



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics and Engineering
Vol. 58, Issue II, June, 2015

THE MECHANICAL BEHAVIOR AND THE MATHEMATICAL MODELING OF AN INTERVERTEBRAL DISC

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Abstract: *In this paper was developed a study about the dynamical of the intervertebral disc. The three main properties of the bone tissue i.e. material damping, anisotropy and non-homogeneity were took into consideration. We consider the torsional vibration of a intervertebral ring under the action of a moment. Using the Laplace transformation method we obtained the general solution of the components of elastic displacement along radial, circumferential and axial direction. Using the Simulink environment from MatLab Software was given the behavior of the solutions. **Key words:** biosolid mechanics, Laplace Transformation, Simulink.*

1. INTRODUCTION

The anisotropy of the bone tissue can be described in two symmetrical ways. Lang, [10], Katz & Ukraincik, [1] and Yoon & Katz, [2], [3] considered the bones being transversely isotropic and the axis of bone symmetry (the third direction) being unique. All the differences between the elastic properties from the radial and transverse axis, were attributed to porosity gradient from the periosteal to the endorseal zone of the bone. These differences occur due to a defect and they do not modify the basis symmetry.

Intervertebral discs provide flexibility of the spine and transmit and distribute large loads through the spine. To carry out these tasks the intervertebral discs have a particularly complex structure consisting of a gelatinous nucleus pulposus (NP) and the annulus fibrous (AF). However, many people show degenerative changes in the intervertebral discs due to aging or pathological process. These changes affect the composition and structure of the intervertebral discs, and their mechanical functions too. Back pain is often a clinical consequence of disc degeneration.

The intervertebral disc is a complex structure,

and its behavior is governed by its biochemical as well as mechanical composition. Simulation of the disc function is therefore challenging and has led to the development of a number of different approaches to represent its behavior, i.e. the NP has often been modeled as a non-linear incompressible solid governed by a Mooney-Rivlin law or a fluid while the AF was modeled as a homogeneous, isotropic, linear-elastic solid. The highly layered and oriented structure of the AF suggests that its material behavior may be significantly anisotropic. The anisotropic behavior of the AF can be taken into account through discrete representation of the collagen fibers embedded within a homogeneous or hyper elastic matrix (the ground substance), [7, 8]

The present study aims at trying to better understand the individual role of the nucleus pulposus and the annulus fibrous using a simplified geometry and constitutive framework compared to the reality. A mathematical model is developed in the paper for studying the dynamical behavior of the intervertebral discs. In accordance with experimental observations, here are taking into account three principal mechanical properties of bone tissues, such as material damping, anisotropy and inhomogeneity.

Frequency spectra and damping coefficients are computed for two different types of vibratory motion of a specific bone specimen representing the vertebral body of a vertebra.

Let us consider (r, θ, z) the cylindrical coordinates of a specific point from the vertebral body. We will note by $(\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \varepsilon_{\theta z}, \varepsilon_{rz}, \varepsilon_{r\theta})$ the components of the strain tensor and by $(\tau_{rr}, \tau_{\theta\theta}, \tau_{zz}, \tau_{\theta z}, \tau_{rz}, \tau_{r\theta})$ the components of the stress tensor. If the vertebral body is in equilibrium then $\tau_{ij} = \tau_{ji}$, where the stress tensor matrix which describes the stress steady in any point of the vertebra is, [4,5,6] :

$$\tau_{ij} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}. \quad (1)$$

Based on the Hooke law we know that the strain and stress tensor are linear dependent:

$$\tau_{ij} = C_{ijkl} \varepsilon_{kl}, i, j, k, l = 1, 2, 3, \quad (2)$$

where C_{ijkl} is the stiffness tensor.

The strain tensor can be expressed as a linear combination of the stress tensor:

$$\varepsilon_{ij} = \sum_{k=1}^3 \sum_{l=1}^3 S_{ijkl} \tau_{kl}, i, j = 1, 2, 3, \quad (3)$$

where S_{ijkl} is the elastic compliance tensor and its elements are named compliant. The stiffness and the compliance tensors have the same symmetry:

$$C_{ijkl} S_{klmn} = I_{ijmn}. \quad (4)$$

Further, we will use the Voight notation to express the stiffness tensor C_{ijkl} as stiffness matrix C . In this situation the index pairs $ij(kl)$ will be replaced by $\alpha(\beta)$, see table 1. Due to the strain and the stress symmetry we have:

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{jilk}. \quad (5)$$

More than that, the existence of a unique strain potential energy implies that:

$$C_{ijkl} = C_{klij}. \quad (6)$$

$ij(kl)$	$\alpha(\beta)$
11	1
22	2
33	3
23,32	4
13,31	5
12,21	6

Table1. Voight notation - index pairs.

In the simple cases of symmetry for the isotropic elastic solids, the material has only two elastic independent modulus named Lamé constants: λ and μ . For such medium the elastic properties in any point are independent by direction. The Lamé constants depend on the stiffness tensor C_{ijkl} :

$$C_{ijkl} = [\lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{kj})] \varepsilon_{kl}, \quad (7)$$

where δ is the Kronecker symbol:

$$\delta = \begin{cases} 0, & \text{for } i \neq j; \\ 1, & \text{for } i = j. \end{cases} \quad i, j = 1, 2, 3$$

The strong elliptic condition for isotropic material are: $\lambda + 2\mu > 0; \mu > 0$.

2. BASIC EQUATIONS

The stiffness tensor of an isotropic medium in Voight notation in a vertical transversal medium is:

$$C_{\alpha\beta} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix} \quad (8)$$

with $C_{44} = C_{55}, C_{12} = C_{11} - 2C_{66}$.

Taking into account the anisotropic properties and the damping material of the tissue, the constitutive equations of the vertebral body are [5]:

$$\begin{aligned}
\tau_{rr} &= C_{11} \frac{\partial \varepsilon_{rr}}{\partial t} + C_{12} \frac{\partial \varepsilon_{\theta\theta}}{\partial t} + C_{13} \frac{\partial \varepsilon_{zz}}{\partial t}; \\
\tau_{\theta\theta} &= C_{12} \frac{\partial \varepsilon_{rr}}{\partial t} + C_{11} \frac{\partial \varepsilon_{\theta\theta}}{\partial t} + C_{13} \frac{\partial \varepsilon_{zz}}{\partial t}; \\
\tau_{zz} &= C_{13} \frac{\partial \varepsilon_{rr}}{\partial t} + C_{13} \frac{\partial \varepsilon_{\theta\theta}}{\partial t} + C_{33} \frac{\partial \varepsilon_{zz}}{\partial t}; \\
\tau_{\theta z} &= C_{55} \frac{\partial \varepsilon_{\theta z}}{\partial t}; \\
\tau_{rz} &= C_{55} \frac{\partial \varepsilon_{rz}}{\partial t}; \\
\tau_{r\theta} &= C_{66} \frac{\partial \varepsilon_{r\theta}}{\partial t} = \frac{C_{11} - C_{12}}{6} \frac{\partial \varepsilon_{r\theta}}{\partial t}.
\end{aligned} \quad (9)$$

We consider the torsional vibration of a intervertebral ring under the action of a moment $M = e^{i\omega t}$. Thus the components of the elastic displacements and the torsional vibrations will be:

$$\begin{aligned}
U_m(r, \theta, t) &= [u_1(r) + u_2(\theta)]e^{i\omega t}; \\
V_m(r, \theta, t) &= [v_1(r) + v_2(\theta)]e^{i\omega t}; \\
W_m(r, \theta, t) &= [w_1(r) + w_2(\theta)]e^{i\omega t},
\end{aligned} \quad (10)$$

where, U_m is the component of the elastic displacement along radial direction, V_m is the component of the elastic displacement along circumferential direction and W_m is the component of the elastic displacement along axial direction.

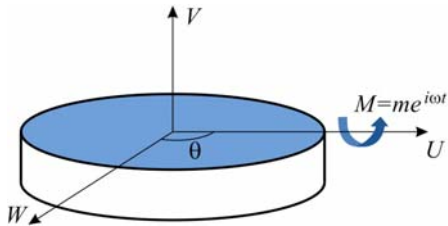


Fig. 1. The model of an intervertebral disc acting by a momentum.

The kinematic relations that connect the components of the displacement vector and those of the strain tensor are:

$$\begin{aligned}
\varepsilon_{rr} &= \frac{\partial U_m}{\partial r} = \frac{\partial u_1}{\partial r} e^{i\omega t}; \\
\varepsilon_{\theta\theta} &= \frac{1}{r} \left(\frac{\partial V_m}{\partial \theta} + U_m \right) \\
&= \frac{1}{r} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) e^{i\omega t};
\end{aligned} \quad (11)$$

$$\begin{aligned}
\varepsilon_{zz} &= \frac{\partial W_m}{\partial z} = 0; \\
\varepsilon_{\theta z} &= \frac{\partial V_m}{\partial z} + \frac{1}{r} \frac{\partial W_m}{\partial \theta} = \frac{1}{r} \frac{\partial w_1}{\partial \theta} e^{i\omega t}; \\
\varepsilon_{zr} &= \frac{\partial W_m}{\partial r} + \frac{\partial U_m}{\partial z} = \frac{\partial w_1}{\partial r} e^{i\omega t}; \\
\varepsilon_{r\theta} &= \frac{1}{r} \left(\frac{\partial U_m}{\partial \theta} - V_m \right) + \frac{\partial V_m}{\partial r} = \frac{1}{r} \left(\frac{\partial u_2}{\partial \theta} - v_1(r) - v_2(\theta) \right) e^{i\omega t} + \frac{\partial v_1}{\partial r} e^{i\omega t}.
\end{aligned} \quad (11)$$

The equations of motion in polar coordinates are:

$$\begin{aligned}
\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{1}{r} (\tau_{rr} - \tau_{\theta\theta}) &= \rho \frac{\partial^2 U_m}{\partial t^2}; \\
\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \left(\frac{\partial \tau_{\theta\theta}}{\partial \theta} + 2\tau_{r\theta} \right) + \frac{\partial \tau_{\theta z}}{\partial z} &= \rho \frac{\partial^2 V_m}{\partial t^2}; \\
\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \left(\frac{\partial \tau_{\theta z}}{\partial \theta} + \tau_{rz} \right) + \frac{\partial \tau_{zz}}{\partial z} &= \rho \frac{\partial^2 W_m}{\partial t^2}.
\end{aligned} \quad (12)$$

Taking into account the derivatives of the strain tensor in report with time, the constitutive equations will be:

$$\begin{aligned}
\tau_{rr} &= \left[C_{11} \frac{\partial u_1}{\partial r} + C_{12} \frac{1}{r} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) \right] i\omega e^{i\omega t}; \\
\tau_{\theta\theta} &= \left[C_{11} \frac{\partial u_1}{\partial r} + C_{11} \frac{1}{r} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) \right] i\omega e^{i\omega t}; \\
\tau_{zz} &= \left[C_{13} \frac{\partial u_1}{\partial r} + C_{13} \frac{1}{r} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) \right] i\omega e^{i\omega t}; \\
\tau_{\theta z} &= C_{55} \frac{1}{r} \frac{\partial w_2}{\partial \theta} i\omega e^{i\omega t}; \\
\tau_{rz} &= C_{55} \frac{1}{r} \frac{\partial w_1}{\partial r} i\omega e^{i\omega t}; \\
\tau_{r\theta} &= \frac{C_{11} - C_{12}}{2} \left[\frac{1}{r} \left(\frac{\partial u_2}{\partial \theta} - v_1(r) - v_2(\theta) \right) + \frac{\partial v_1}{\partial r} \right] i\omega e^{i\omega t}.
\end{aligned} \quad (13)$$

The equations of motion in polar coordinates have the new form:

$$\begin{aligned} & \left[C_{11} \frac{\partial^2 u_1}{\partial r^2} + \frac{C_{12}}{r^2} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) \right] + \\ & + \frac{C_{12}}{r} \frac{\partial u_1}{\partial r} + C_{12} 2r^2 \left(\frac{\partial^2 u_2}{\partial \theta^2} - \frac{\partial v_2}{\partial \theta} \right) + \\ & + \frac{C_{12} - C_{11}}{r^2} \left(\frac{\partial v_2}{\partial \theta} + u_1(r) + u_2(\theta) \right) \Big] i\omega = \\ & = -\rho\omega^2 (u_1(r) + u_2(\theta)); \end{aligned} \quad (14)$$

$$\begin{aligned} & \left\{ \frac{C_{11} - C_{12}}{2} \left[\frac{1}{r^2} \left(\frac{\partial u_2}{\partial \theta} - v_1(r) - v_2(\theta) \right) - \right. \right. \\ & - \frac{1}{r} \frac{\partial v_1}{\partial r} + \frac{\partial^2 v_1}{\partial r^2} \Big] + \frac{1}{r} \left[C_{11} \frac{1}{r} \left(\frac{\partial^2 v_2}{\partial \theta^2} + \frac{\partial u_2}{\partial \theta} \right) + \right. \\ & + (C_{11} - C_{12}) \left[\frac{1}{r} \left(\frac{\partial u_2}{\partial \theta} - v_1(r) - v_2(\theta) \right) + \right. \\ & \left. \left. \left. + \frac{\partial v_1}{\partial r} \right] \right] \right\} i\omega = -\rho\omega^2 (v_1(r) + v_2(\theta)); \end{aligned} \quad (15)$$

$$\begin{aligned} & \left[C_{55} \frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r} \left(C_{55} \frac{1}{r} \frac{\partial^2 w_2}{\partial \theta^2} + \right. \right. \\ & \left. \left. + C_{55} \frac{\partial w_1}{\partial r} \right) \right] i\omega = -\rho\omega^2 (w_1(r) + w_2(\theta)). \end{aligned} \quad (16)$$

3. MAIN RESULTS

In the system formed by the equation (14), (15), (16) we will consider that: $u_1(r) = A \cdot e^\theta$, $v_1(r) = B \cdot e^\theta$; $w_1(r) = A \cdot e^\theta$ with A, B, C real parameters. The new form of the system will be:

$$\begin{aligned} & -\frac{C_{12}}{r^2} \left(\frac{\partial v_2}{\partial \theta} + A e^\theta + u_2(\theta) \right) + \frac{C_{11} - C_{12}}{2r^2} \\ & \left(\frac{\partial^2 u_2}{\partial \theta^2} - \frac{\partial v_2}{\partial \theta} \right) + \frac{C_{12} - C_{11}}{r^2} \left(\frac{\partial v_2}{\partial \theta} + A e^\theta + \right. \\ & \left. + u_2(\theta) \right) - \rho i \omega (A e^\theta + u_2(\theta)) = 0; \end{aligned} \quad (17)$$

$$\begin{aligned} & -\frac{C_{11} - C_{12}}{2r^2} \left(\frac{\partial u_2}{\partial \theta} - B e^\theta - v_2(\theta) \right) + \\ & + \frac{1}{r} \left[\frac{C_{11}}{r} \left(\frac{\partial^2 v_2}{\partial \theta^2} + \frac{\partial u_2}{\partial \theta} \right) + \frac{C_{11} - C_{12}}{r} \left(\frac{\partial u_2}{\partial \theta} - \right. \right. \\ & \left. \left. - B e^\theta - v_2(\theta) \right) \right] - \rho i \omega (B e^\theta + v_2(\theta)) = 0; \\ & \frac{C_{55}}{r^2} \frac{\partial^2 w_2}{\partial \theta^2} - \rho i \omega [C e^\theta + w_2(\theta)] = 0. \end{aligned} \quad (17)$$

The third equation of the above system is a differential equation with complex coefficients:

$$w_2'' - \alpha^2 w_2 = \beta e^\theta \quad (18)$$

$$\text{where: } \alpha^2 = \frac{\rho i \omega r^2}{C_{55}}; \beta = \frac{\rho i \omega C r^2}{C_{55}}.$$

The eigenvalues of the differential equation (20) are: $\lambda_{1,2} = \pm \alpha i$, and the homogeneous solution are: $w_{2o} = C_1 \cos \alpha \theta + C_2 \sin \alpha \theta$.

The particular solution of the differential equation (20) is:

$$w_{2p} = \frac{\beta}{1 + \alpha^2} e^\theta.$$

Therefore, the general solution of the differential equation (20) is:

$$W_2 = C_1 \cos \alpha \theta + C_2 \sin \alpha \theta + \frac{\beta}{1 + \alpha^2} e^\theta. \quad (19)$$

Using the initial conditions: $w_2(0) = 0$; $w_2'(0) = 0$, we obtain for the constants values:

$$C_1 = -\frac{\beta}{1 + \alpha^2}; C_2 = -\frac{\beta}{\alpha(1 + \alpha^2)}.$$

Thus, the component of elastic displacement along axial direction is:

$$\begin{aligned} W_m(r, \theta, t) = & \left(C e^\theta - \frac{\beta}{1 + \alpha^2} \cos \alpha \theta - \right. \\ & \left. - \frac{\beta}{\alpha(1 + \alpha^2)} \sin \alpha \theta + \sin \alpha \theta + \frac{\beta}{1 + \alpha^2} e^\theta \right) e^{i\omega t}. \end{aligned} \quad (20)$$

The first and the second equation lead us to the following system:

$$\begin{aligned}
& \frac{C_{12}-3C_{11}}{2r^2} \frac{\partial v_2}{\partial \theta} + \frac{C_{11}-C_{12}}{2r^2} \frac{\partial^2 u_2}{\partial \theta^2} + \\
& + u_2(\theta) \left(\frac{C_{11}-2C_{12}}{r^2} - \rho i \omega \right) + \\
& + A e^{\theta} \left(-\frac{C_{11}}{r^2} - \rho i \omega \right) = 0 \\
& - \frac{C_{12}-3C_{11}}{2r^2} \frac{\partial u_2}{\partial \theta} + \frac{C_{11}}{r^2} \frac{\partial^2 v_2}{\partial \theta^2} + \\
& + v_2(\theta) \left(\frac{-C_{11}+C_{12}}{r^2} - \rho i \omega \right) + \\
& + B e^{\theta} \left(\frac{-C_{11}+C_{12}}{2r^2} - \rho i \omega \right) = 0.
\end{aligned} \tag{21}$$

The system will be solved using the Laplace transformation with the initial conditions: $v_2(0) = v_2'(0) = 0; u_2(0) = u_2'(0) = 0$.

Using the following notations:

$$\begin{aligned}
a &= \frac{C_{12}-3C_{11}}{2r^2}; b = \frac{C_{11}-C_{12}}{2r^2}; \\
c &= \frac{C_{11}-2C_{12}}{r^2} - \rho i \omega; d = A \left(-\frac{C_{11}}{r^2} - \rho i \omega \right); \\
m &= \frac{C_{11}}{r^2}; n = \frac{C_{12}-C_{11}}{2r^2} - \rho i \omega.
\end{aligned}$$

The system in the Laplace image will be:

$$\begin{aligned}
asV(s) + (bs^2 + c)U(s) &= -\frac{d}{s-1}; \\
(ms^2 + c)V(s) - asU(s) &= -\frac{n}{s-1},
\end{aligned} \tag{22}$$

where $U(s) = L[u_2(\theta)](s)$, $V(s) = L[v_2(\theta)](s)$ are the images of the Laplace transformation. Solving the system (22) we obtain:

$$\begin{aligned}
U(s) &= \frac{-dms^2 + asn - dc}{(s-1)[bms^4 + (bc + cm + a^2)s^2 + c^2]}; \\
V(s) &= \frac{-nbs^2 - ads - nc}{(s-1)[bms^4 + (bc + cm + a^2)s^2 + c^2]}.
\end{aligned} \tag{23}$$

Using the inverse Laplace transformation we obtain the general solution of the elastic

component along radial and circumferential direction.

4. NUMERICAL RESULTS

The stiffness coefficients $C_{\alpha\beta}$ (in GPa) for the human bones were obtained by Van Buskirk & Asman, [9] by measuring the elastic properties of the anisotropic bone, using the technique of the ultrasonic waves propagation:

$$\begin{aligned}
C_{11} &= 20, C_{22} = 21.7, C_{33} = 30, C_{44} = 6.56, \\
C_{55} &= 5.85, C_{66} = 4.74, C_{12} = 10.9, \\
C_{13} &= 11.5, C_{23} = 11.5.
\end{aligned}$$

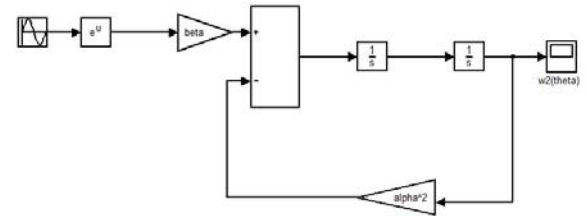


Fig. 2. The Simulink scheme for the elastic displacement along axial direction w_m .

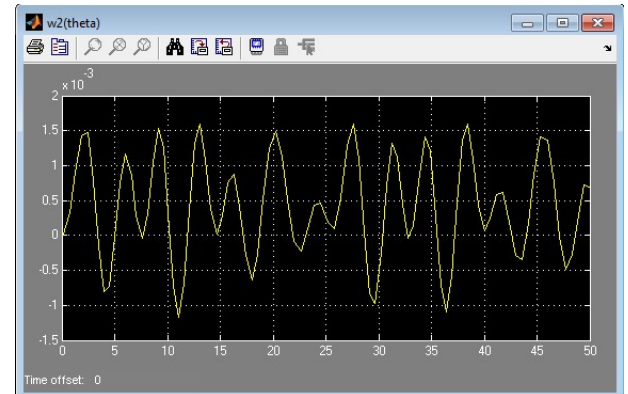


Fig. 3. The variation of elastic displacement along axial direction w_m .

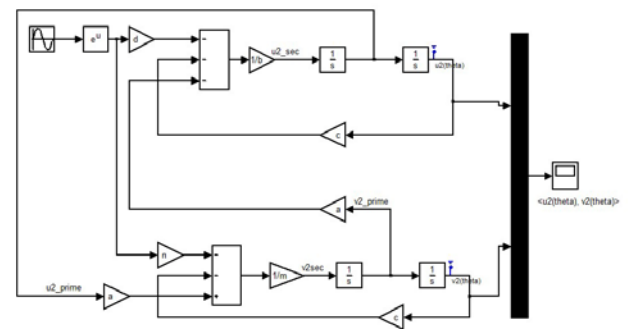


Fig. 4. The Simulink scheme for u_m and v_m .

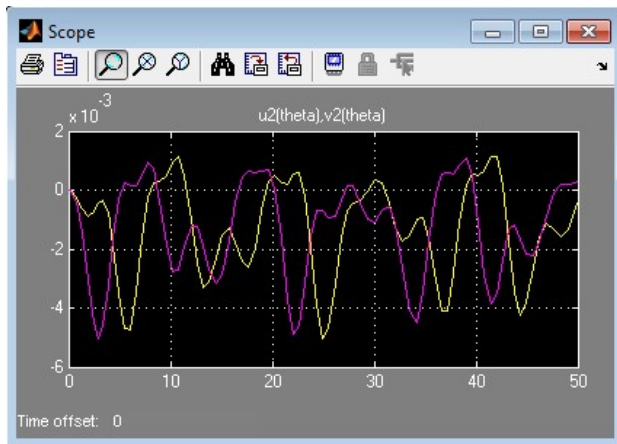


Fig. 5. The variation of elastic displacement along radial u_m and circumferential v_m directions.

5. ACKNOWLEDGEMENT

This paper is supported by the Sectorial Operational Program Human Resources Development (SOP HRD), Financed from the European Social Fund and by the Romanian Government under the project number POSDRU/159/1.5/S/134378.

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Comportamentul mecanic și modelarea matematică a unui disc intervertebral.

Rezumat: În această lucrare a fost dezvoltat un studiu asupra dinamicii discurilor intervertebrale. Principalele trei proprietăți ale osului, precum rezistența materialului, anizotropia și neomogenitatea au fost luate în considerare. A fost abordată vibrația torsională a unui inel intervertebral sub acțiunea unui moment. Utilizând metoda transformatei Laplace s-a obținut soluția generală a componentelor deplasărilor elastice de-a lungul direcțiilor radiale, circumferențiale și axiale. Utilizând mediul Simulink din cadrul pachetului de simulare numerică Matlab au fost reprezentate grafic soluțiile sistemului.

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