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MATHEMATICAL MODELING OF A CNC MACHINE STRUCTURE WITH 3+2 AXIS

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Abstract: *In this article the mathematical modeling of a CNC milling machine is presented. The machine is modelled from geometrical perspective and then the movements of the rotary centers are correlated one to others.*
Key words: *3+2 axis CNC mathematical modeling, mathematical modeling.*

1. INTRODUCTION

Mathematical modeling is the description of a system using mathematical concepts and terms. It is used on a large scale to describe systems, relationships and phenomena in both the natural sciences and in industrial engineering and economics. From a mathematical modeling process must follow a mathematical model of case (situation) study the pattern used in technical or economic decisions.

Types of mathematical models:

• Linear models and nonlinear models;
• Deterministic and probabilistic models;
• Discrete and continuous models;
• Algorithmic models;
• Intelligent models;
• Stationary and dynamic models;
• Axiomatic models.

• Linear models operate with mathematical relationships at which variables have exponents equal to unity.

• Nonlinear models operate with mathematical relationships involve variables with exponents above unit.

• Deterministic and probabilistic models are built both for clear processes and for processes with a high degree of uncertainty in the events.

• Deterministic models can be derived from the probabilistic ones when the probabilities of development of events in the analyzed system are equal to unity.

• Discrete or continuous modeling is applied to complete math problems in which phenomena are equally discrete or continuous distribution.

• Static models include events described by variables that do not depend on time.

2. GEOMETRIC MODELING

2.1 Direct geometry equations for 3T structure

To determine the direct geometry equations matrices we will apply the algorithm described in the following location according to [Neg08].

The first main part of the algorithm is devoted to establishing direct geometry equations (equations MGD) corresponding to the initial configuration of the machine tool. Takes up the following steps:

- Consider a structure type of a machine tool: $\{3R;\ RTTR;\ RRTR;\ 5R;\ RTT3R;\ 6R;\cdots\}$

- To establish the kinematic scheme of the machine tools is used nominal geometry matrix:

$$M_{vn}^{(0)} = \underset{[(n+1)\times 6]}{Matrix}\left\{ \begin{bmatrix} \overline{p}_i^{(0)T} & k_i^{(0)T} \end{bmatrix} i = 1 \rightarrow n+1 \right\}^T \quad (1)$$

- Kinematic machine tool scheme is represented in the initial configuration (zero configurations):

$$\overline{\theta}^{(0)} = \begin{bmatrix} q_i = 0 \ ;\ i = 1 \rightarrow n \end{bmatrix}^T \quad (2)$$

- Corresponding to initial configuration of the robot mechanical structure, matrices are established:

$$\left\{ \left\{ i^{(0)} \right\};\ R_{ii-1};\ {}^{i-1}\overline{p}^{(0)}_{ii-1};\ T_{ii-1};\ T_{i0}\quad i=1\rightarrow n+1 \right\} \quad (3)$$

- The geometric center of each motor couplings is attached the mobile system. The orientation of this system coincides with the orientation of fixed reference system, i.e.: $\left\{ i^{(0)} \right\}_{OR} \equiv \left\{ 0 \right\}_{OR}$. This restriction is an important advantage provided by this algorithm. The matrix of rotation and reference position vector of the neighboring system are calculated as follows:

$$R_{ii-1}=I_3;\ R_{i0}=I_3;\quad {}^{i-1}\overline{p}^{(0)}_{ii-1}=\overline{p}^{(0)}_{ii-1}=\overline{p}^{(0)}_{i}-\overline{p}^{(0)}_{i-1} \quad (4);$$

The location of the expression matrix systems is determinate $\{i-1\}\rightarrow\{i\}$ and $\{0\}\rightarrow\{i\}$:

$$T_{ii-1}=\begin{bmatrix} R_{ii-1} & {}^{i-1}\overline{p}^{(0)}_{ii-1} \\ 0\ 0\ 0 & 1 \end{bmatrix};\quad T_{i0}=\begin{bmatrix} R_{i0} & \overline{p}^{(0)}_{i0} \\ 0\ 0\ 0 & 1 \end{bmatrix} \quad (5)$$

For $i=n+1$, location arrays between systems $\{n\}\rightarrow\{n+1\}$ and $\{0\}\rightarrow\{n+1\}$ are:

$$T_{n+1n}=\begin{bmatrix} R_{n+1n} & {}^{n}\overline{p}^{(0)}_{n+1n} \\ 0\ 0\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \overline{n}^{(0)} & \overline{s}^{(0)} & \overline{a}^{(0)} & {}^{n}\overline{p}^{(0)}_{n+1n} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

invariance with $\overline{\theta}_k \neq \overline{\theta}^{(0)}$;

$$T_{n+10}=T_{n0}\cdot T_{n+1n}=\begin{bmatrix} \overline{n}^{(0)} & \overline{s}^{(0)} & \overline{a}^{(0)} & \overline{p}^{(0)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

The second part of the algorithm is established for equations in MGD case: $\overline{\theta}_k \neq \overline{\theta}^{(0)}$

- From the picture of the input data, matrix of the nominal configuration is called:

$$\underset{(m\times n)}{M_{\theta n}}=\left\{ \left[\overline{\theta}^T_k\ k=1\rightarrow m \right]\ \ unde\ \ \overline{\theta}^T_k=\left[q_{ik}\ i=1\rightarrow n \right] \right\} \quad (8)$$

The notations are implemented as follows:

$$\overline{k}_i=\left\{ \overline{x}_i;\overline{y}_i;\overline{z}_i \right\} \quad (9);$$

$$\Delta_i=\left\{ \left\{ 1; i=R \right\};\left\{ 0; i=T \right\} \right\} \quad (10)$$

- $i=1\rightarrow n$ The matrix of rotation and reference position vector of the neighboring system $\{i-1\}\rightarrow\{i\}$ are calculated with the next expressions:

$${}^{i-1}_{i}[R]=R\left(\overline{k}_i;q_i\cdot\Delta_i\right)=\left\{ R\left(\overline{x}_i;q_i\cdot\Delta_i\right);\ R\left(\overline{y}_i;q_i\cdot\Delta_i\right);\ R\left(\overline{z}_i;q_i\cdot\Delta_i\right) \right\} \quad (11)$$

$${}^{i-1}\overline{r}_i={}^{i-1}\overline{p}_{ii-1}={}^{i-1}\overline{p}^{(0)}_{ii-1}+\left(1-\Delta_i\right)\cdot q_i\cdot{}^{i-1}\overline{k}_i={}^{i-1}\overline{p}^{(0)}_{ii-1}+\left(1-\Delta_i\right)\cdot q_i\cdot{}^{i}\overline{k}_i \quad (12)$$

- $i=1\rightarrow n$ The location matrix between neighboring systems are calculated with the next expressions:

$$T_\Delta\left(\overline{k}_i;q_i\right)=\left\{ T_T\left(\overline{k}_i;\left(1-\Delta_i\right)\cdot q_i\right);T_R\left(\overline{k}_i;q_i\cdot\Delta_i\right) \right\}=\begin{bmatrix} R\left(\overline{k}_i;q_i\cdot\Delta_i\right) & \left(1-\Delta_i\right)\cdot q_i\cdot{}^{i}\overline{k}_i \\ 0\ 0\ 0 & 1 \end{bmatrix} \quad (13)$$

$${}^{i-1}_{i}[T]=T_{ii-1}\cdot T_\Delta\left(\overline{k}_i;q_i\right)=\begin{bmatrix} R\left(\overline{k}_i;q_i\cdot\Delta_i\right) & {}^{i-1}\overline{p}^{(0)}_{ii-1}+\left(1-\Delta_i\right)\cdot q_i\cdot{}^{i}\overline{k}_i \\ 0\ 0\ 0 & 1 \end{bmatrix} \quad (14)$$

- $i=1\rightarrow n$ Location matrix system regard to the system is defined by:

$$\left\{ \begin{matrix} {}^{0}_{i}[T] \\ \prod_{j=1}^{i} {}^{j-1}_{j}[T] \end{matrix} \right\}=\begin{bmatrix} {}^{0}_{i}[R] & \overline{p}_i \\ 0\ 0\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{ix} & \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iy} & \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iz} & \begin{matrix}p_{xi}\\p_{yi}\\p_{zi}\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

For i=n+1 the location matrix between systems $\{0\}\rightarrow\{n+1\}$ is determined by the expression:

$$\left\{ \begin{matrix} {}^{0}_{i}[T] \\ \prod_{j=1}^{i} {}^{j-1}_{j}[T] \end{matrix} \right\}=\begin{bmatrix} {}^{0}_{i}[R] & \overline{p}_i \\ 0\ 0\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{ix} & \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iy} & \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iz} & \begin{matrix}p_{xi}\\p_{yi}\\p_{zi}\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$${}^{0}_{n+1}[T]={}^{0}_{n}[T]\cdot T_{n+1n}=\begin{bmatrix} \overline{n} & \overline{s} & \overline{a} & \overline{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} {}^{0}_{n+1}[R] & \overline{p} \\ 0\ 0\ 0 & 1 \end{bmatrix} \quad (17)$$

For $i=1\rightarrow n$ and $j=1\rightarrow i$ location matrix of the mobile system $\{i\}$ in relation to another system $\{j\}$ is determined, using the expression shown below, as follows:

$$\left\{ \begin{matrix} {}^{j}_{i}[T]=\prod_{k=j+1}^{i} {}^{k-1}_{k}[T]\equiv\begin{bmatrix} \prod_{k=j+1}^{i} {}^{k-1}_{k}[R] & \sum_{k=j+1}^{i} {}^{0}_{j}[R]^T\cdot\overline{p}_{kk-1} \\ 0\ 0\ 0 & 1 \end{bmatrix}=\begin{bmatrix} {}^{j}_{i}[R] & {}^{j}\overline{p}_{ij} \\ 0\ 0\ 0 & 1 \end{bmatrix} \\ {}^{j}_{i}[T]=\prod_{k=j+1}^{i} {}^{k-1}_{k}[T]\equiv\begin{bmatrix} {}^{j}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{ix} & {}^{j}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iy} & {}^{j}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}_{iz} & \begin{matrix}{}^{j}p_{xij}\\{}^{j}p_{yij}\\{}^{j}p_{zij}\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \right\} \quad (18)$$

Orientation vector is determined by the identity matrix:

$$\left\{ R\left(\gamma_C-\beta_B-\alpha_A\right)\equiv R\left(\alpha_A-\beta_B-\gamma_C\right) \right\}=\left\{ {}^{0}_{n}[R];{}^{0}_{n+1}[R] \right\};\quad \overline{\psi}=\left(\alpha_A\ \ \beta_B\ \ \gamma_C\right)^T \quad (19)$$

In the last step, the MGD equations will be included in the following generalized matrixes:

$$
{}^{0}\overline{X}=\begin{bmatrix}\overline{p}\\ \overline{\psi}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}p_x & p_y & p_z\end{bmatrix}^T\\ \begin{bmatrix}\alpha_A & \beta_B & \gamma_C\end{bmatrix}^T\end{bmatrix}=\left\{\begin{matrix}\begin{bmatrix}f_j\left(q_i\cdot\delta_i;i=1\rightarrow n\right) & j=1\rightarrow 6\end{bmatrix}^T\\ \delta_i=\left\{\left\{1;j=1\rightarrow 3\right\};\left\{\Delta_i;j=4\rightarrow 6\right\}\right\}\end{matrix}\right\} \quad (20)
$$

…

$$
\overline{p}_T=\begin{pmatrix}p_{xT}\\ p_{yT}\\ p_{zT}\end{pmatrix}=\begin{pmatrix}q_1\\ q_2-b_1-b_2\\ q_3+d_1-d_2\end{pmatrix} \quad (21)
$$

;

$$
\overline{\psi}_T=\begin{pmatrix}\alpha_{xT}\\ \beta_{yS}\\ \gamma_{zT}\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix} \quad (22).
$$

Above equation expresses the position and orientation of the tool $\{S\}$, trough the vectors $\overline{p}_T$ and $\overline{\psi}_T$ compared with the fixed base attached, assuming known movements (generalized coordinates) of couplings engines.

2.2 Direct geometry equations of 2R structure

To determine the direct geometry equations it will be applied location matrices algorithm.

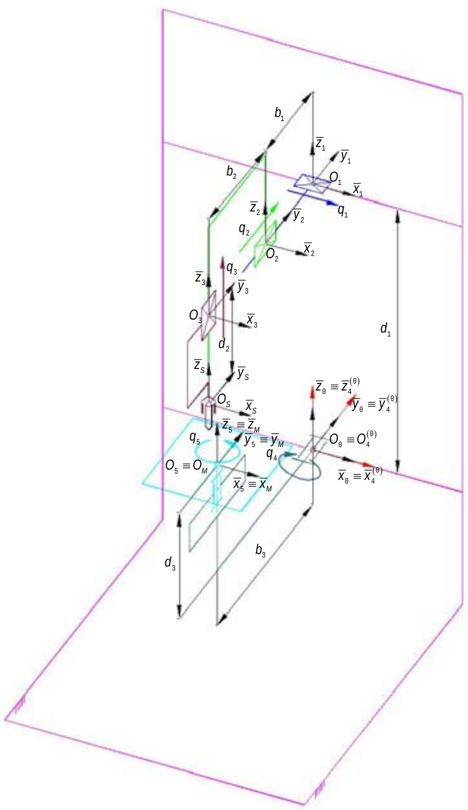

Fig. 1. Kinematic scheme of the mechanical structure at zero configuration.

According to this algorithm, first of all, the kinematic scheme of the configuration at zero is represented (see Fig. 1). This configuration requires that the column vector of the generalized coordinates is:

In this configuration the mobile reference systems are represented, noting that their orientation is identical to orientation of the fixed system, attached to the fixed base and symbolized by $O_0x_0y_0z_0\equiv\{0\}$. As a result, in the initial configuration there identity between the reference systems:

$$
\left\{\left\{i^{(0)}\right\}\ unde\ \ i=4\rightarrow 5\right\}_{OR}\equiv\{0\} \quad (23)
$$

and

$$
\left\{M^{(0)}\right\}\equiv\left\{i^{(0)}\right\}\equiv\{0\} \quad (24)
$$

Where $\{M\}$ represent the mass processing system located in the extreme mechanical structure type 2R.

According location matrices algorithm, matrix transformations are written between reference systems $\{i-1\}\rightarrow\{i\}$ and $\{0\}\rightarrow\{i\}$, where $i=4\rightarrow 5$, and between $\{5\}\rightarrow\{M\}$ respectively $\{0\}\rightarrow\{M\}$.

The location matrix of the system $\{M\}$ attached to the mass processing with respect to a fixed system $\{0\}$.

$$
{}^{0}_{M}[T]={}^{0}_{5}[T]\cdot{}^{5}_{M}[T]=\begin{bmatrix}{}^{0}_{M}[R] & \overline{p}_M\\ 0\ 0\ 0 & 1\end{bmatrix}=\begin{bmatrix}\cos q_4\cdot\cos q_5 & -\cos q_4\cdot\sin q_5 & \sin q_4 & d_3\cdot\sin q_4\\ \sin q_5 & \cos q_5 & 0 & -b_3\\ -\sin q_4\cdot\cos q_5 & \sin q_4\cdot\sin q_5 & \cos q_4 & d_3\cdot\cos q_4\\ 0 & 0 & 0 & 1\end{bmatrix} \quad (25)
$$

Direct geometry equations are obtained from the identity matrix, according to the following:

$$
\overline{p}_M=\begin{pmatrix}p_{xM}\\ p_{yM}\\ p_{zM}\end{pmatrix}=\begin{pmatrix}d_3\cdot\sin q_4\\ -b_3\\ d_3\cdot\cos q_4\end{pmatrix} \quad (26)
$$

$$
\overline{p}_M=\begin{pmatrix}p_{xM}\\ p_{yM}\\ p_{zM}\end{pmatrix}=\begin{pmatrix}d_3\cdot\sin q_4\\ -b_3\\ d_3\cdot\cos q_4\end{pmatrix} \quad (27)
$$

$$
\overline{\psi}_M=\begin{pmatrix}\alpha_{yM}\\ \beta_{xM}\\ \gamma_{zM}\end{pmatrix}=\begin{pmatrix}q_4\\ 0\\ q_5\end{pmatrix} \quad (28)
$$

The equations express the position and orientation of the mass, through the vectors, compared with the fixed base attached, assuming known movements (generalized coordinates) of couplings engines.

2.3 Equations of direct geometry of cooperation between 3T structure and 2R structure.

In the cooperative action of the two structures 3T and 2R is intended that work piece, represented by the system $\{M\}$ to be processed by the tool $\{S\}$, after a given technological process. Given the location arrays between the two systems $\{M\}$ and $\{S\}$ is writing a series of matrix transformations, as follows:

$$ {}_{S}^{0}[T] = {}_{3}^{0}[T] \cdot {}_{S}^{3}[T] \quad (29)$$

$$ {}_{G}^{0}[T] = {}_{5}^{0}[T] \cdot {}_{M}^{5}[T] \quad (30)$$

$$ {}_{M}^{0}[T] = {}_{S}^{0}[T] \cdot {}_{M}^{S}[T] \quad (31)$$

$$ {}_{3}^{0}[T] \cdot {}_{S}^{3}[T] \cdot {}_{M}^{S}[T] = {}_{5}^{0}[T] \cdot {}_{M}^{5}[T] \quad (32)$$

Location matrix array between the systems is explained from the matrix equations:

$$ {}_{S}^{0}[T]^{-1} = \begin{bmatrix} 1 & 0 & 0 & -q_1 \\ 0 & 1 & 0 & b_1 + b_2 - q_2 \\ 0 & 0 & 1 & d_2 - d_1 - q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (33)$$

Performing matrix calculation, we obtain the following expressions for the location matrix components:

$$ {}_{M}^{S}[T] = \begin{bmatrix} \cos q_4 \cdot \cos q_5 & -\cos q_4 \cdot \sin q_5 & \sin q_4 & d_3 \cdot \sin q_4 - q_1 \\ \sin q_5 & \cos q_5 & 0 & b_1 + b_2 - b_3 - q_2 \\ -\sin q_4 \cdot \cos q_5 & \sin q_4 \cdot \sin q_5 & \cos q_4 & d_2 - d_1 + d_3 \cdot \cos q_4 - q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (34)$$

Where ${}_{M}^{S}[R]$ is orientation matrix of the system $\{M\}$ compared to $\{S\}$. The orientation vector ${}^{S}\overline{\psi}_{M}$ is obtained from the identity matrix below, in accordance with the orientation functions algorithm:

$$ {}_{M}^{S}[R] \equiv R(\alpha_y - \beta_x - \gamma_z) = \begin{bmatrix} s\alpha_y \cdot s\beta_x \cdot s\gamma_z + c\alpha_y \cdot c\gamma_z & s\alpha_y \cdot s\beta_x \cdot c\gamma_z - c\alpha_y \cdot s\gamma_z & s\alpha_y \cdot c\beta_x \\ c\beta_x \cdot s\gamma_z & c\beta_x \cdot c\gamma_z & -s\beta_x \\ c\alpha_y \cdot s\beta_x \cdot s\gamma_z - s\alpha_y \cdot c\gamma_z & c\alpha_y \cdot s\beta_x \cdot c\gamma_z + s\alpha_y \cdot s\gamma_z & c\alpha_y \cdot c\beta_x \end{bmatrix} \quad (35)$$

Location matrix between $\{M\}$ and $\{S\}$ is characterized by:

$$ {}_{S}^{M}[R] = \begin{bmatrix} \cos q_4 \cdot \cos q_5 & \sin q_5 & -\sin q_4 \cdot \cos q_5 & \left\{ \begin{matrix} q_1 \cdot \cos q_4 \cdot \cos q_5 + (d_2 - d_1 - q_3) \cdot \sin q_4 \cdot \cos q_5 - \\ -(b_1 + b_2 - b_3 - q_2) \cdot \sin q_5 \end{matrix} \right\} \\ -\cos q_4 \cdot \sin q_5 & \cos q_5 & \sin q_4 \cdot \sin q_5 & \left\{ \begin{matrix} -q_1 \cdot \cos q_4 \cdot \sin q_5 - (d_2 - d_1 - q_3) \cdot \sin q_4 \cdot \sin q_5 - \\ -(b_1 + b_2 - b_3 - q_2) \cdot \cos q_5 \end{matrix} \right\} \\ \sin q_4 & 0 & \cos q_4 & q_1 \cdot \sin q_4 - (d_2 - d_1 - q_3) \cdot \cos q_4 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (36)$$

where ${}_{S}^{M}[R]$ is the orientation matrix of the system $\{S\}$ with respect to $\{M\}$. The orientation vector ${}^{M}\overline{\psi}_{S}$ is obtained from the identity matrix, below, written in accordance with the algorithm of the guidance functions:

$$ {}^{M}\overline{\psi}_{S} = \left({}^{M}\alpha_{zS} \quad {}^{M}\beta_{xS} \quad {}^{M}\gamma_{yS} \right)^T = \left[-q_5 \quad 0 \quad -q_4 \right]^T \quad (37)$$

To determine the position vector ${}^{M}\overline{p}_{SM}$ are written following equations transfer matrix:

$$ {}^{M}\overline{p}_{SM} = -{}_{S}^{M}[R] \cdot {}^{S}\overline{p}_{MS} \equiv {}_{M}^{0}[R]^{-1} \cdot \overline{p}_{S} - {}_{M}^{0}[R]^{-1} \cdot \overline{p}_{M} \equiv {}_{M}^{0}[R]^{-1} \cdot (\overline{p}_{S} - \overline{p}_{M}) \quad (38)$$

$$ {}^{M}\overline{p}_{SM} = \begin{bmatrix} q_1 \cdot \cos q_4 \cdot \cos q_5 + (d_2 - d_1 - q_3) \cdot \sin q_4 \cdot \cos q_5 - \\ -(b_1 + b_2 - b_3 - q_2) \cdot \sin q_5 \\ \hdashline -q_1 \cdot \cos q_4 \cdot \sin q_5 - (d_2 - d_1 - q_3) \cdot \sin q_4 \cdot \sin q_5 - \\ -(b_1 + b_2 - b_3 - q_2) \cdot \cos q_5 \\ \hdashline q_1 \cdot \sin q_4 - (d_2 - d_1 - q_3) \cdot \cos q_4 - d_3 \end{bmatrix} \quad (39)$$

The equations express the position and orientation of the tool $\{S\}$ relative to the reference system attached to work piece $\{M\}$.

2.4 Inverse geometry equations in cooperation of 3T and 2R structures.

In order to determine the geometrical features of the control the above equations are used. The input data are: process tool position and orientation relative to the work piece system are known through numeric values. Thus, the column vector $\{S\}$ with respect to situating system $\{M\}$ is known:

$$ {}^{M}\overline{X}_{SM} = \begin{pmatrix} {}^{M}\overline{p}_{SM} \\ \cdots\cdots\cdots \\ {}^{M}\overline{\psi}_{S} \end{pmatrix} = \begin{bmatrix} \left({}^{M}p_{xSM} \quad {}^{M}p_{ySM} \quad {}^{M}p_{zSM} \right)^T \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \left({}^{M}\alpha_{zS} \quad {}^{M}\beta_{xS} \quad {}^{M}\gamma_{yS} \right)^T \end{bmatrix} \quad (40)$$

The unknown parameters are the geometric control functions contained in the column vector:

$$\overline{\theta} = \left[\left(q_1 \quad q_2 \quad q_3\right) \quad \left(q_4 \quad q_5\right)\right]^T \qquad (41)$$

$$\overline{\theta}_{poz} = \left[\left(q_1 \quad q_2 \quad q_3\right)\right]^T \qquad (42)$$

$$\overline{\theta}_{or} = \left[\left(q_4 \quad q_5\right)\right]^T \qquad (43)$$

$$^{M}\overline{p}_{SM} = \begin{bmatrix} ^{M}p_{xSM} \\ ^{M}p_{ySM} \\ ^{M}p_{zSM} \end{bmatrix} = \begin{bmatrix} q_1 \cdot \cos q_4 \cdot \cos q_5 + \left(d_2 - d_1 - q_3\right) \cdot \sin q_4 \cdot \cos q_5 - \\ -\left(b_1 + b_2 - b_3 - q_2\right) \cdot \sin q_5 \\ \hline -q_1 \cdot \cos q_4 \cdot \sin q_5 - \left(d_2 - d_1 - q_3\right) \cdot \sin q_4 \cdot \sin q_5 - \\ -\left(b_1 + b_2 - b_3 - q_2\right) \cdot \cos q_5 \\ \hline q_1 \cdot \sin q_4 - \left(d_2 - d_1 - q_3\right) \cdot \cos q_4 - d_3 \end{bmatrix} \qquad (44)$$

Generalized coordinate q_1 is determined by the relationship:

$$q_1 = \left[^{M}p_{xSM} \cdot \cos\left(^{M}\alpha_{zS}\right) + {}^{M}p_{ySM} \cdot \sin\left(^{M}\alpha_{zS}\right)\right] \cdot \cos\left(^{M}\gamma_{yS}\right) - \qquad (45)$$
$$- {}^{M}p_{zSM} \cdot \sin\left(^{M}\gamma_{yS}\right) - d_3 \cdot \sin\left(^{M}\gamma_{yS}\right)$$

3. KINEMATIC MODELING

In the cooperative action of the two structures 3T and 2R intended that the work piece, represented by the system $\{M\}$ to be processed by the tool $\{S\}$, after a technological process. For the kinematic modeling of the structure are used an iterative algorithm with starting equations according to Negrean [2].

3.1 Direct kinematics equations of cooperation between 3T and 2R structure.

In the first stage, when $i = 1$ absolute kinematic parameters corresponding to the fixed mechanical structure base are replaced with iterative expressions:

$$\left\{ {}^{0}\overline{\omega}_0 = \overline{0},\ {}^{0}\dot{\overline{\omega}}_0 = \overline{0},\ {}^{0}\overline{v}_0 = 0 \ si \ {}^{0}\dot{\overline{v}}_0 = 0 \right\} \qquad (46)$$

In the last stage, when $i = n$ determinates kinematic parameters that characterize the movement of the end effector absolute:

$$\left\{ {}^{(n)0}\overline{\omega}_n ;\ {}^{(n)0}\dot{\overline{\omega}}_n ;\ {}^{(n)0}\overline{v}_n ;\ {}^{(n)0}\dot{\overline{v}}_n \right\} \qquad (47)$$

These parameters characterize the direct kinematic modeling equations. Angular velocity and linear element i is determined by the relationship:

$$^{0}\overline{\omega}_i = {}^{0}\overline{\omega}_{i-1} + \Delta_i \cdot \dot{q}_i \cdot {}^{0}\overline{k}_i \equiv {}^{0}\overline{\omega}_{i-1} + \Delta_i \cdot {}^{0}_{i}\left[R\right] \cdot \dot{q}_i \cdot {}^{i}\overline{k}_i$$
$$^{0}\overline{v}_i = \left({}^{0}\overline{v}_{i-1} + {}^{0}\overline{\omega}_{i-1} \times \overline{p}_{ii-1}\right) + \left(1 - \Delta_i\right) \cdot \dot{q}_i \cdot {}^{0}\overline{k}_i \qquad (48)$$

To determine the direct kinematic equations of the structure starts from the attached of the mass processing system $\{M\}$ and determine the velocities and relative accelerations of the following items $\{M\}$ according to the tool attachment system $\{S\}$.

In this case orientation of couples to a fixed reference system is taken with negative sense.

Given location matrices between the two systems $\{M\}$ and $\{S\}$ is writing a series of matrix transformations, as follows:

$$^{0}_{S}\left[T\right] = {}^{0}_{M}\left[T\right] \cdot {}^{M}_{S}\left[T\right] \qquad (49)$$

$$^{M}_{S}\left[T\right] = {}^{0}_{M}\left[T\right]^{-1} \cdot {}^{0}_{S}\left[T\right] \qquad (50)$$

The location matrix between the systems $\{M\} \to \{S\}$ is characterized by the following components:

$$^{M}_{S}\left[T\right] = \begin{bmatrix} ^{M}_{S}\left[R\right] & ^{M}\overline{p}_{SM} \\ 0\ 0\ 0 & 1 \end{bmatrix} \qquad (51)$$

$$^{M}_{S}\left[R\right] = {}^{0}_{M}\left[R\right]^{-1} \cdot {}^{0}_{S}\left[R\right] = \begin{bmatrix} \cos q_4 \cdot \cos q_5 & \sin q_5 & -\sin q_4 \cdot \cos q_5 \\ -\cos q_4 \cdot \sin q_5 & \cos q_5 & \sin q_4 \cdot \sin q_5 \\ \sin q_4 & 0 & \cos q_4 \end{bmatrix} \qquad (52)$$

3.2 The angular velocity of rotation

The angular velocity of rotation $\{S\}$ is determined in relation to the system $\{M\}$ as follows:

$$^{4}_{5}\left[R\right]^{-1} \cdot {}^{4}\overline{\omega}_4 = \begin{bmatrix} \cos q_5 & \sin q_5 & 0 \\ -\sin q_5 & \cos q_5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \cdot \dot{q}_4 \qquad (53)$$

The final expression for the angular speed of rotation $\{S\}$ is compared with the system $\{M\}$:

$$^{M}\overline{\omega}_S = \begin{pmatrix} ^{M}\omega_{Sx} \\ ^{M}\omega_{Sy} \\ ^{M}\omega_{Sz} \end{pmatrix} = \begin{pmatrix} -\sin q_5 & 0 \\ -\cos q_5 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \dot{q}_4 \\ \dot{q}_5 \end{pmatrix} = \begin{pmatrix} -\dot{q}_4 \cdot \sin q_5 \\ -\dot{q}_4 \cdot \cos q_5 \\ -\dot{q}_5 \end{pmatrix} \quad (54)$$

3. CONCLUSIONS

The mathematical model offers an overview of the movement for all axis of the machine. This model constitutes the basis for command and control of the machine. The programming of the machine is based on this model, offering the best description of the machining capabilities for a certain machine.

The model can be an universal model and restricting certain axis of movement can be reduce to the desired machine configuration or it could be a specific model for a family of machines.

The current model is based on a 3+2 axis machine, general for the 3+2 case, but the configuration of the machine is generally described and it is adapted trough restrictions to current case machine. In the section with cooperation between 3 translations (3T) and 2 rotations (2R) any 3+2 axis CNC machine with has 3 translations and 2 rotations could be modeled.

4. REFERENCES

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Modelarea matematică a unei maşini CNC cu structura în 3+2 axe

Rezumat: În acest articol este prezentată modelarea matematica a unei maşini de frezat CNC avand 3+2 axe. Maşina este modelata din punct de vedere geometric urmând ca apoi deplasarea centrilor motrici sa fie corelați unul cu celălalt pentru a obtine cazul specific urmărit.

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