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VIBRATION ANALYSIS OF A GEAR TRANSMISSION

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Abstract: *This paper is a survey of the literature on gear vibration and is designed to offer a better understanding of the need for predictive maintenance for safe operation of machine-tools and minimizing idle time and reducing repair costs. Vibration monitoring is the most effective technique for detecting mechanical faults rotating machinery. Transmission error (TE) is considered to be an important excitation mechanism for the gear noise and vibration.*
Key words: *spur gear, noise, gear vibration, transmission error.*

1. INTRODUCTION

Vibration analysis is one of the most used methods for detection and diagnosis of faults in electromechanical systems. Based on system vibration values through this method is determined usually with accelerometer [1].

Through vibration can be detect many defects such as unbalance, slide bearings problems, structural resonance, electrical machinery rotor faults and eccentricities. Measurements are fast and noninvasive because the functioning of system is not disturbed through test. For each electromechanical system shall be defined its own level of vibration, any derivation in this indicates a problem so that it can intervene before the system is damaged. Data can be collected periodically or continuously using a portable system, by setting up a system for online monitoring [2].

The vast majority of vibration measuring devices working in 10 Hz -1 kHz. This domain is considered the best interval for problems such as imbalance, eccentricity, additional efforts.

2. MECHANICAL GEAR TRANSMISION SYSTEM MODEL

For modeling a mechanical system, the transmission gear is considered a system with three degrees of freedom presented in figure 1. This system consists of an electric motor for drive and two meshing spur gears.

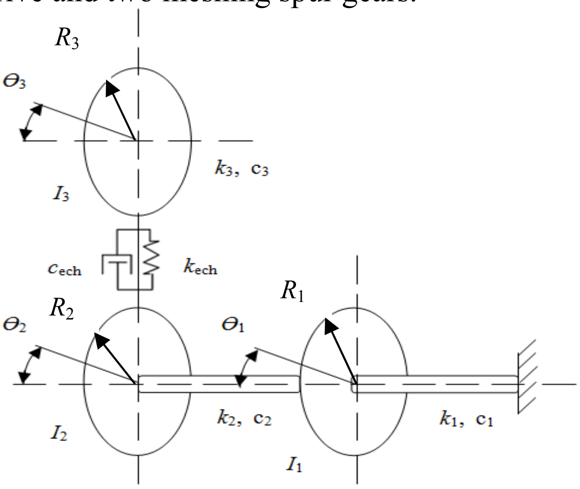

Fig.1. Mechanical system of the transmission gear with 3 degrees of freedom - 3MPR

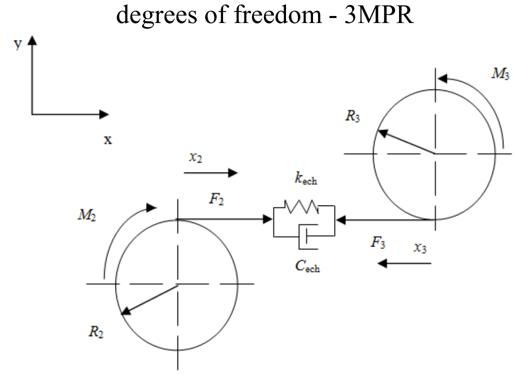

Fig.2. Scheme of cylindrical spur-gear 3MPR mechanical model

Input data for gear mesh calculation are: power P [kW], speed n_1 [rot/min], torques M_1, M_2, M_3 [Nm] and the gear ratio i. It is also know the number of teeth z_1 and z_2, the moments of inertia I_1, I_2, I_3 [kg m^2], the coefficients of elasticity k_1, k_2, k_3 [N/m], and damping coefficients c_1, c_2, c_3 [Ns/m].

In determining the vibration parameters, it is assumed no friction forces. It also ignores the impact forces between teeth are in meshing gears and gearing force is approximated to be equal to the tangential component. The gear transmission system motion law is a system of differential equations of second order nonhomogeneous with constant coefficients where:

- I_1, I_2, I_3 represents the moments of inertia of the drive motor shaft, pinion and driven gear [kg m^2];
- θ_1, θ_2 are relative angular displacement of the drive motor shaft and pinion [rad];
- $\dot{\theta}_1, \dot{\theta}_2$ are relative angular speeds of the drive motor shaft and pinion [rad/s];
- $\ddot{\theta}_1, \ddot{\theta}_2, \ddot{\theta}_3$ are relative angular accelerations of the drive motor shaft, pinion and driven gear [rad/s^2];
- F_2, F_3 are the mesh forces of the pinion respectively of the driven gear [N];
- r_2, r_3 are the radii of the basic circles for pinion and driven gear [mm];
- k_1, k_2, k_3 are the elasticity coefficients of the shaft drive motor, pinion respectively of the driven gear [N/m];
- c_1, c_2, c_3 are the damping coefficients of shaft drive motor, pinion respectively of the driven gear [N·s/m].
- M_1, M_2, M_3 are the torques corresponding to shaft drive motor, pinion respectively of the driven gear [Nm];

Mesh forces, F_2 and F_3 depends on the elasticity and damping of the system.

Relative linear displacement can be determined, depending on the angle of rotation of the gears, differential equations are written as:

$$\begin{cases} I_1\ddot{\theta}_1 + c_1\left(\dot{\theta}_1 - \dot{\theta}_2\right) + k_1\left(\theta_1 - \theta_2\right) = M_1 \\ I_2\ddot{\theta}_2 - c_{ech}r_2\left(\dot{\theta}_3 r_3 - \dot{\theta}_2 r_2\right) - k_{ech}r_2\left(\theta_3 r_3 - \theta_2 r_2\right) = M_2 \\ I_3\ddot{\theta}_3 + c_{ech}r_3\left(\dot{\theta}_3 r_3 - \dot{\theta}_2 r_2\right) + k_{ech}r_3\left(\theta_3 r_3 - \theta_2 r_2\right) = M_3 \end{cases} \quad (1)$$

where: - c_{ech} represents the equivalent damping coefficient [Ns/m] gear teeth dependent damping, which is calculated as [3]:

- k_{ech} is the equivalent elasticity coefficient [N/m] gear teeth dependent elasticity, which is calculated as [3]:
- x_2, x_3 are relative displacements, along the line of gearing, for pinion, respectively driven gear [m];
- $\dot{x}_2, \dot{x}_3$ are the relative velocities, along the line of meshing, for pinion, driven gear respectively [m/s].

2.1 MATHEMATICAL MODELING OF 3MPR SYSTEM WITH TRANSMISSION ERROR

If the meshing gear is functioning with transmission error, TE has the following form:

$$TE = r_3\theta_3 - r_2\theta_2 \quad (2)$$

From equation (2) is determined displacement, velocity and angular acceleration of the driven gear, according to the movement, speed and angular acceleration of the pinion and the transmission error:

$$\theta_3 = \frac{TE + r_2\theta_2}{r_3} = \frac{TE}{r_3} + \frac{r_2}{r_3}\theta_2 = \frac{TE}{r_3} + \frac{\theta_2}{i} \quad (3)$$

$$\dot{\theta}_3 = \frac{\dot{TE}}{r_3} + \frac{\dot{\theta}_2}{i} \quad (4)$$

$$\ddot{\theta}_3 = \frac{\ddot{TE}}{r_3} + \frac{\ddot{\theta}_2}{i} \quad (5)$$

Through replacement $\theta_3, \dot{\theta}_3, \ddot{\theta}_3$. by (3), (4) and (5) in system (1), we obtain the system of differential equations of second order inhomogeneous with constant coefficients, for modeling the mechanical system with error transmission, 3MPR:

$$\begin{cases} I_1\ddot{\theta}_1 + c_1\left(\dot{\theta}_1 - \dot{\theta}_2\right) + k_1\left(\theta_1 - \theta_2\right) = M_1 \\ I_2\ddot{\theta}_2 - c_{ech}r_2\dot{TE} - k_{ech}r_2 TE = M_2 \\ \frac{I_3}{i}\ddot{\theta}_2 + \frac{I_3\ddot{TE}}{r_3} + c_{ech}r_3\dot{TE} + k_{ech}r_3 TE = iM_2 \end{cases} \quad (6)$$

Multiplying the second relation of equation system (6) with i and equating it with the last one resulting system of two differential equations of second order inhomogeneous with constant coefficients:

$$\begin{cases} I_1\ddot{\theta}_1 + c_1\left(\dot{\theta}_1 - \dot{\theta}_2\right) + k_1\left(\theta_1 - \theta_2\right) = M_1 \\ \left(I_2 - \frac{I_3}{i}\right)\ddot{\theta}_2 - \frac{I_3\ddot{TE}}{r_3} - c_{ech}\dot{TE}\left(r_2 - r_3\right) \\ -k_{ech}TE\left(r_2 - r_3\right) = 0 \end{cases} \quad (7)$$

From the system of differential equations (7) resulting accelerations expressions $\ddot{\theta}_1$ and $\ddot{\theta}_2$:

$$\ddot{\theta}_1 = \frac{M_1}{I_1} - \frac{c_1\left(\dot{\theta}_1 - \dot{\theta}_2\right) + k_1\left(\theta_1 - \theta_2\right)}{I_1} \quad (8)$$

$$\ddot{\theta}_2 = \frac{\left(c_{ech}\dot{TE}\left(r_2 - r_3\right) + k_{ech}TE\left(r_2 - r_3\right) + I_3\ddot{TE}\right)}{r_2\left(iI_2 - I_3\right)} \quad (9)$$

With MathLAB, Simulink, by Runghe Kutta 4.5^{th} order method 4.5 order, isobtain the value of parameters $\theta_1, \dot{\theta}_1, \ddot{\theta}_1, \theta_2, \dot{\theta}_2, \ddot{\theta}_2, \theta_3, \dot{\theta}_3, \ddot{\theta}_3$.

Transmission error is a function of the time which can have an expression, for example, for the deviation profile:

$$TE(t) = A_e \sin\left(\omega_z t + \alpha\right) \quad (10)$$

$$\dot{TE}(t) = A_e \omega_z \cos\left(\omega_z t + \alpha\right) \quad (11)$$

$$\ddot{TE}(t) = -A_e \omega_z^2 \sin\left(\omega_z t + \alpha\right) \quad (12)$$

where:

- ω_z – frequency of mesh [rad/s];
- α – pressure angle [rad];
- A_e – error amplitude [m].

Natural frequency of mashing has the following expression:

$$\omega_z = \sqrt{\frac{k_{ech}}{I_{ech}}} \quad (13)$$

where: - I_{ech} - equivalent mechanical inertia module of gear can by written as:

$$I_{ech} = \frac{I_2 I_3}{r_3^2 I_2 + r_2^2 I_3} \quad (14)$$

3. EXPERIMENTAL RESULTS

The gear ratio between the follower (F) and base (B) is steadily increases from 1:1 to 2:1.

Table 1

Main parameters.

$I_2 = 4000$ Kg m^2	$k_2 = 1.6$ MNm/rad	$C_2 = 36$ Nms/rad	$N_2 = 111$	$\Omega_z = 48$ Hz
$I_3 = 1300$ Kg m^2	$k_3 = 0.29$ MNm/rad	$C_3 = 5.8$ Nms/rad	$N_3 = 56$	

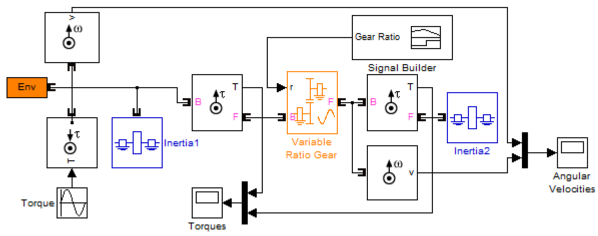

Fig.1 MathLAB Simulink model of 3MPR.

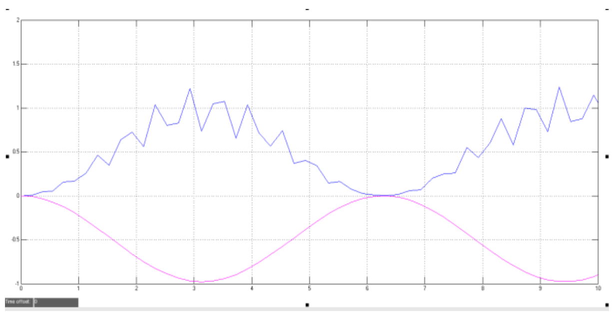
Fig.2 Angular velocities of Shaft 1 and Shaft 2.

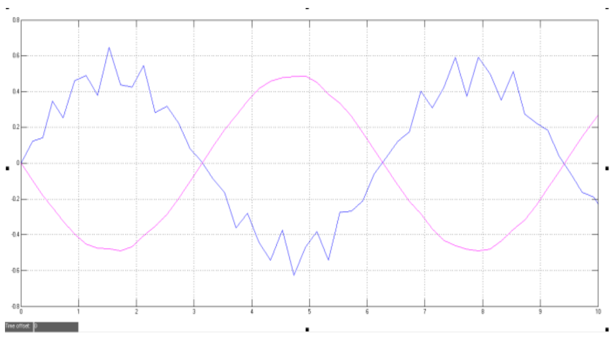
Fig.3 Torques in Shaft 1 and Shaft 2.

4. CONCLUSION

Transmission error (TE) is considered as the main mechanism of noise and vibration excitation for gear. If a gear is load gears teeth undergoes a elastic deformation. Also, elastic deformation takes place at the shaft, bearings and the housing.

Still transmission errors occur under the influence of the load and depend on the elastic properties of all the components

By requiring contact and bending rigidity of gear it deforms depending of teeth who are in mesh. This means that by rotating the gears, the teeth which come into contact is changing, thereby also the stiffness is changing.

5. REFERENCES

[1] Țepeş-Bobescu F., Arghir Mariana, *Mentenanța Angrenajelor Maşinilor-Unelte*, Prevenirea riscurilor în activitățile de mentenanță (revizii, reparații şi curățenie), pag. 243-250, ISBN 978-606-543-174-4, Editura Mega, Cluj-Napoca 2011

[2] Țepeş-Bobescu F., Țepeş-Bobescu Alina-Sabina, Arghir Mariana, *Machine Tools Maintenance of Gears by Vibration Analysis,* 84th Annual Meeting of the International Association of Applied Mathematics and Mechanics, Novi Sad, Serbia 2013.

[3] Litak G., Friswell M.I., *Dynamics of a Gear System with Faults in Meshing Stiffness,* Vol. 41, Issue 4, pp. 415-421, ISSN: 0924-090X, Springer Link Germania, 2005.

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Analiza vibrațiilor unei transmisii cu roți dințate

Rezumat: Această lucrare este un studiu de literatura de specialitate privind vibrațiile angrenajelor şi este conceput pentru a oferi o mai bună înțelegere a nevoii de mentenanță predictivă pentru funcționarea în siguranță a maşinilor-unelte şi minimizarea timpului de inactivitate şi pentru a reduce costurile de reparație. Monitorizarea vibrațiilor este tehnica cea mai eficientă pentru detectarea defectelor mecanice. Eroare de transmisie (TE) este considerată a fi un mecanism important de excitație pentru zgomotele şi vibrațiile produse de angrenajele maşinilor-unelte.

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