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BEARING LIFE APPROXIMATION USING KRIGING

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Abstract: *One of the main trends in nowadays industry is to develop end products that are optimal with respect to some given criteria. Often the criteria are expressed using complex functions that cannot be used within an optimization algorithm due to several reasons. Therefore the evaluation of the end products has to be performed using approximate values for those criteria. The approximation of bearing life using Kriging is presented in this paper.*
Key words: *approximation technique, Kriging, Ordinary Kriging, rolling bearing life, DACE*

1. INTRODUCTION

Real-world optimization problems sometimes deal with unknown functions of multiple variables for which only some experimental data is available. Other times the function is so complex that it cannot be evaluated efficiently. There is also the case when the model of a complex engineering process is based on nonlinear systems that do not have solution for certain combinations of input data. In this case the model cannot be used within an optimization program based on an evolutive strategy. In all above mentioned cases it is necessary to use an approximation of the function obtained through different interpolation methods. Even if interpolation errors are usually present, the gain in simplicity overcomes the resultant loss in accuracy.

When interpolation on multidimensional scattered data is required, Kriging, also called optimal interpolation, is frequently used. Kriging is a technique named after the South African mining engineer D.G. Krige who developed it in order to determine true ore-bodies, based on samples. The predictions regarding the quantity of ore-bodies in an untested location were weighted averages of the measurements obtained in the observed locations. The weights depended on the distances between the input location to be predicted and the input locations already observed and they were chosen such that they minimized the prediction variance [10].

This method is basically a form of linear prediction which is also able to provide an estimation of the prediction error [7]. One of the features that makes it an attractive interpolation technique is given by the fact that predicted output values for locations that have already been observed equal the observed values. When predicting the output for a location that has not been observed Kriging takes into consideration that the shorter the distance is between the input data, the larger the positive correlation is between the prediction errors. According to [10], this is modeled through “a second-order stationary covariance process” which implies constant expectations of the observations regardless of the existing input data and a dependence of the observations covariances only on the distances between the directly related input data. The result of the process is an estimated metamodel that uses a predictor based on the criterion of minimum mean squared prediction errors which gives larger weights to the input data closer to the prediction point.

The relationship between input data and interpolation weights is modeled through a function called variogram which describes the variance of the difference between the measurements obtained at two locations. Kriging has proven to give more accurate predictions than other estimating methods because it allows correlated errors and adapts the used weights whenever a prediction for a new location is required [11]. Although it was initially developed and used in geostatistical mapping in the mining industry, it was later successfully used in other areas such as meteorology [2] and oceanography [3]. Kriging has also been applied in engineering in order to create accurate global approximations to facilitate design optimization [8] or in order to conduct some parametric studies for certain system characteristics that can be estimated only using surrogate models [6].

There are several types of Kriging: Simple Kriging, Ordinary Kriging, Kriging with a Trend and Co-Kriging. Simple Kriging assumes that the trend component is a constant and known mean over the entire component, while Ordinary Kriging assumes that the mean is constant only in the local neighborhood of each estimation point. Kriging with a Trend, also known as Universal Kriging, is similar to Ordinary Kriging, except that instead of fitting just a local mean in the neighborhood of the estimation point, it fits a linear or higher-order trend in the coordinates of the data points. Co-Kriging uses information from one or more correlated secondary variables. Also, it requires developing new models for cross-covariance between two variables as a function of lag [1]. Ordinary Kriging is of interest for the current paper.

2. MATHEMATICAL MODEL

All Kriging estimators have their origin in the the basic linear regression estimator $Z^*(u)$, given by [1]:

$$Z^*(u) = m(u) + \sum_{k=1}^{n(u)} \lambda_k \left[Z(u_k) - m(u) \right] \quad (1)$$

with:

- u: location vector for estimation point;
- u_k : location vestors of the neighboring data points, indexed by k;
- $n(u)$: number of data points in local neighborhood used for estimation of $Z^*(u)$;
- $m(u)$, $m(u_k)$: expected values (means) of $Z(u)$ and $Z(u_k)$;
- $\lambda_k(u)$: Kriging weight assigned to for estimating the location u; $Z(u_k)$ will be assigned a different weight when estimating a different location.

The variable $Z(u)$ is treated as a random variable with two main components: a trend component, $m(u)$, and a residual component, $R(u) = Z(u) - m(u)$. As it can be seen in (1), Kriging uses a weighted sum of the residual components at surrounding data points for estimating the residual at location u. The interpolation weights used by Kriging, λ_k, are derived from the semivariogram, which describes the variance of the difference between the measurements obtained at two locations.

Ordinary Kriging is based on the assumption that the mean is constant in the neighborhood of every point that needs to be estimated. Therefore, $m(u_k) = m(u)$ for each close by location u_k, whose value $Z(u_k)$ is used to estimate $Z(u)$ and the Kriging estimator can be written as:

$$\begin{aligned} Z^*(u) &= m(u) + \sum_{k=1}^{n(u)} \lambda_k \left[Z(u_k) - m(u) \right] \\ &= \sum_{k=1}^{n(u)} \lambda_k(u) Z(u_k) + \left[1 - \sum_{k=1}^{n(u)} \lambda_k(u) \right] m(u) \end{aligned} \quad (2)$$

If the condition that the sum of the interpolation weights equals 1 is imposed, then the following Kriging estimator is obtained, estimator that does not depend on the unknown local mean:

$$Z^*_{OK}(u) = \sum_{k=1}^{n(u)} \lambda_k^{OK} Z(u_k) \quad (3)$$

with

$$\sum_{k=1}^{n(u)} \lambda_k^{OK} = 1 \quad (4)$$

The goal is to determine weights, λ_k^{OK}, that minimize the variance of the estimator

$$\sigma_E^2(u) = Var[Z_{OK}^*(u) - Z(u)] \quad (5)$$

taking into consideration that $E\left[Z_{OK}^*(u) - Z(u)\right] = 0$. In order to achieve this, the following function involving a Lagrange parameter, $\mu_{OK}(u)$, is written:

$$L = \sigma_E^2(u) + 2\mu_{OK}(u)\left[1 - \sum_{k=1}^{n(u)} \lambda_k(u)\right] \quad (6)$$

so that minimization with respect to the Lagrange parameter imposes the constraint:

$$\frac{1}{2}\frac{\partial L}{\partial \mu} = 1 - \sum_{k=1}^{n(u)} \lambda_k(u) = 0 \quad (7)$$

In this case, the system of equations for the kriging weights becomes:

$$\sum_{j=1}^{n(u)} \lambda_j^{OK}(u) C_R(u_k - u_j) + \mu_{OK}(u) = C_R(u_k - u),$$

$$\sum_{j=1}^{n(u)} \lambda_j^{OK}(u) = 1$$

where $k = 1,2,\ldots,n(u)$ and $C_R(h)$ is the covariance function for the residual component of the variable. The interpolation weights and Lagrange parameter can be obtained and the Ordinary Kriging error variance can be computed as follows approximating the covariance function $C(h)$ for $Z(u)$ with the one for the residual component, $C_R(h)$:

$$\sigma_{OK}^2(u) = C(0) - \sum_{k=1}^{n(u)} \lambda_k^{OK}(u) C(u_k - u) - \mu_{OK}(u) \quad (8)$$

In general, Kriging offers more realistic predictions than most other interpolation methods due to the fact that is more mathematically robust. Usually, most interpolation techniques estimate the predicted value in a point that has not been evaluated as a weighted sum of the input data from its neighborhood. Almost all methods use functions that assign weights inversely proportional to the distance between the prediction point and input data. However, Kriging assigns weights according to not just an arbitrary function, but a data-driven weighting function [1].

Ordinary Kriging works under the assumption that the mean value is constant only in the local neighborhood of each estimation point which is usually the standard operating assumption. When the input data come in clusters separated by large gaps between them and therefore, unreliable estimates are expected, Ordinary Kriging has proven to perform better than most interpolation techniques because it treats clusters more like single points by assigning lower weights to individual points within a cluster than the weights assigned to isolated inputs.

More than just reducing the effect of data clustering, this method also offers an error estimation together with the predicted value. The fact that an estimation error is provided is an important feature because it allows stochastic simulation of possible realizations of the predicted value.

3. DACE

3.1 Instrument description

DACE (Design and Analysis of Computer Experiments) is a Matlab toolbox designed for applications that require Kriging approximations. This software is used for building a kriging surrogate model based on the input data that allows making predictions for unevaluated points. This toolbox offers the possibility of choosing between regression models with polynomials of orders 0, 1 or 2. Also, the correlation model can be chosen from the seven models presented in Table 1. The correlations are of the following form:

$$C(\theta, a, b) = \prod_{j=1}^{n} C_j(\theta, a_j - b_j) \quad (9)$$

where a and b are two n-dimensional points and θ is a n-dimensional parameter.

The spline correlation model is given by the following function:

$$S(\xi j) = \begin{cases} 1 - 15\xi_j + 30\xi_j^3, & if\ 0 \le \xi_j \le 0.2 \\ 1.25(1 - \xi_j)^3, & if\ 0.2 < \xi_j < 1 \\ 0 & if\ \xi_j \ge 1 \end{cases}$$

Table 1

Correlation models [5]

Name	**$C_j(\theta, d_j)$, where $d_j = a_j - b_j$**
EXP	$\exp(-\theta_j \lvert d_j \rvert)$
EXPG	$\exp(-\theta_j \lvert d_j \rvert^{\theta_{n+1}})$, where $0<\theta_{n+1} \le 2$
GAUSS	$\exp(-\theta_j d_j^2)$
LIN	$\max\{0, 1-\theta_j \lvert d_j \rvert\}$
SPHERICAL	$1-1.5\xi_j + 0.5\xi_j^3$, where $\xi_j = \min\{1, \theta_j \lvert d_j \rvert\}$
CUBIC	$1-3\xi_j^2 + 2\xi_j^3$, where $\xi_j = \min\{1, \theta_j \lvert d_j \rvert\}$
SPLINE	$S(\xi_j)$, where $\xi_j = \theta_j \lvert d_j \rvert$

The point coordinates and the corresponding values of the function which provide the input data are read from a file. The instrument checks the correctness of the input data and then creates the design and analysis model based on that data. The model is created by normalizing the data, computing the distances and the regression matrix. Based on the created model the function value in a new point is predicted.

The instrument usage procedure:

Call: val = KrigingTest (x)

Input: x - the array that contains the coordinates of the point required to be evaluated

Output: val - predicted value for the given point

3.2 Instrument testing

The applied test was conducted for 2695 4-dimensional points and their corresponding function values obtained using the RKB software for bearing life calculation.

The RKB software is briefly presented in [9] and its usage is exemplified on a real case of a bevel pinion shaft supported by a bearing arrangement consisting of two tapered roller bearings in back-to-back arrangement and a cylindrical roller bearing. The RKB software provides the values of the slopes and deformations of each node used to discretize the shaft. This means that the values of the angles of rotation of the shaft in supports (due to the loads and shaft elastic deformations) are known and can be used in bearing life calculation. The bearing life was calculated according to ISO 16281:2008 [4].

Table 2

Kriging data test points

No.	**4-dimensional test point**	**Codification**
1	(50, -43, 43, -130)	TP1
2	(50, -43, 43, -45)	TP2
3	(50, -43, 43, 5)	TP3
4	(70, -17, 85, 75)	TP4
5	(70, -24, 74, -75)	TP5
6	(70, -24, 74, -110)	TP6
7	(90, -61, 11, -110)	TP7
8	(90, -61, 11, -60)	TP8
9	(90, -17, 43, 40)	TP9

The application considered for bearing life calculation consists in a hollow bevel pinion-shaft resting on two similar single-row angular ball bearings (basic designation 7048) in back-to-back (O-) arrangement. All the details regarding the shaft geometry, the shaft loading, the mechanical and thermal properties of used materials, the bearing temperature distribution, the lubricant and lubrication system type and cleanliness, and the mounting dimensions and tolerances are known. Consequently, the life of each bearing can be expressed as a function of four variables: the bench bearing arrangement preload/clearance, the interference between the shaft and the bearing inner ring bore, the clearance between the housing bore and the bearing outer ring, and the operating temperature. The bearing arrangement life is the minimum between the lives of the two bearings on which the shaft is rested.

First, 2695 4-dimensional points consisting of values for the four variables necessary for the bearing life calculation were considered. For each point the corresponding bearing life was computed using the RKB software. Then, from the given 2695 data points, 2686 were used as input and the other 9 were used as test data in order to identify the best correlation and regression models. The points used as test data are given in Table 2.

The best predictions with an average percentage error of 0.6564% were obtained when using regression model with polynomial of order 1 and spherical correlation model (Table 3).

Table 3

Best results obtained using Kriging for the applied test

Poly1 CorrSpherical					
Data test point	Actual function value	Predicted value	Absolute error	% error	Average % error
TP1	13028	13216	188	0.01443	0.6564
TP2	39355	39263	92	0.00233	
TP3	33590	32679	911	0.02712	
TP4	11279	11277	2	0.00017	
TP5	7218	7312.6	94.6	0.01310	
TP6	3414	3410.7	3.3	0.00096	
TP7	3366	3364.8	1.2	0.00035	
TP8	3085	3084.4	0.6	0.00019	
TP9	2846	2844.9	1.1	0.00038	

4. BEARING LIFE APPROXIMATION USING KRIGING

The computation of the bearing life is an essential aspect of the bearing industry. Sophisticated programs have been developed for determining the bearing life in all stages of a certain project. However, when the computed values of the bearing life have to be used within an optimization program based on an evolutionary algorithm additional aspects have to be considered. An important one is the communication between the programming environments in which the optimization program and the program for the bearing life calculation are implemented. Even if this aspect is solved, that might not be sometimes enough. In this case the bearing life computation using the RKB software is based on a system of nine non-linear equations that does not have a solution for all variable value combinations.

If the aim is, for example, the maximization of the bearing life of the bearing life briefly presented in section 3.2 and the maximization is conducted with an evolutionary algorithm that uses random possible solutions, combinations for which the solution of the mentioned system cannot be found are likely to be generated. In order to avoid blocking the evolutionary process or using a time-consuming handling procedure for those combinations, Kriging interpolation is an efficient solution. The use of approximation is a convenient compromise because the exact value of the bearing life is not essential for solving the mentioned optimization problem and, moreover, Kriging has a low average percentage error as shown in Table 3.

When using the Kriging for approximating the bearing life of the already mentioned bearing arrangement, all 2695 4-dimensional points were used to build the design and analysis model. Based on this model the bearing arrangement life was approximated within the optimization algorithm for each 4-dimensional point randomly generated.

5. CONCLUSION

Bearing life calculation is a major aspect of the rolling bearing industry and the development of rolling bearings whose life is maximum, this is the goal of every respectable company from this industry.

The accurate bearing life calculation requires a complex computation based on solving a system of nonlinear equations that does not have a solution for certain configurations. Therefore, the algorithm for calculating the bearing life cannot be included within an optimization program. Thus, the need for an efficient approximation technique is obvious.

Kriging has proven to be a fast and reliable technique for approximating the bearing life in order to use the values within an optimization program.

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APROXIMAREA DURABILITĂȚII RULMENȚILOR FOLOSIND KRIGING

Abstract: Unul din principalele curente din industria zilelor noastre este dezvoltarea de produse finite care sunt optime în raport cu unele criterii date. Adesea criteriile sunt exprimate folosind funcții complexe care, din diferite motive, nu pot fi folosite în cadrul algoritmilor de optimizare. De aceea, evaluarea produselor finite trebuie realizată folosind valori aproximative pentru acele criterii. În această lucrare este prezentată aproximarea durabilității rulmenților folosind tehnica de interpolare Kriging

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