end{Bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -iJ_P\Omega \end{pmatrix}\begin{pmatrix} \dot{r}_C \\ \dot{\varphi}_C \end{pmatrix} +$$
$$+ \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}\begin{pmatrix} r_C \\ \varphi_C \end{pmatrix} = 0 \quad (22)$$

The equation (22) is homogeneous equation, in matricial form, of second differential order, and has the solution in general expressions as:

$$r_C = R_C e^{i\varpi t} \qquad \varphi_C = \Phi_C e^{i\varpi t} \quad (23)$$

For obtaining the solution, need to derive two times the unknown quantities, the solution, and its derivatives are introduced into matricial equation (22) obtaining an homogeneous following equation:

$$\begin{pmatrix} k_{11} - m\varpi^2 & k_{12} \\ k_{21} & k_{22} - J_T\varpi^2 + J_P\Omega\omega \end{pmatrix}\begin{Bmatrix} R_C \\ \varphi_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (24)$$

To have different solutions of the routine, it is necessary that the system determinant will be zero:

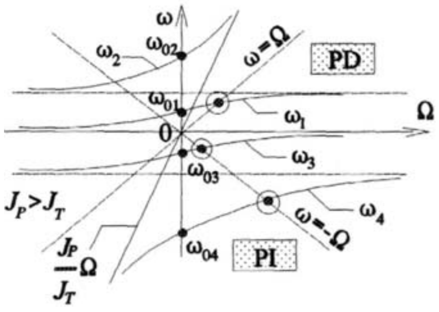
$$\begin{vmatrix} k_{11} - m\varpi^2 & k_{12} \\ k_{21} & k_{22} - J_T\varpi^2 + J_P\Omega\omega \end{vmatrix} = 0 \qquad (25)$$

The solution is:

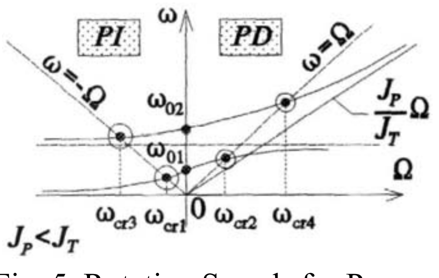
$$\Omega = \frac{k_{12}k_{21} - \left(k_{11} - m\omega^2\right)\left(k_{22} - J_T\omega^2\right)}{J_P\omega\left(k_{11} - m\omega^2\right)} \qquad (26)$$

This is the owner pulsation of the system, or the critical speed for the mechanical system formed with an elastic rotor, none symmetrically placed between bearings, without dumping. Looking at this relations the owner pulsation of the system depends of the elastically characteristics of the shaft, of the mechanical moments of inertia related to the main axes of the system.

Representation of the function $\omega = \omega(\Omega)$ curves resulting from the variation of its own pulses (rotation speeds the precession motion) as a function of speed of rotation for the (26) relation, they can realize the following figures:



a)



b)

Fig. 5. Rotation Speeds for Precession Motion

4. CONCLUSIONS

In the study of the elastic rotor subjected on non symetrically bearing, without damping, there are the following conclusions:

1. For $J_P>J_T$, there is a critical speed in the field of the direct precession motion (PD), and two critical speeds in the field of the reverse precession motion (PI);
2. For $J_P<J_T$, there are two critical speeds in each field, and in the PD, and in the PI;
3. With the increase of Ω shaft speed, the effect of gyroscopic disk give rise to an increase in their own pulses in the movement of direct precession and a decrease in the movement of their precession inversa.
4. Another representation of the graph in figure 5 is shown in figure 6 in which it can be seen much more clearly the sequence critical speeds.

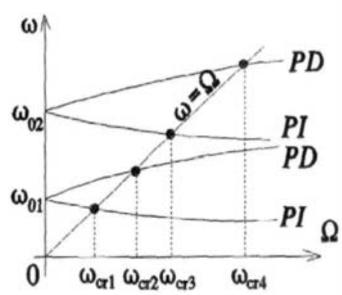


Fig. 6. The Sequence of Critical Speeds

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Studiu asupra mentenanţei predictive pentru un rotor elastic în lagăre elastice cu amortizare

Rezumat: *Lucrarea conţine fundamentarea teoretică pentru masinile si utilajele, cu organe de masini in miscare de rotatie. Se analizează masinile dinamice rotative, prevazute cu rotor elastic in lagare elastic cu amortizare, precum si rotorul simetric cu miscarea de rotatie in lagare elastic cu amortizare. Principalele defecte la echipamentele dinamice rotative sunt eliminate prin echilibrarea statica si dinamica, prin care se aplica direct mentenanta predictivă.*

Marinel CÎMPAN; Phd**.** Stud. Eng., Department of Engineering Mechanical Systems, Technical University of Cluj-Napoca, E-mail: marinel_cimpan@yahoo.com, Phone: 0264.401.759.

Mariana ARGHIR, Prof. Dr. Eng., Department of Engineering Mechanical Systems, UTCN, E-mail: Mariana.Arghir@mep.utcluj.ro , Office Phone 0264.401.657.

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