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NEW FORMULATIONS ON THE ACCELERATION ENERGY OF FIRST AND SECOND ORDER APPLIED IN ANALYTICAL MECHANICS

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Abstract: *This paper is devoted to the presentation of new formulations on the energies of higher order that are used in the dynamic study of mechanical systems. Integral part of these mechanical systems is the mechanical robot structures, on which an application will be presented in order to highlight the importance of the energies of higher order regarding the dynamic behavior.*
All this leads to a more precise control on the transitory motion phases.

Key words: *matrix exponentials; dynamics; energies of higher order; multibody systems; robot;*

1. INTRODUCTION

The first one, this paper, is focused on the acceleration energies of higher order. Known aspects, but also new ones are presented about them. Acceleration energies of first order and of second order are given in both *explicit and matrix forms*. It is important of mentioned that, this application regarding the theoretical and experimental aspects, presented in this paper, are based on the differential equations of arm motion for a 5R serial robot FANUC, that equations are not presented here.

2. ADVANCED SYSTEMS KINEMATICS

2.1 Forward in Advanced Kinematics

Based on new formulations by the first author, regarding matrix exponentials in according to [1], [2] and [4], in the following are defined the forward geometry and the kinematics equations. There are considerate the aspects from variation principles, applied in advanced dynamics of the mechanical system.

Therefore, the matrix exponentials [1] and their associated transformations are included in the Algorithm of Matrix Exponential in Forward Kinematics (abbreviated MEK), whose main steps are described in the following section.

2.2 The Algorithm of Matrix Exponentials

The matrix of the nominal geometry [1], [2], corresponding to the initial configuration of MBS (Fig.1) with the screw parameters, also named homogeneous coordinates, is completed as:

$$M_{vn}^{(0)**} \underset{\left[(n+1)\times 9\right]}{=Matrix} \left\{ \left[\bar{p}_i^{(0)T} \quad k_i^{(0)T} \quad \bar{v}_i^{(0)} \right] i = 1 \to n+1 \right\}^T \quad (1)$$

where $M_{vn}^{(0)**}$ - represents the nominal geometry matrix; $\left\{ \bar{p}_i^{(0)T} = \left[x_i \quad y_i \quad z_i \right]^T \right\}$ is the position vector of the origin of the system $\{i\}$ with respect to $\{0\}$ frame; $\left\{ \bar{k}_i^0 ; \bar{v}_i^{(0)} \right\}$- screw parameters or homogenous coordinate corresponding moving axis; $\bar{k}_i^{(0)}$ represents the unit vector corresponding to each driving axis, $i = 1 \to n$, while:

$$\bar{v}_i^{(0)} = \left\{ \bar{p}_i^{(0)} \times \right\} \bar{k}_i^{(0)} \cdot \Delta_i + \left(1 - \Delta_i\right) \cdot \bar{k}_i^{(0)} \qquad (2)$$

$\Delta_i = \left\{ (1, if\ i = R);(0, if\ i = T) \right\}$ is an operator which marks out the type of joint:
(R-rotation; T-prismatic joint), according to figure 1.
- the matrix of the screw parameters A_i (rel. (3)) is unchangeable for any MBS configuration. This property is an important advantage in the kinematical study of a multibody structure (MBS).

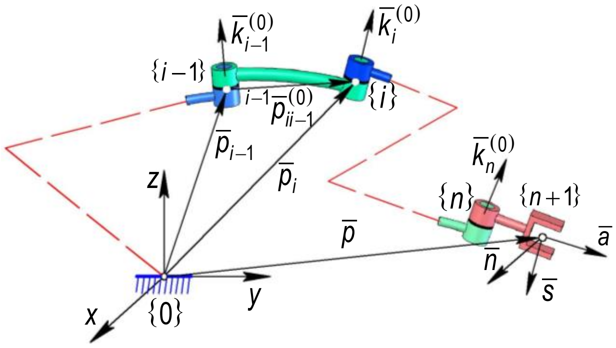

Fig. 1 Sequence from a MBS (MultiBody System).

The expression for this screw matrix is:

$$A_i=\left[\begin{array}{c:c}\left\{\bar{k}_i^{(0)}\times\right\}\Delta_i & \bar{v}_i^{(0)}\\ \hdashline 0\ 0\ 0 & 0\end{array}\right] \tag{3}$$

Throughout the paper, the following notations are implemented:

$$q_{i,j,k,m}\cdot\Delta_{i,j,k,m}=q^{*}_{i,j,k,m}\ ,$$
$$c\left(q_i^{*}\right)\equiv\cos\left(q_i^{*}\right);\ s\left(q_i^{*}\right)\equiv\sin\left(q_i^{*}\right) \tag{4}$$

and $\bar{q}_i$ - angular coordinate in the driving joint; $\bar{\theta}=\left(q_i, for\ i=1\rightarrow n\right)^T$, which defines the column vector of the generalized coordinates, expressing the configuration space, according to analytical mechanics.

The *exponential of the rotation matrix* is defined of the following expression:

$$\left\{\begin{array}{l}R\left(\bar{k}_i\ ;\ q_i^{*}\right)=\exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}\cdot q_i^{*}\right\}=I_3\cdot c\left(q_i^{*}\right)+\\ \quad+\left\{\bar{k}_i^{(0)}\times\right\}s\left(q_i^{*}\right)+\bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\left[1-c\left(q_i^{*}\right)\right]\end{array}\right\} \tag{5}$$

where $\left\{\bar{k}_i^{(0)}\times\right\}$ is the skew-symmetric matrix associated to the unit vector belonging to every kinematical axis. In the position study based on matrix exponentials, a new column vector is established, according to [1] and [2]:

$$\begin{aligned}\bar{b}_i=&\left\{I_3\cdot q_i+\left\{\bar{k}_i^{(0)}\times\right\}\ \left[1-c\left(q_i^{*}\right)\right]+\right.\\ &\left.+\bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\cdot\left[q_i-s\left(q_i^{*}\right)\right]\right\}\cdot\bar{v}_i^{(0)}\end{aligned} \tag{6}$$

where I_3 represents the unit matrix.

The exponentials for the locating matrices (homogeneous transformation matrix), which define the position and the orientation of the frames $\{n\}$ and $\{n+1\}$ with respect to fixed frame $\{0\}$ are given us by relation (7)-(12).

The $\bar{b}_i$ - represents the exponential function colon vector.

$$\begin{aligned}T_{x_0}&=\prod_{i=1}^{x}T_{i\,i-1}=\begin{bmatrix}R_{x_0} & \bar{p}\\ 000 & 1\end{bmatrix}=\left\{\left\{\prod_{i=1}^{n}e^{A_iq_i}\right\}\cdot T_{x_0}^{(0)}=\right.\\ &=e^{\sum_{i=1}^{n}A_iq_i}\cdot T_{x_0}^{(0)}=\left\{\exp\left\{\sum_{i=1}^{n}A_iq_i\right\}\right\}\cdot T_{x_0}^{(0)}\end{aligned} \tag{7}$$

$$where\ x=\{n,n+1\}$$

$$\begin{aligned}&\left\{T_{x0}^{-1}=\prod_{i=x}^{1}T_{ii-1}^{-1}=\begin{bmatrix}R_{x0}^{T} & -R_{x0}^{T}\cdot\bar{p}\\ 0\,0\,0 & 1\end{bmatrix}\right\}=\\ &\left\{\left\{T_{x0}^{(0)}\right\}^{-1}\cdot\prod_{i=n}^{1}\exp\left(-A_i\cdot q_i\right)=\left\{T_{x0}^{(0)}\right\}^{-1}\cdot\exp\left\{-\sum_{i=n}^{1}A_i\cdot q_i\right\}\right\}\end{aligned} \tag{8}$$

where $T_{x_0};T_{x_0}^{-1}$ - represent the exponential expression that characterized the position matrix and $e^{A_iq_i}$ - is rotation exponential matrix.

$$\begin{aligned}R_{x0}&=\left\{\exp\left\{\sum_{i=1}^{n}\left[\bar{k}_i^{(0)}\times\right]\cdot q_i\cdot\Delta_i\right\}\right\}\cdot R_{x0}^{(0)}=\\ &=\prod_{i=1}^{n}\exp\left\{\left[\bar{k}_i^{(0)}\times\right]\cdot q_i\cdot\Delta_i\right\}\cdot R_{x0}^{(0)}\end{aligned} \tag{9}$$

$$R_{x0}^{T}=\left\{R_{x0}^{(0)}\right\}^{T}\cdot\exp\left\{-\left\{\sum_{i=n}^{1}\left\{\bar{k}_i^{(0)}\times\right\}q_i\Delta_i\right\}\right\} \tag{10}$$

where $R_{x_0};R_{x_0}^{T}$ - represent the exponential expression that characterized the rotation matrix, and $\bar{p}$ - position vector reported of the fix system.

$$\bar{p}=\left\{\sum_{i=1}^{n}\begin{array}{l}\left\{\exp\left\{\sum_{j=0}^{i-1}\left[\bar{k}_i^{(0)}\times\right]\cdot q_j\cdot\Delta_j\right\}\right\}\cdot\bar{b}_i+\\ \quad+\left\{\exp\left\{\sum_{i=1}^{n}\left[\bar{k}_i^{(0)}\times\right]\cdot q_i\cdot\Delta_i\right\}\right\}\cdot\bar{p}^{(0)}\cdot\delta_x\end{array}\right\} \tag{11}$$

$$iar\quad\delta_x=\left\{\left(0;\ x=n\right);\quad\left(1;\ x=n+1\right)\right\}$$

$$-R_{x0}^{T}\cdot\bar{p}=\left\{\begin{array}{l}-\sum_{i=n}^{1}\left\{\exp\left\{\sum_{j=i-1}^{0}\left[\bar{k}_i^{(0)}\times\right]\cdot q_j\cdot\Delta_j\right\}\right\}\bar{b}_i-\\ -\exp\left\{-\left\{\sum_{i=n}^{1}\left[\bar{k}_i^{(0)}\times\right]\cdot q_i\cdot\Delta_i\right\}\right\}\bar{p}^{(0)}\cdot\delta_x\\ \\ and\quad\delta_x=\left\{\left\{0;\ x=n\right\};\left\{1;\ x=n+1\right\}\right\}\end{array}\right\} \tag{12}$$

The previous results are further used to determine the forward kinematic equations for any robot structure [6]. In the following, a few expressions from the matrix exponentials algorithm in kinematics [7] (MEK) are presented.

First, an external loop $(i=1\rightarrow n)$ is opened, this yielding to:

$$\underset{(3\times3)}{ME}(V_{i1})=\exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j^*\right\} \tag{13}$$

$$\underset{(3\times6)}{ME}(V_{i2})=\left[\begin{array}{c:c} I_3 & \Delta_i\cdot\left\{\bar{k}_i^{(0)}\times\right\}\end{array}\right] \tag{14}$$

$$\underset{\{6\times[9+3\cdot(n-i)]\}}{ME(V_{i3})}\equiv\left[ME\left(V_{i31}^*\right)\quad ME\left(V_{i32}^*\right)\quad ME\left(V_{i33}^*\right)\right] \tag{15}$$

Inside expression (15), the terms have the following meaning:

$$\left\{\begin{array}{c} ME\left(V_{i31}^*\right)=\begin{bmatrix} I_3 \\ [0]_{3\times3}\end{bmatrix} \\ ME\left(V_{i33}^*\right)=\begin{bmatrix} [0]_{3\times3} \\ \exp\left\{\sum\limits_{k=i}^{n}\left\{\bar{k}_k^{(0)}\times\right\}q_k^*\right\}\end{bmatrix} \\ ME\left(V_{i32}^*\right)=\begin{bmatrix} [0]_{3\times3} \\ \exp\left\{\sum\limits_{m=i-1}^{k-1}\left\{\bar{k}_m^{(0)}\times\right\}q_m^*\cdot\delta_m\right\}\end{bmatrix}\end{array}\right\} \tag{16}$$

Applying a series of matrix transformations, it could obtain the following exponential expressions:

$$\begin{array}{l}\underset{(6\times6)}{ME}\{J_{i1}\}=\begin{bmatrix} ME\{V_{i1}\} & [0] \\ [0] & ME\{V_{i1}\}\end{bmatrix} \\ \underset{(6\times9)}{ME}\{J_{i2}\}=\begin{bmatrix} ME\{V_{i2}\} & [0] \\ [0] & I_3\end{bmatrix}\end{array} \tag{17}$$

$$\underset{\{9\times[12+3\cdot(n-i)]\}}{ME\{J_{i3}\}}=\begin{bmatrix} ME\{V_{i3}\} & [0] \\ [0] & I_3\end{bmatrix} \tag{18}$$

$$M_{iv\omega}=\left\{\begin{array}{c} M_{iv}^T=\left(\left[\bar{v}_i^{(0)T}\ \bar{b}_k{}^T\ \bar{p}_n^{(0)T}\right]\right) \\ \Delta_i\cdot\bar{k}_i^{(0)T}\end{array}\right\} \tag{19}$$

Equation (19) contains the column vector of screw parameters, as well as the position and orientation parameters of the robot's end-effector. They are included [2] in the Jacobian matrix (rel. 21), according to the expressions:

$$\underset{(6\times n)}{{}^0J}\left[\bar{\theta}(t)\right]=\left\{{}^0J_i\left[\bar{\theta}_i(t)\right],\ i=1\rightarrow n\right\} \tag{20}$$

where every column is defined as follows:

$${}^0J_i\left[\bar{\theta}_i(t)\right]=\left[ME\{J_{i1}\}\cdot\begin{bmatrix} ME(V_{i2})ME(V_{i3}) & [0] \\ [0] & I_3\end{bmatrix}\{M_{iv\omega}\}\right] \tag{21}$$

where ${}^0J_i\left[\bar{\theta}_i(t)\right]$- represents the Jacobian matrix of the {i} system reported of the fix {0} system.

2.3 Kinematical Parameters

The absolute values for angular and linear velocities and accelerations, corresponding to any kinetic link $(i=1\rightarrow n)$ from a MBS (Fig. 2) projected on $\{0\}$ frame, are determined in the two variants: *in the explicit* and *matrix form* and all these use the *Exponential Matrix (ME).*

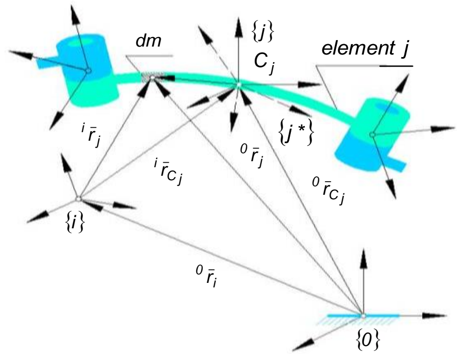

Fig.2 A kinetic ensemble from MBS.

In the follow it will present the kinematical parameters in the *explicit form* used in the moving study.

$$\begin{array}{l}\bar{\omega}_i=\bar{\omega}_0+\sum\limits_{j=1}^{i}\Delta_j\cdot\bar{\omega}_{jj-1} \\ \dot{\bar{\omega}}_i=\dot{\bar{\omega}}_0+\sum\limits_{j=1}^{i}\Delta_j\cdot\left[\bar{\omega}_{j-1}\times\bar{\omega}_{jj-1}+\dot{\bar{\omega}}_{jj-1}\right] \\ \ddot{\bar{\omega}}_i=\ddot{\bar{\omega}}_0+\sum\limits_{j=1}^{i}\Delta_j\cdot\left[\dot{\bar{\omega}}_{j-1}\times\bar{\omega}_{jj-1}+\bar{\omega}_{j-1}\times\dot{\bar{\omega}}_{jj-1}+\ddot{\bar{\omega}}_{jj-1}\right]\end{array} \tag{22}$$

where $\bar{\omega}_i$, $\dot{\bar{\omega}}_i$ and $\ddot{\bar{\omega}}_i$ represent the angular velocity, acceleration of first order and acceleration of second order, according to the rotation of the kinetic ensemble (i), expresses the absolute acceleration of second order of the mass center C_i.

$${}^i\dot{\bar{v}}_i={}^i\dot{\bar{v}}_{C_i}+{}^i\dot{\bar{\omega}}_i\times{}^i\bar{r}_i^*+{}^i\bar{\omega}_i\times{}^i\bar{\omega}_i\times{}^i\bar{r}_i^*$$

$$\left\{\begin{array}{l}{}^i\ddot{\bar{v}}_i={}^i\ddot{\bar{v}}_{C_i}+{}^i\ddot{\bar{\omega}}_i\times{}^i\bar{r}_i^*+2\cdot{}^i\dot{\bar{\omega}}_i\times{}^i\bar{\omega}_i\times{}^i\bar{r}_i^*+{}^i\bar{\omega}_i\times{}^i\dot{\bar{\omega}}_i\times{}^i\bar{r}_i^*+ \\ +{}^i\bar{\omega}_i\times{}^i\bar{\omega}_i\times{}^i\bar{\omega}_i\times{}^i\bar{r}_i^*\end{array}\right\} \tag{23}$$

where $\bar{r}_i,\bar{r}_i^T$ - represent the position and transpose vector of the moving; ${}^i\bar{v}_i$ is the absolute velocity of the elements {i} of multibody; ${}^i\dot{\bar{v}}_i$ the absolute acceleration of the elements {i} of multibody; ${}^i\ddot{\bar{v}}_i$ the absolute acceleration of the second order, according to the moving of the kinetic ensemble (i).

$$\begin{gathered}\bar{v}_{C_i} = \bar{v}_i + \bar{\omega}_i \times \bar{r}_{C_i} = \\ \bar{v}_0 + \sum_{j=1}^{i}\left[\bar{\omega}_{j-1} \times \bar{p}_{jj-1} + \sigma_j \cdot \bar{v}_{jj-1}\right] + \bar{\omega}_i \times \bar{r}_{C_i}\end{gathered} \tag{24}$$

$$\left\{\begin{aligned}&\dot{\bar{v}}_{C_i} = \dot{\bar{v}}_i + \dot{\bar{\omega}}_i x\, \bar{r}_{C_i} + \bar{\omega}_i x \bar{\omega}_i x \bar{r}_{C_i} = \\ &= \dot{\bar{v}}_0^{*} + \sum_{j=1}^{i}\left[\dot{\bar{\omega}}_{j-1} \times \bar{p}_{j\,j-1} + \bar{\omega}_{j-1} \times \bar{\omega}_{j-1} x \bar{p}_{\mathbf{j}\,j-1}\right] + \\ &+ \sum_{j=1}^{i}\sigma_j \cdot \left\{\left[2 \cdot \bar{\omega}_j \times \bar{v}_{j\,j-1} + \dot{\bar{v}}_{j\,j-1}\right] + \right. \\ &\left. + \dot{\bar{\omega}}_i \times \bar{r}_{C_i} + \bar{\omega}_i \times \bar{\omega}_i \times \bar{r}_{C_i}\right\}\end{aligned}\right\} \tag{25}$$

$$\left\{\begin{aligned}&\ddot{\bar{v}}_{C_i} = \sum_{j=1}^{i}\left[\ddot{\bar{\omega}}_{j-1} \times \bar{p}_{j\,j-1} + 2 \cdot \dot{\bar{\omega}}_{j-1} \times \bar{\omega}_{j-1} \times \bar{p}_{j\,j-1} + \right. \\ &\left. + \bar{\omega}_{j-1} \times \dot{\bar{\omega}}_{j-1} \times \bar{p}_{j\,j-1} + \bar{\omega}_{j-1} \times \bar{\omega}_{j-1} \times \bar{\omega}_{j-1} \times \bar{p}_{j\,j-1}\right] + \\ &+ \sum_{j=1}^{i}\sigma_j \cdot \left[2 \cdot \dot{\bar{\omega}}_j \times \bar{v}_{j\,j+1} + 2 \cdot \bar{\omega}_j \times \dot{\bar{v}}_{j\,j+1} + \ddot{\bar{v}}_{j\,j+1}\right] + \\ &+ \ddot{\bar{\omega}}_i \times \bar{r}_{C_i} + 2 \cdot \dot{\bar{\omega}}_i \times \bar{\omega}_i \times \bar{r}_{C_i} + \bar{\omega}_i \times \dot{\bar{\omega}}_i \times \bar{r}_{C_i} + \\ &+ \bar{\omega}_i \times \bar{\omega}_i \times \bar{\omega}_i \times \bar{r}_{C_i}\end{aligned}\right\} \tag{26}$$

The relations (23)-(26) are represented: ${}^{(i)}\bar{v}_{C_i}$ the absolute velocity of the mass center; ${}^{(i)}\dot{\bar{v}}_{C_i}$ the absolute acceleration of the mass center; ${}^{(i)}\ddot{\bar{v}}_{C_i}$ the absolute acceleration of the second order of the mass center, according to the moving of the kinetic ensemble (i), expresses the absolute acceleration of second order of the mass center C_i.

The next explanations are a presentation of the kinematical parameters in the *matrix form*. So, regarding the matrix form of acceleration energy of higher order, it is necessary the presents the follow notions:

$${}^{0}\bar{\omega}_i = \sum_{j=1}^{i}\left\{exp\left\{\sum_{k=1}^{j-1}\left[\bar{k}_k^{(0)} \times\right] \cdot q_k \cdot \Delta_k\right\}\right\} \cdot \bar{k}_j^{(0)} \cdot \dot{q}_j \cdot \Delta_j \tag{27}$$

$${}^{0}\dot{\bar{\omega}}_i = \left\{\sum_{j=1}^{i}\left\{ME\left\{V_{j1}\right\}\ddot{q}_j + ME\left\{\dot{V}_{j1}\right\} \cdot \dot{q}_j\right\}\bar{k}_j^{(0)} \cdot \Delta_j\right\} \tag{28}$$

$${}^{0}\ddot{\bar{\omega}}_i = \left\{\begin{aligned}&\sum_{j=1}^{i}\left\{ME\left\{V_{j1}\right\} \cdot \dddot{q}_j + 2 \cdot ME\left\{\dot{V}_{j1}\right\} \cdot \ddot{q}_j + \right. \\ &\left. + ME\left\{\ddot{V}_{j1}\right\} \cdot \dot{q}_j\right\} \cdot \bar{k}_j^{(0)} \cdot \Delta_j\end{aligned}\right\} \tag{29}$$

$$and\ {}^{0}\bar{v}_i = \sum_{j=1}^{i}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv} \cdot \dot{q}_j\right\} \tag{30}$$

$$\left\{\begin{aligned}&{}^{0}\dot{\bar{v}}_i = \sum_{j=1}^{i}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv} \cdot \ddot{q}_j\right\} + \\ &+ \sum_{j=1}^{i}\left\{\frac{d}{dt}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv}\right\} \cdot \dot{q}_j\right\}\end{aligned}\right\} \tag{31}$$

$$\left\{\begin{aligned}&{}^{0}\ddot{\bar{v}}_i = \sum_{j=1}^{i}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv} \cdot \dddot{q}_j\right\} + \\ &+ 2\sum_{j=1}^{i}\left\{\frac{d}{dt}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv} \cdot \ddot{q}_j\right\} + \right. \\ &\left. + \sum_{j=1}^{i}\left\{\frac{d^2}{dt^2}\left\{\left\{\prod_{k=1}^{3} ME\left\{J_{jk}\right\}\right\} \cdot M_{jv}\right\} \cdot \dot{q}_j\right\}\right\}\end{aligned}\right\} \tag{32}$$

Finally, the exponential matrix (ME) form is:

$${}^{i}\bar{V}_i = ME\left({}^{i}R\right) \cdot {}^{0}\bar{V}_i; \quad where\ \bar{V} = \left\{\bar{\omega}; \dot{\bar{\omega}}; \ddot{\bar{\omega}}; \bar{v}; \dot{\bar{v}}; \ddot{\bar{v}}\right\} \tag{33}$$

For the transfer of the above kinematical parameters from $\{0\}$ in $\{i\}$ moving frame, the next matrix exponential expression is applied in:

$$ME\left({}^{i}R\right) = \left[R_{i0}^{(0)}\right]^{-1} \cdot \prod_{j=i}^{1} \exp\left[-\left(\bar{k}_j^{(0)} \times\right) \cdot q_j \cdot \Delta_j\right] \tag{34}$$

where $ME\left({}^{i}R\right)$- represents the transfer matrix component in the exponential form; $ME\left(V_{ji}\right)$- is the transfer matrix component of the linear velocity, in the exponential form; $ME\left(J_{ji}\right)$- the components of the *i* colon included in the Jacobian matrix of the exponential form; $\bar{q}_j, \dot{\bar{q}}_j, \ddot{\bar{q}}_j, \dddot{\bar{q}}_j$- are represent the general components of the moving, respective of the velocity, and acceleration of the first and second order.

3. ENERGIES OF HIGHER ORDER

The kinetic energy is a fundamental notion in systems dynamics [7] being included in the Lagrange – Euler equations, based on which the dynamic control functions are achieved.

3.2 The acceleration energy of first order

The acceleration energy of first order is defined with:

$$E_A^{(1)i}=\frac{1}{2}\cdot\int\dot{\bar{v}}_i^T\cdot\dot{\bar{v}}_i\cdot dm=\frac{1}{2}\cdot\int Trace\left(\ddot{\bar{r}}_i\cdot\ddot{\bar{r}}_i^T\right)\cdot dm \qquad (35)$$

where $\dot{\bar{v}}_i=\ddot{\bar{r}}_i$ represents the absolute acceleration of the elementary and infinitesimal mass dm, belonging to body, where $i=1\rightarrow n$, and the symbol $Trace$ corresponds to a squared matrix. Performing a few differentials and matrix transformations, it is obtained the expression for the acceleration energy of first order, corresponding to a rigid body, which is in fact, a component of a MBS.

The symbol Δ_m from (36) has the meaning:

$\Delta_m=\{\{\text{-1;General motion}\};\{\text{0;Translation}\};\{\text{1;Rotation}\}\}$

Regarding the energy of first order this could be present in two forms, one in the explicit form and one in the matrix form. In follow, it will be presented in the *explicit form*:

$$\left\{\begin{array}{l}
E_A^{(1)i}=(-1)^{\Delta_m}\cdot\dfrac{1-\Delta_m}{1+3\cdot\Delta_m}\left\{\dfrac{1}{2}\cdot M_i\cdot{}^{(i)}\dot{\bar{v}}_{C_i}^T\cdot{}^{(i)}\dot{\bar{v}}_{C_i}\right\}+\\
+\Delta_m^2\cdot\left\{\dfrac{1}{2}\cdot{}^{(i)}\dot{\bar{\omega}}_i^T\cdot\left[{}^{(i)}I_i^*\cdot{}^{(i)}\dot{\bar{\omega}}_i+\left({}^{(i)}\bar{\omega}_i\times{}^{(i)}I_i^*\cdot{}^{(i)}\bar{\omega}_i\right)\right]+\right.\\
+\Delta_m^2\cdot\left\{\dfrac{1}{2}\cdot{}^{(i)}\dot{\bar{\omega}}_i^T\cdot\left({}^{(i)}\bar{\omega}_i\times{}^{(i)}I_i^*\cdot{}^{(i)}\bar{\omega}_i\right)\right\}+\\
+\Delta_m^2\cdot\left\{\dfrac{1}{2}\cdot{}^{(i)}\bar{\omega}_i^T\cdot\left[{}^{(i)}\bar{\omega}_i^T\cdot Trace\left({}^{(i)}I_{pi}^*\right)\cdot{}^{(i)}\bar{\omega}_i-\right.\right.\\
\left.\left.-{}^{(i)}\bar{\omega}_i^T\cdot{}^{(i)}I_{pi}^*\cdot{}^{(i)}\bar{\omega}_i\right]\cdot{}^{(i)}\bar{\omega}_i\right\}
\end{array}\right\} \qquad (36)$$

In the equation (36) are marked out the mass distribution properties, where M_i is the mass corresponding to each kinetic link of the robot, ${}^iI_i^*$ is the axial and centrifugal inertial tensor and ${}^iI_{pi}^*$ represents the planar centrifugal inertial tensor corresponding to the entire kinetic assembly (i), relative to frame $\{i\}$, applied in the mass center of each link C_i:

$$\left\{{}^iI_i^*=\int\left\{{}^i\bar{r}_i^*\times\right\}\cdot\left\{{}^i\bar{r}_i^*\times\right\}^T dm\right\}=$$

$$=\begin{bmatrix} {}^iI_x^* & -{}^iI_{xy}^* & -{}^iI_{xz}^* \\ -{}^iI_{yx}^* & {}^iI_y^* & -{}^iI_{yz}^* \\ -{}^iI_{zx}^* & -{}^iI_{zy}^* & {}^iI_z^* \end{bmatrix} \qquad (37)$$

$$\left\{{}^iI_{pi}^*=\int{}^i\bar{r}_i^*\cdot{}^i\bar{r}_i^{*T}dm\right\}=\begin{bmatrix} {}^iI_{xx}^* & {}^iI_{xy}^* & {}^iI_{xz}^* \\ {}^iI_{yx}^* & {}^iI_{yy}^* & {}^iI_{yz}^* \\ {}^iI_{zx}^* & {}^iI_{zy}^* & {}^iI_{zz}^* \end{bmatrix} \qquad (38)$$

In the same expression ${}^i\dot{\bar{v}}_{C_i}$ defines the acceleration of the mass center, while ${}^i\bar{\omega}_i$, ${}^i\dot{\bar{\omega}}_i$ and ${}^i\dot{\bar{v}}_i$ are substituted by (22) – (26).

The expression of the acceleration energy of the first order development in the *matrix form* is:

$$\left\{\begin{array}{l}
E_A^{(1)}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t);\ddot{\bar{\theta}}(t)\right]=\\
=\dfrac{1}{2}\cdot\left\{\dddot{\bar{\theta}}^T(t)\cdot M[\bar{\theta}(t)]\cdot\ddot{\bar{\theta}}(t)+\dddot{\bar{\theta}}^T(t)\cdot V\left[\bar{\theta}(t);\dot{\bar{\theta}}^2(t)\right]+\right.\\
\left.\left[\dot{\bar{\theta}}^T(t)\cdot D\left[\bar{\theta}(t);\dot{\bar{\theta}}^2(t)\right]\cdot\dot{\bar{\theta}}(t)\right]\right\}
\end{array}\right\} \qquad (39)$$

where $M(\bar{\theta})$ is the inertial mass matrix, $V\left(\bar{\theta},\dot{\bar{\theta}},\ddot{\bar{\theta}}\right)$ is the Coriolis matrix, and $D\left(\bar{\theta},\dot{\bar{\theta}},\ddot{\bar{\theta}}\right)$ is pseudoinertial matrix correspond of the acceleration energy, and they are the follow relations:

$$M_{ij}=\sum_{k=\max(i;j)}^{n}Trace\left[A_{ki}\cdot{}^kI_{psk}\cdot A_{kj}^T\right]$$

$$M(\bar{\theta})=\underset{(nxn)}{Matrix}\left\{\begin{array}{l} M_{ij}=M_{ji}= \\ \quad=\sum\limits_{k=1}^{n}Trace\left\{A_{ki}\cdot{}^kI_{psk}\cdot A_{kj}^T\right\} \end{array}\quad \begin{array}{l} i=1\rightarrow n \\ j=1\rightarrow n \end{array}\right\} \qquad (40)$$

$$\left\{V_{ijm}=V_{imj}\right\}=\sum_{k=\max(i;j;m)}^{n}Trace\left[A_{ki}\cdot{}^kI_{psk}\cdot A_{kjm}^T\right]$$

$$V\left(\bar{\theta};\dot{\bar{\theta}};\ddot{\bar{\theta}}\right)=\underset{(nxn)}{Matrix}\left\{V_i \quad i=1\rightarrow n\right\}$$

$$V_{i=}\left\{\ddot{\bar{\theta}}^T\cdot\begin{bmatrix}\left\{V_{ijm}=V_{imj}\right\} & j=1\rightarrow n\\ & m=1\rightarrow n\end{bmatrix}\cdot\dot{\bar{\theta}}\right\}^T \tag{41}$$

$$D_{ijlm}=\sum_{k=\max(i;j;l;m)}^{n}Trace\left[A_{kij}\cdot{}^{k}I_{psk}\cdot A_{klm}^{T}\right]$$

$$D\left(\bar{\theta};\dot{\bar{\theta}};\ddot{\bar{\theta}}\right)=\underset{(nxn)}{Matrix}\left\{D_{ij}\quad\begin{matrix}i=1\rightarrow n\\ j=1\rightarrow n\end{matrix}\right\}$$

$$D_{ij}=\ddot{\bar{\theta}}^T\cdot\begin{bmatrix}D_{ijlm} & l=1\rightarrow n\\ & m=1\rightarrow n\end{bmatrix}\cdot\dot{\bar{\theta}} \tag{42}$$

3.3 The acceleration energy of second order

Therefore, the dynamic analysis requires higher order differential, of at least third order. Corresponding to those equations, in the following the new formulation, by the first author, for the acceleration energy of second order is presented here:

$$E_A^{(2)}=\frac{1}{2}\cdot\int{}^{i}\ddot{\bar{v}}_i^T\cdot{}^{i}\ddot{\bar{v}}_i\cdot dm=\frac{1}{2}\cdot Trace\left(\dddot{\bar{r}}_i\cdot\dddot{\bar{r}}_i^T\right)\cdot dm \tag{43}$$

where $\ddot{\bar{v}}_i=\dddot{\bar{r}}_i$ is the absolute acceleration of second order for the elementary mass dm, belonging to body $\left(S_i\right)$, and $i=1\rightarrow n$:

$$\left\{\begin{array}{l}
E_A^{(2)}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t);\ddot{\bar{\theta}}(t);\dddot{\bar{\theta}}(t)\right]=\\
=(-1)^{\Delta_m}\cdot\dfrac{1-\Delta_m}{1+3\cdot\Delta_m}\cdot\displaystyle\sum_{i=1}^{n}\left\{\frac{1}{2}\cdot M_i\cdot{}^{i}\ddot{\bar{v}}_{C_i}^T\cdot{}^{i}\ddot{\bar{v}}_{C_i}\right\}+\\
+\Delta_m^2\cdot\displaystyle\sum_{i=1}^{n}\left\{\frac{1}{2}\cdot{}^{i}\ddot{\bar{\omega}}_i^T\cdot{}^{i}I_i^*\cdot{}^{i}\ddot{\bar{\omega}}_i+2\cdot{}^{i}\bar{\omega}_i^T\cdot\left({}^{i}\ddot{\bar{\omega}}_i\times{}^{i}I_{pi}^*\cdot{}^{i}\dot{\bar{\omega}}_i\right)+\right.\\
+{}^{i}\dot{\bar{\omega}}_i^T\cdot\left({}^{i}\ddot{\bar{\omega}}_i\times{}^{i}I_{pi}^*\cdot{}^{i}\bar{\omega}_i\right)-{}^{i}\bar{\omega}_i^T\cdot\left({}^{i}\ddot{\bar{\omega}}_i^T\cdot{}^{i}I_i^*\cdot{}^{i}\bar{\omega}_i\right)\cdot{}^{i}\bar{\omega}_i+\\
+2\cdot{}^{i}\dot{\bar{\omega}}_i^T\cdot\left({}^{i}\bar{\omega}_i^T\cdot{}^{i}I_i^*\cdot{}^{i}\dot{\bar{\omega}}_i\right)\cdot{}^{i}\bar{\omega}_i+\\
+2\cdot{}^{i}\bar{\omega}_i^T\cdot\left[{}^{i}\dot{\bar{\omega}}_i^T\cdot{}^{i}I_{pi}^*\cdot{}^{i}\dot{\bar{\omega}}_i\right]\cdot{}^{i}\bar{\omega}_i-\\
-5\cdot\left({}^{i}\dot{\bar{\omega}}_i^T\cdot{}^{i}I_{pi}^*\right)\cdot\left({}^{i}\dot{\bar{\omega}}_i^T\cdot{}^{i}\bar{\omega}_i\right)\cdot{}^{i}\bar{\omega}_i+\\
+\dfrac{5}{2}\cdot\left({}^{i}\dot{\bar{\omega}}_i^T\cdot{}^{i}\bar{\omega}_i\right)\cdot Trace\left({}^{i}I_{pi}^*\right)\cdot\left({}^{i}\dot{\bar{\omega}}_i^T\cdot{}^{i}\bar{\omega}_i\right)+\\
+\dfrac{1}{2}\cdot{}^{i}\dot{\bar{\omega}}_i^T\cdot\left[{}^{i}\bar{\omega}_i^T\cdot{}^{i}I_{pi}^*\cdot{}^{i}\bar{\omega}_i\right]\cdot{}^{i}\dot{\bar{\omega}}_i+\\
+{}^{i}\bar{\omega}_i^T\cdot\left[{}^{i}\bar{\omega}_i^T\cdot\left({}^{i}\dot{\bar{\omega}}_i\times{}^{i}I_{pi}^*\cdot{}^{i}\bar{\omega}_i\right)\right]\cdot{}^{i}\bar{\omega}_i+\\
\left.+\dfrac{1}{2}\cdot{}^{i}\bar{\omega}_i^T\cdot\left[{}^{i}\bar{\omega}_i^T\cdot\left({}^{i}\bar{\omega}_i^T\cdot{}^{i}I_i^*\cdot{}^{i}\bar{\omega}_i\right)\cdot{}^{i}\bar{\omega}_i\right]\cdot{}^{i}\bar{\omega}_i\right\}
\end{array}\right\} \tag{44}$$

The relation (44) explain the acceleration energy of second order $E_A^{(2)}$ in the *explicit form.*

The same relation (44) has the *matrix form* given us by relation (45) too.

In the relation (45) the notion have the follow explanation: $M\left(\bar{\theta}\right)$, $V\left(\bar{\theta},\dot{\bar{\theta}},\ddot{\bar{\theta}}\right)$ and $D\left(\bar{\theta},\dot{\bar{\theta}},\ddot{\bar{\theta}}\right)$ are the dynamical matrix form (relation 40-42),

$$\left\{\begin{array}{l}
E_A^{(2)}\left[\bar{\theta}(t);\dot{\bar{\theta}}(t);\ddot{\bar{\theta}}(t);\dddot{\bar{\theta}}(t)\right]=\\
+\dfrac{1}{2}\cdot\dddot{\bar{\theta}}^T(t)\cdot M\left[\bar{\theta}(t)\right]\cdot\dddot{\bar{\theta}}(t)+\\
+3\cdot\dddot{\bar{\theta}}^T(t)\cdot V\left[\bar{\theta}(t);\dot{\bar{\theta}}(t);\ddot{\bar{\theta}}(t)\right]+\\
+\dddot{\bar{\theta}}^T(t)\cdot H\left[\bar{\theta}(t);\dot{\bar{\theta}}^2(t)\right]\cdot\dot{\bar{\theta}}(t)+\\
+3\cdot\ddot{\bar{\theta}}^T(t)\cdot K\left[\bar{\theta}(t);\dot{\bar{\theta}}^4(t)\right]+\\
+\dfrac{9}{2}\cdot\ddot{\bar{\theta}}^T(t)\cdot D\left[\bar{\theta}(t);\dot{\bar{\theta}}(t);\ddot{\bar{\theta}}(t)\right]\cdot\dot{\bar{\theta}}(t)+\\
+\dfrac{1}{2}\cdot\dot{\bar{\theta}}^T(t)\cdot N\left[\bar{\theta}(t);\dot{\bar{\theta}}^4(t)\right]\cdot\dot{\bar{\theta}}(t)
\end{array}\right\} \tag{45}$$

and $H\left(\bar{\theta},\dot{\bar{\theta}}^2\right)$, $K\left(\bar{\theta},\dot{\bar{\theta}}^4\right)$, $N\left(\bar{\theta},\dot{\bar{\theta}}^4\right)$ have the follow expressions:

$$\left\{H_{ijlm}=H_{imlj}\right\}=\sum_{k=\max(i;j;l;m)}^{n}Trace\left[A_{ki}\cdot{}^{k}I_{psk}\cdot A_{kjlm}^{T}\right]$$

$$H\left(\bar{\theta};\dot{\bar{\theta}}^2\right)=\underset{(nxn)}{Matrix}\left\{H_{ij}\quad\begin{matrix}i=1\rightarrow n\\ j=1\rightarrow n\end{matrix}\right\}$$

$$H_{ij}=\dot{\bar{\theta}}^T\cdot\begin{bmatrix}H_{ijlm} & l=1\rightarrow n\\ & m=1\rightarrow n\end{bmatrix}\cdot\dot{\bar{\theta}} \tag{46}$$

$$K_{ijlmp}=\sum_{k=\max(i;j;l;m;p)}^{n}Trace\left[A_{kij}\cdot{}^{k}I_{psk}\cdot A_{klmp}^{T}\right]$$

$$K\left(\bar{\theta};\dot{\bar{\theta}}^4\right)=\underset{(nx1)}{Matrix}\left\{K_i\quad i=1\rightarrow n\right\}$$

$$K_i=\dot{\bar{\theta}}^T\cdot\left\{\dot{\bar{\theta}}^T\cdot\begin{bmatrix}K_{ijlmp} & m=1\rightarrow n\\ & p=1\rightarrow n\end{bmatrix}\cdot\dot{\bar{\theta}};\quad\begin{matrix}j=1\rightarrow n\\ l=1\rightarrow n\end{matrix}\right\}\dot{\bar{\theta}} \tag{47}$$

$$N_{ijlmpr}=\sum_{k=\max(i;j;l;m;p;r)}^{n}Trace\left[A_{kijl}\cdot{}^{k}I_{psk}\cdot A_{kmpr}^{T}\right]$$

$$N\left(\bar{\theta};\dot{\bar{\theta}}^4\right)=\underset{(nxn)}{Matrix}\left\{N_{ij}\quad\begin{matrix}i=1\rightarrow n\\ j=1\rightarrow n\end{matrix}\right\}$$

$$N_{ij}=\dot{\bar{\theta}}^T\cdot\left\{\dot{\bar{\theta}}^T\cdot\begin{bmatrix}N_{ijlmpr} & p=1\rightarrow n\\ & r=1\rightarrow n\end{bmatrix}\cdot\dot{\bar{\theta}};\quad\begin{matrix}l=1\rightarrow n\\ m=1\rightarrow n\end{matrix}\right\}\cdot\dot{\bar{\theta}} \tag{48}$$

4. EXPERIMENTAL METHOD

In the following step, it will be presents an experimental study regarding the demonstration of this accelerations energies of higher-order (1-st and 2-nd) on the FANUC Robot (Fig. 3).

In this way, it will present a correlation between the experimental and theoretical study, that are validated them. The mono-axial accelerometer measured the tangential acceleration of arm moving (0-π rad) of the robot and, it was fixed on the gripper, with a magnet (Fig. 3).

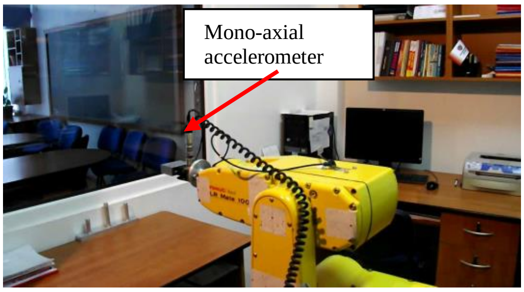

Fig. 3 The Kinematical Structure for the FANUC (5R) Robot.

The generalized variable:

$$\boxed{\left\{q_3(\tau)\,;\,\dot{q}_3(\tau)\,;\,\ddot{q}_3(\tau)\,;\,\dddot{q}_3(\tau)\,;\,\ddddot{q}_3(\tau)\right\}}$$

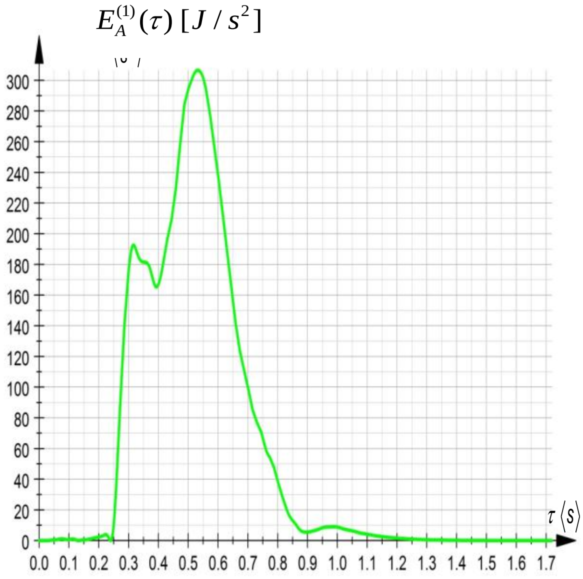

Fig. 4 Time Variation Law of the Acceleration Energy of First Order

The curve of the figure 4, respective acceleration energy of first order $E_A^{(1)}$ have been determinate in the theoretical form with the interpolation function of 5-th order, using the relation (49) and then, this is conform with tangential acceleration measurements introduced in the relation (50):

$$\ddddot{q}_{jik}(\tau)=\frac{\tau_i-\tau}{t_i}\cdot\ddddot{q}_{jik}(\tau_{i-1})+\frac{\tau-\tau_{i-1}}{t_i}\cdot\ddddot{q}_{jik}(\tau_i)\quad(49)$$

$$\dddot{q}_{jik}(\tau)=-\frac{(\tau_i-\tau)^2}{2\cdot t_i}\cdot\ddddot{q}_{ji-1k}+\frac{(\tau-\tau_{i-1})^2}{2\cdot t_i}\cdot\ddddot{q}_{jik}+a_{jik1}$$

$$\ddot{q}_{jik}(\tau)=\frac{(\tau_i-\tau)^3}{6\cdot t_i}\cdot\ddddot{q}_{ji-1k}+\frac{(\tau-\tau_{i-1})^3}{6\cdot t_i}\cdot\ddddot{q}_{jik}+a_{jik1}\cdot\tau+a_{jik2}$$

$$\dot{q}_{jik}(\tau)=-\frac{(\tau_i-\tau)^4}{24\cdot t_i}\cdot\ddddot{q}_{ji-1k}+\frac{(\tau-\tau_{i-1})^4}{24\cdot t_i}\cdot\ddddot{q}_{jik}+a_{jik1}\cdot\frac{\tau^2}{2}+a_{jik2}\cdot\tau+a_{jik3}$$

$$q_{jik}(\tau)=\frac{(\tau_i-\tau)^5}{120\cdot t_i}\cdot\ddddot{q}_{ji-1k}+\frac{(\tau-\tau_{i-1})^5}{120\cdot t_i}\cdot\ddddot{q}_{jik}+a_{jik1}\cdot\frac{\tau^3}{6}+a_{jik2}\cdot\frac{\tau^2}{2}+$$
$$+a_{jik3}\cdot\tau+a_{jik4}$$

In the end of the paper, it will present the acceleration energy of second order applied the robot arm in the rotation moving with 0-π rad.

The presentation corresponds $E_A^{(2)}$ in the explicit form (rel.(50)) and graphical form (Fig.5).

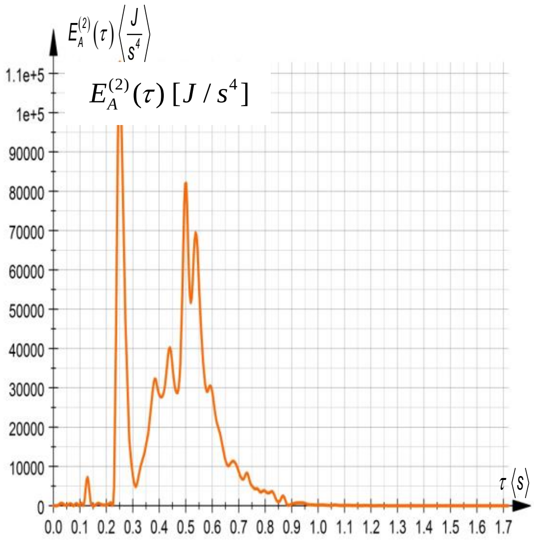

Fig. 5 Time Variation Law of the Acceleration Energy of Second Order

$$E_{Aik}^{(1)}(\tau)=\frac{1}{2}\cdot\left(M_1\cdot x_{C1}^2+M_1\cdot z_{C1}^2+{}^3I_y\right)\cdot\left(\ddot{q}_{3ik}^2(\tau)+\dot{q}_{3ik}^4(\tau)\right)\quad(50)$$

$$\left\{\begin{aligned}&E_{Aik}^{(2)}(\tau)=\frac{1}{2}\cdot\left(M_1\cdot x_{c1}^2+M_1\cdot z_{c1}^2+{}^3I_y\right)\cdot\left[\dddot{q}_{3ik}^2(\tau)-2\cdot\dot{q}_{3ik}^3(\tau)\cdot\dddot{q}_{3ik}(\tau)\right]+\\&+\frac{1}{2}\cdot\left(M_1\cdot x_{c1}^2+M_1\cdot z_{c1}^2+{}^3I_y\right)\cdot\left[9\cdot\dot{q}_{3ik}^2(\tau)\cdot\ddot{q}_{3ik}^2(\tau)+\dot{q}_{3ik}^6(\tau)\right]\end{aligned}\right\}$$

It could observe in the figures 4 and 5, too, that $E_A^{(1)}$ and $E_A^{(2)}$ are the positive values, started in zero and finished in zero. This aspect relived the importance of the transition moving of the multibody system, respective by the start and the stop of the moving, and it relieves an innovative idea of the dynamical mechanical field by the first author.

5. CONCLUSIONS

Within this paper, a few new formulations by the first author, regarding advanced dynamics of multibody systems have been presented. In order to achieve this aim, in the first part of the paper, the forward kinematics equations of multibody systems have been presented as an algorithm. These equations have been developed using matrix exponentials that have undeniable advantages in the matrix study of any complex mechanical system. The kinematic parameters expressions from the first part of the paper have been used to express the energies of higher order. Therefore, new formulations by the first author, for acceleration energy of first and second order have been presented in this paper in the explicit and matrix form. In the fourth part of the paper, an application regarding dynamics equations in the case of a serial structure, for a robot of 5R FANUC type, was presented. As a result of this application, the expressions of generalized driving forces of second order that have the generalized accelerations of second order as components, have been established in the symbolical form.

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Noi formulări privind energia acceleraţiilor de ordinul I-îi şi al II-lea aplicate în mecanica analitică

În cadrul acestei lucrări sunt analizate, pe baza unor noi formulări, noţiuni importante de dinamică, cum ar fi energia acceleraţiilor de ordinul întâi şi de ordinul al doilea, precum şi aplicaţii ale acestora în mecanica analitică. Acestea au fost aplicate pentru sisteme mecanice multicorp, cum ar fi roboţii. La începutul lucrării a fost prezentată pe scurt, aplicarea funcţiilor exponenţiale de matrice. În partea finală a acestei lucrări, formulările au fost aplicate asupra structurii robotice de tipul FANUC.

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