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DIRECT KINEMATICS MODELING OF TRTRR SERIAL MODULAR ROBOT

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Abstract: *There is a modular serial robot noted TRTRR with two translation modules and three rotational modules, and that is inside the manufacturing cell. In this paper is presented the kinematics modeling using matrix study. For the five modules are given step by step the position matrix and the final result if they work as an assembly. In this paper there is the direct kinematics modeling and this is the continuing of the geometric modeling of the TRTRR robot.*
Keywords: *direct kinematics modeling, TRTTR robot, serial modular robot.*

1. INTRODUCTION

The robot kinematics diagram type industrial TRTRR, is shown in figure 1. This is made up of the following: The vertical translation module 1 (MTV), module 2 of robot arm rotation (MRB), the module 3 for translation in the horizontal direction (MT), module 4 for the horizontal rotation (MR), the guidance module 5 (MO) of the prehensiune tool noted 6.

2. DIRECT KINEMATICS MODELING

After shaping the robot direct geometric TRTRR as shown in Figure 1, has been obtained from the vector column of the operational coordinates.

Homogeneous transformation matrixes can be determined using the rotation matrixes and of position vectors obtained in geometric modeling. These matrixes of homogeneous transformation are set out below, in accordance with [Pop78], [Gui10e] and [ISP04]:

$$[T]_1^0(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&l_1+q_1\\\hline 0&0&0&1\end{array}\right];\quad [T]_2^1(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&0\\0&0&1&l_2\\\hline 0&0&0&1\end{array}\right];\tag{1}$$

$$[T]_3^2(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&l_4+q_3\\0&0&1&l_3\\\hline 0&0&0&1\end{array}\right];\quad [T]_4^3(t)=\left[\begin{array}{ccc|c}cq_4&0&sq_4&0\\0&1&0&l_5\\-sq_4&0&cq_4&0\\\hline 0&0&0&1\end{array}\right];\tag{2}$$

$$[T]_5^4(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&cq_5&-sq_5&l_6\\0&sq_5&cq_5&0\\\hline 0&0&0&1\end{array}\right];\quad [T]_6^5(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&l_7\\0&0&1&0\\\hline 0&0&0&1\end{array}\right];\tag{3}$$

$$[T]_2^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&0\\0&0&1&l_1+l_2+q_1\\\hline 0&0&0&1\end{array}\right];$$
$$[T]_3^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&-(l_4+q_3)sq_2\\sq_2&cq_2&0&(l_4+q_3)cq_2\\0&0&1&l_1+l_2+l_3+q_1\\\hline 0&0&0&1\end{array}\right];\tag{4}$$

$$[T]_4^0(t)=\left[\begin{array}{ccc|c}cq_2cq_4&-sq_2&cq_2sq_4&-(l_4+l_5+q_3)sq_2\\sq_2cq_4&cq_2&sq_2sq_4&(l_4+l_5+q_3)cq_2\\-sq_4&0&cq_4&l_1+l_2+l_3+q_1\\\hline 0&0&0&1\end{array}\right];\tag{5}$$

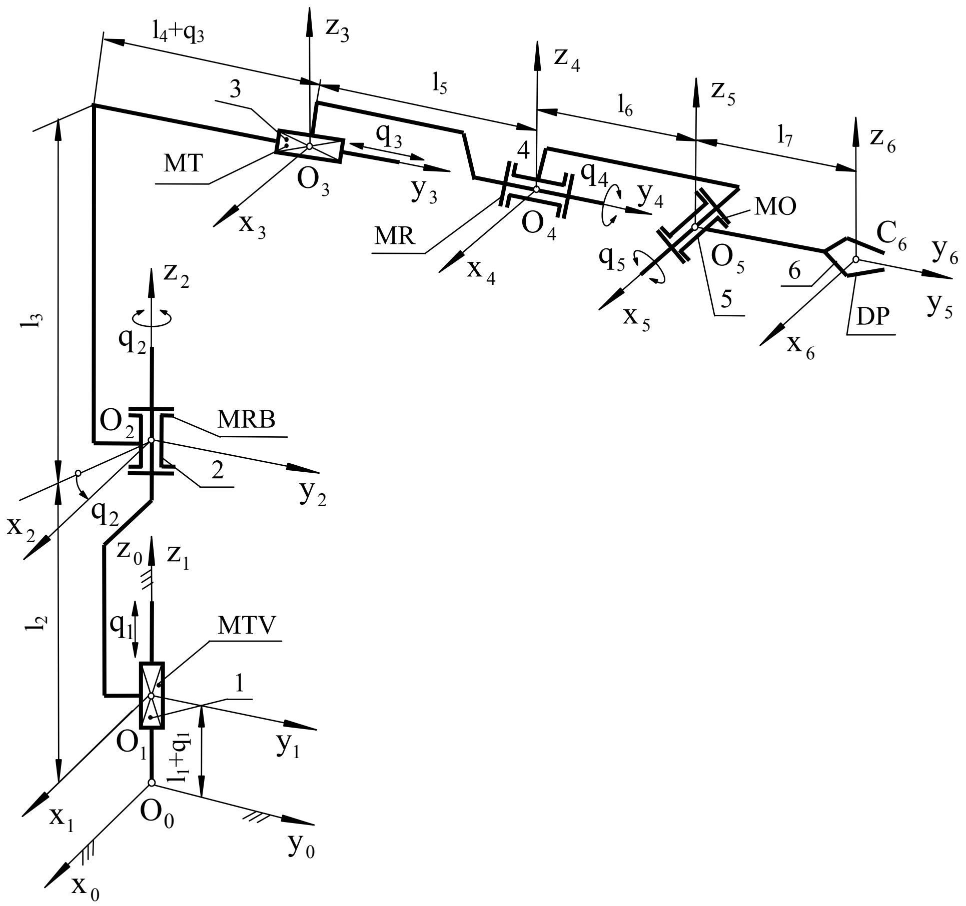

Fig.1. Structural Scheme of Kinematics for the Industrial Modular Serial Robot TRTRR

$$[T]_5^0(t)=\begin{bmatrix} cq_2cq_4 & -sq_2cq_5+cq_2sq_4sq_5 & sq_2sq_5+cq_2sq_4cq_5 & -(l_4+l_5+l_6+q_3)sq_2 \\ sq_2cq_4 & cq_2cq_5+sq_2sq_4sq_5 & -cq_2sq_5+sq_2sq_4cq_5 & (l_4+l_5+l_6+q_3)cq_2 \\ -sq_4 & cq_4sq_5 & cq_4cq_5 & l_1+l_2+l_3+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \tag{6}$$

$$[T]_6^0(t)=\begin{bmatrix} cq_2cq_4 & -sq_2cq_5+cq_2sq_4sq_5 & sq_2sq_5+cq_2sq_4cq_5 & -(l_4+l_5+l_6+q_3)sq_2+ \\ & & & +l_7(-sq_2cq_5+cq_2sq_4sq_5) \\ sq_2cq_4 & cq_2cq_5+sq_2sq_4sq_5 & -cq_2sq_5+sq_2sq_4cq_5 & (l_4+l_5+l_6+q_3)cq_2+ \\ & & & +l_7(cq_2cq_5+sq_2sq_4sq_5) \\ -sq_4 & cq_4sq_5 & cq_4cq_5 & l_1+l_2+l_3+q_1+l_7cq_4sq_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

Homogeneous transformation matrixes established following geometric modeling is used to determine the kinematics parameters $\overline{\omega}_i^i, \overline{v}_i^i, \overline{\varepsilon}_i^i, \overline{a}_i^i$. Using the relationship below is likely to determine the inverse of rotation matrixes:

$$[R]_{i-1}^{i}=\left[R_i^{i-1}\right]^{-1}=\left[R_i^{i-1}\right]^{T}. \tag{8}$$

Then, result:

$$[R]_0^1=[I_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\ [R]_1^2=\begin{bmatrix}cq_2&sq_2&0\\-sq_2&cq_2&0\\0&0&1\end{bmatrix};$$

$$[R]_2^3=[I_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}; \tag{9}$$

$$[R]_3^4=\begin{bmatrix}cq_4 & 0 & -sq_4\\ 0 & 1 & 0\\ sq_4 & 0 & cq_4\end{bmatrix};\ [R]_4^5=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_5 & sq_5\\ 0 & -sq_5 & cq_5\end{bmatrix};$$

$$[R]_5^6=[I_3]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}. \qquad (10)$$

The unit vectors of kinematics axes (Fig. 1), are shown with matrix as follows:

$$[\bar{k}]_1^1=\begin{bmatrix}0\\0\\1\end{bmatrix};\quad [\bar{k}]_2^2=\begin{bmatrix}0\\0\\1\end{bmatrix};\quad [\bar{j}]_3^3=\begin{bmatrix}0\\1\\0\end{bmatrix};$$

$$[\bar{j}]_4^4=\begin{bmatrix}0\\1\\0\end{bmatrix};\quad [\bar{i}]_5^5=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad [\bar{j}]_6^6=\begin{bmatrix}0\\1\\0\end{bmatrix}. \qquad (11)$$

Kinematics elements expressions, transposed with matrix, corresponding robot base, are the following:

$$[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ [\bar{v}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ [\bar{\varepsilon}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ [\bar{a}]_0^0=\begin{bmatrix}0\\0\\g\end{bmatrix}. \qquad (12)$$

Operational rotation speeds are determined by inverse matrixes and with the kinematics parameters. So:

$$[\bar{\omega}]_1^1=[R]_0^1\cdot[\bar{\omega}]_0^0=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}; \qquad (13)$$

$$[\bar{\omega}]_2^2=[R]_1^2\cdot[\bar{\omega}]_1^1+\dot{q}_2\cdot[\bar{k}]_2^2=\begin{bmatrix}cq_2 & sq_2 & 0\\ -sq_2 & cq_2 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\end{bmatrix}+\dot{q}_2\cdot\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix}; \qquad (14)$$

$$[\bar{\omega}]_3^3=[R]_2^3\cdot[\bar{\omega}]_2^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix}=\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix}; \qquad (15)$$

$$[\bar{\omega}]_4^4=[R]_3^4\cdot[\bar{\omega}]_3^3+\dot{q}_4\cdot[\bar{j}]_4^4=\begin{bmatrix}cq_4 & 0 & -sq_4\\ 0 & 1 & 0\\ sq_4 & 0 & cq_4\end{bmatrix}\cdot$$

$$\cdot\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix}+\dot{q}_4\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-sq_4\dot{q}_2\\ \dot{q}_4\\ cq_4\dot{q}_2\end{bmatrix}; \qquad (16)$$

$$[\bar{\omega}]_5^5=[R]_4^5\cdot[\bar{\omega}]_4^4+\dot{q}_5\cdot[\bar{i}]_5^5=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_5 & sq_5\\ 0 & -sq_5 & cq_5\end{bmatrix}\cdot$$

$$+\dot{q}_5\cdot\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}-sq_4\dot{q}_2+\dot{q}_5\\ \dot{q}_4cq_5+\dot{q}_2sq_5cq_4\\ -\dot{q}_4sq_5+\dot{q}_2cq_5cq_4\end{bmatrix} \qquad (17)$$

$$[\bar{\omega}]_6^6=[R]_5^6\cdot[\bar{\omega}]_5^5=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}-sq_4\dot{q}_2+\dot{q}_5\\ \dot{q}_4cq_5+\dot{q}_2sq_5cq_4\\ -\dot{q}_4sq_5+\dot{q}_2cq_5cq_4\end{bmatrix}=$$

$$=\begin{bmatrix}-sq_4\dot{q}_2+\dot{q}_5\\ \dot{q}_4cq_5+\dot{q}_2sq_5cq_4\\ -\dot{q}_4sq_5+\dot{q}_2cq_5cq_4\end{bmatrix}. \qquad (18)$$

In accordance with [Ber93], [Neg01] and [Neg08], with opposite symmetric matrix 3x3 of the type $\{\bar{\omega}\times\}$, in which $\bar{\omega}$ represents the vector angular speed, it is of the form:

$$\{\bar{\omega}\times\}=\begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix}. \qquad (19)$$

Vector products and double-vector products which still appear shall be expressed in accordance with the relationship (19). Linear speeds expressions transformed into operational matrix are the following:

$$[\bar{v}]_1^1=[R]_0^1\cdot\{[v]_0^0+[\omega]_0^0\times\bar{r}_1^0\}+\dot{q}_1\cdot[\bar{k}]_1^1;$$

$$[\bar{v}]_1^1=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\l_1+q_1\end{bmatrix}\right\}+\dot{q}_1\cdot\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}; \qquad (20)$$

$$[\bar{v}]_2^2=[R]_1^2\cdot\{\bar{v}_1^1+\bar{\omega}_1^1\times\bar{r}_2^1\};$$

$$[\bar{v}]_2^2=\begin{bmatrix}cq_2 & sq_2 & 0\\ -sq_2 & cq_2 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\l_2\end{bmatrix}\right\}=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}; \qquad (21)$$

$$[\bar{v}]_3^3=[R]_2^3\cdot\{\bar{v}_2^2+\bar{\omega}_2^2\times\bar{r}_3^2\}+\dot{q}_3\cdot[\bar{j}]_3^3;$$

$$[\bar{v}]_3^3=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}+\begin{bmatrix}0 & -\dot{q}_2 & 0\\ \dot{q}_2 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}0\\l_4+q_3\\l_3\end{bmatrix}\right\}+$$

$$+\dot{q}_3\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-\dot{q}_2(l_4+q_3)\\ \dot{q}_3\\ \dot{q}_1\end{bmatrix}; \qquad (22)$$

$$[\bar v]_4^4 = [R]_3^4 \cdot \{\bar v_3^3 + \bar\omega_3^3 \times \bar r_4^3\};$$

$$[\bar v]_4^4 = \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -\dot q_2(l_4+q_3) \\ \dot q_3 \\ \dot q_1 \end{bmatrix} + \begin{bmatrix} 0 & -\dot q_2 & 0 \\ \dot q_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} [-\dot q_2(l_4+l_5+q_3)]cq_4 - \dot q_1 sq_4 \\ \dot q_3 \\ [-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 \end{bmatrix}; \tag{23}$$

$$[\bar v]_5^5 = [R]_4^5 \cdot \{\bar v_4^4 + \bar\omega_4^4 \times \bar r_5^4\};$$

$$[\bar v]_5^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix} \cdot \left\{ \begin{bmatrix} [-\dot q_2(l_4+l_5+q_3)]cq_4 - \dot q_1 sq_4 \\ \dot q_3 \\ [-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 \end{bmatrix} + \begin{bmatrix} 0 & -\dot q_2 cq_4 & \dot q_4 \\ \dot q_2 cq_4 & 0 & \dot q_2 sq_4 \\ -\dot q_4 & -\dot q_2 sq_4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix} \right\} =$$

$$= \begin{bmatrix} [-\dot q_2(l_4+l_5+q_3)]cq_4 - \dot q_1 sq_4 - \dot q_2 l_6 cq_4 \\ \dot q_3 cq_5 + sq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} \\ -\dot q_3 sq_5 + cq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} \end{bmatrix}; \tag{24}$$

$$[\bar v]_6^6 = [R]_5^6 \cdot \{\bar v_5^5 + \bar\omega_5^5 \times \bar r_6^5\};$$

$$[\bar v]_6^6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} [-\dot q_2(l_4+l_5+q_3)]cq_4 - \dot q_1 sq_4 - \dot q_2 l_6 cq_4 \\ \dot q_3 cq_5 + sq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} \\ -\dot q_3 sq_5 + cq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} 0 & \dot q_4 sq_5 - \dot q_2 cq_5 cq_4 & \dot q_4 cq_5 + \dot q_2 sq_5 cq_4 \\ -\dot q_4 sq_5 + \dot q_2 cq_5 cq_4 & 0 & \dot q_2 sq_4 - \dot q_5 \\ -\dot q_4 cq_5 - \dot q_2 sq_5 cq_4 & -\dot q_2 sq_4 + \dot q_5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_7 \\ 0 \end{bmatrix} \right\} =$$

$$= \begin{bmatrix} [-\dot q_2(l_4+l_5+q_3)]cq_4 - \dot q_1 sq_4 - \dot q_2 l_6 cq_4 + (\dot q_4 sq_5 - \dot q_2 cq_5 cq_4)l_7 \\ \dot q_3 cq_5 + sq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} \\ -\dot q_3 sq_5 + cq_5\{[-\dot q_2(l_4+l_5+q_3)]sq_4 + \dot q_1 cq_4 - \dot q_2 l_6 sq_4\} + (-\dot q_2 sq_4 + \dot q_5)l_7 \end{bmatrix}. \tag{25}$$

Similarly, can be expressed angular accelerations transformed into operational matrix. So:

$$[\bar\varepsilon]_1^1 = [R]_0^1 \cdot [\bar\varepsilon]_0^0; \quad [\bar\varepsilon]_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \tag{26}$$

$$[\bar\varepsilon]_2^2 = [R]_1^2 \cdot [\bar\varepsilon]_1^1 + \{[R]_1^2 \cdot \bar\omega_1^1 \times \dot q_2 \cdot \bar k_2^2 + \ddot q_2 \cdot \bar k_2^2\}; \quad [\bar\varepsilon]_2^2 = \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} +$$

$$+ \left\{ \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot q_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \ddot q_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot q_2 \end{bmatrix}; \tag{27}$$

$$[\bar\varepsilon]_3^3 = [R]_2^3 \cdot [\bar\varepsilon]_2^2; \quad [\bar\varepsilon]_3^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot q_2 \end{bmatrix}; \tag{28}$$

$$[\bar\varepsilon]_4^4 = [R]_3^4 \cdot [\bar\varepsilon]_3^3 + \{[R]_3^4 \cdot \bar\omega_3^3 \times \dot q_4 \cdot \bar j_4^4 + \ddot q_4 \cdot \bar j_4^4\}; \quad [\bar\varepsilon]_4^4 = \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot q_2 \end{bmatrix} +$$

$$+ \left\{ \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot q_2 & 0 \\ \dot q_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \dot q_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \ddot q_4 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} -\ddot q_2 sq_4 - \dot q_2 \dot q_4 cq_4 \\ \ddot q_4 \\ \ddot q_2 cq_4 - \dot q_2 \dot q_4 sq_4 \end{bmatrix}; \tag{29}$$

$$[\bar{\varepsilon}]_5^5 = [R]_4^5 \cdot [\bar{\varepsilon}]_4^4 + \left\{ [R]_4^5 \cdot \bar{\omega}_4^4 \times \dot{q}_5 \cdot \bar{i}_5^5 + \ddot{q}_5 \cdot \bar{i}_5^5 \right\};$$

$$[\bar{\varepsilon}]_5^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix} \cdot \begin{bmatrix} -\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4 \\ \ddot{q}_4 \\ \ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4 \end{bmatrix} + \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_2 cq_4 & \dot{q}_4 \\ \dot{q}_2 cq_4 & 0 & \dot{q}_2 sq_4 \\ -\dot{q}_4 & -\dot{q}_2 sq_4 & 0 \end{bmatrix} \cdot \right.$$

$$\left. \cdot \begin{bmatrix} \dot{q}_5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \ddot{q}_5 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} -\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4 + \ddot{q}_5 \\ \ddot{q}_4 cq_5 + sq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(\dot{q}_2 cq_5 cq_4 - \dot{q}_4 sq_5) \\ -\ddot{q}_4 sq_5 + cq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5) \end{bmatrix}; \tag{30}$$

$$[\bar{\varepsilon}]_6^6 = [R]_5^6 \cdot [\bar{\varepsilon}]_5^5;$$

$$[\bar{\varepsilon}]_6^6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4 + \ddot{q}_5 \\ \ddot{q}_4 cq_5 + sq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(\dot{q}_2 cq_5 cq_4 - \dot{q}_4 sq_5) \\ -\ddot{q}_4 sq_5 + cq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5) \end{bmatrix} =$$

$$= \begin{bmatrix} -\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4 + \ddot{q}_5 \\ \ddot{q}_4 cq_5 + sq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(\dot{q}_2 cq_5 cq_4 - \dot{q}_4 sq_5) \\ -\ddot{q}_4 sq_5 + cq_5(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4) + \dot{q}_5(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5) \end{bmatrix}. \tag{31}$$

In accordance with the relationship [Isp04], expressions of operational linear acceleration are as follows:

$$[\bar{a}]_1^1 = [R]_0^1 \cdot \left\{ \bar{a}_0^0 + \bar{\varepsilon}_0^0 \times \bar{r}_1^0 + \bar{\omega}_0^0 \times \left(\bar{\omega}_0^0 \times \bar{r}_1^0 \right) \right\} + \left\{ 2\bar{\omega}_1^1 \times \dot{q}_1 \cdot \bar{k}_1^1 + \ddot{q}_1 \cdot \bar{k}_1^1 \right\};$$

$$[\bar{a}]_1^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix} \right\} +$$

$$+ \left\{ 2 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot{q}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \ddot{q}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix}; \tag{32}$$

$$[\bar{a}]_2^2 = [R]_1^2 \cdot \left\{ \bar{a}_1^1 + \bar{\varepsilon}_1^1 \times \bar{r}_2^1 + \bar{\omega}_1^1 \times \left(\bar{\omega}_1^1 \times \bar{r}_2^1 \right) \right\}; \quad [\bar{a}]_2^2 = \begin{bmatrix} cq_2 & sq_2 & 0 \\ -sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot$$

$$\cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix}; \tag{33}$$

$$[\bar{a}]_3^3 = [R]_2^3 \cdot \left\{ \bar{a}_2^2 + \bar{\varepsilon}_2^2 \times \bar{r}_3^2 + \bar{\omega}_2^2 \times \left(\bar{\omega}_2^2 \times \bar{r}_3^2 \right) \right\} + \left\{ 2\bar{\omega}_3^3 \times \dot{q}_3 \cdot \bar{j}_3^3 + \ddot{q}_3 \cdot \bar{j}_3^3 \right\};$$

$$[\bar{a}]_3^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 + g \end{bmatrix} + \begin{bmatrix} 0 & -\ddot{q}_2 & 0 \\ \ddot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_4 + q_3 \\ l_3 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \right.$$

$$\left. \cdot \begin{bmatrix} 0 \\ l_4 + q_3 \\ l_3 \end{bmatrix} \right\} + \left\{ 2 \cdot \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \dot{q}_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \ddot{q}_3 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} -\ddot{q}_2(l_4 + q_3) - 2\dot{q}_2\dot{q}_3 \\ -\dot{q}_2^2(l_4 + q_3) + \ddot{q}_3 \\ \ddot{q}_1 + g \end{bmatrix}; \tag{34}$$

$$[\bar{a}]_4^4 = [R]_3^4 \cdot \left\{ \bar{a}_3^3 + \bar{\varepsilon}_3^3 \times \bar{r}_4^3 + \bar{\omega}_3^3 \times \left(\bar{\omega}_3^3 \times \bar{r}_4^3 \right) \right\};$$

$$[\bar{a}]_4^4 = \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -\ddot{q}_2 (l_4 + q_3) - 2\dot{q}_2 \dot{q}_3 \\ -\dot{q}_2^2 (l_4 + q_3) + \ddot{q}_3 \\ \ddot{q}_1 + g \end{bmatrix} + \begin{bmatrix} 0 & -\ddot{q}_2 & 0 \\ \ddot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \right.$$

$$\left. \cdot \begin{bmatrix} 0 & -\dot{q}_2 & 0 \\ \dot{q}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} cq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] - sq_4 (\ddot{q}_1 + g) \\ -\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 \\ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) \end{bmatrix}; \qquad (35)$$

$$[\bar{a}]_5^5 = [R]_4^5 \cdot \left\{ \bar{a}_4^4 + \bar{\varepsilon}_4^4 \times \bar{r}_5^4 + \bar{\omega}_4^4 \times \left(\bar{\omega}_4^4 \times \bar{r}_5^4 \right) \right\};$$

$$[\bar{a}]_5^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix} \cdot \left\{ \begin{bmatrix} cq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] - sq_4 (\ddot{q}_1 + g) \\ -\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 \\ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) \end{bmatrix} + \right.$$

$$+ \begin{bmatrix} 0 & -\ddot{q}_2 cq_4 + \dot{q}_2 \dot{q}_4 sq_4 & \ddot{q}_4 \\ \ddot{q}_2 cq_4 - \dot{q}_2 \dot{q}_4 sq_4 & 0 & \ddot{q}_2 sq_4 + \dot{q}_2 \dot{q}_4 cq_4 \\ -\ddot{q}_4 & -\ddot{q}_2 sq_4 - \dot{q}_2 \dot{q}_4 cq_4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix} +$$

$$\left. + \begin{bmatrix} 0 & -\dot{q}_2 cq_4 & \dot{q}_4 \\ \dot{q}_2 cq_4 & 0 & \dot{q}_2 sq_4 \\ -\dot{q}_4 & -\dot{q}_2 sq_4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_2 cq_4 & \dot{q}_4 \\ \dot{q}_2 cq_4 & 0 & \dot{q}_2 sq_4 \\ -\dot{q}_4 & -\dot{q}_2 sq_4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix} \right\} =$$

$$= \begin{bmatrix} cq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] - sq_4 (\ddot{q}_1 + g) + (\ddot{q}_2 cq_4 + \dot{q}_2 \dot{q}_4 sq_4) l_6 - \dot{q}_2 \dot{q}_4 l_6 sq_4 \\ \hdashline cq_5 \left[-\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2 \right) l_6 \right] + sq_5 \{ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - \\ - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) + (-\ddot{q}_2 sq_4 - \dot{q}_2 \dot{q}_4 cq_4) l_6 + \dot{q}_2 \dot{q}_4 l_6 cq_4 \} \\ \hdashline -sq_5 \left[-\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2 \right) l_6 \right] + cq_5 \{ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - \\ - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) + (-\ddot{q}_2 sq_4 - \dot{q}_2 \dot{q}_4 cq_4) l_6 + \dot{q}_2 \dot{q}_4 l_6 cq_4 \} \end{bmatrix}; \qquad (36)$$

$$[\bar{a}]_6^6 = [R]_5^6 \cdot \left\{ \bar{a}_5^5 + \bar{\varepsilon}_5^5 \times \bar{r}_6^5 + \bar{\omega}_5^5 \times \left(\bar{\omega}_5^5 \times \bar{r}_6^5 \right) \right\};$$

$$[\bar{a}]_6^6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} cq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] - sq_4 (\ddot{q}_1 + g) + \\ + (\ddot{q}_2 cq_4 + \dot{q}_2 \dot{q}_4 sq_4) l_6 - \dot{q}_2 \dot{q}_4 l_6 sq_4 \\ \hdashline cq_5 \left[-\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2 \right) l_6 \right] + \\ + sq_5 \{ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) + \\ + (-\ddot{q}_2 sq_4 - \dot{q}_2 \dot{q}_4 cq_4) l_6 + \dot{q}_2 \dot{q}_4 l_6 cq_4 \} \\ \hdashline -sq_5 \left[-\dot{q}_2^2 (l_4 + l_5 + q_3) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2 \right) l_6 \right] + \\ + cq_5 \{ sq_4 [-\ddot{q}_2 (l_4 + l_5 + q_3) - 2\dot{q}_2 \dot{q}_3] + cq_4 (\ddot{q}_1 + g) + \\ + (-\ddot{q}_2 sq_4 - \dot{q}_2 \dot{q}_4 cq_4) l_6 + \dot{q}_2 \dot{q}_4 l_6 cq_4 \} \end{bmatrix} + \right.$$

$$
+\left[\begin{array}{c|c|c}
0 & \begin{array}{l}\ddot{q}_4 sq_5 - \\ - cq_5\left(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4\right) - \\ - \dot{q}_5\left(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5\right)\end{array} & \begin{array}{l}\ddot{q}_4 cq_5 + \\ + sq_5\left(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4\right) + \\ + \dot{q}_5\left(\dot{q}_2 cq_5 cq_4 - \dot{q}_4 sq_5\right)\end{array} \\ \hline
\begin{array}{l}-\ddot{q}_4 sq_5 + \\ + cq_5\left(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4\right) + \\ + \dot{q}_5\left(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5\right)\end{array} & 0 & \ddot{q}_2 sq_4 + \dot{q}_2\dot{q}_4 cq_4 - \ddot{q}_5 \\ \hline
\begin{array}{l}-\ddot{q}_4 cq_5 - \\ - sq_5\left(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4\right) - \\ - \dot{q}_5\left(\dot{q}_2 cq_5 cq_4 - \dot{q}_4 sq_5\right)\end{array} & -\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4 + \ddot{q}_5 & 0
\end{array}\right] \cdot \begin{bmatrix} 0 \\ l_7 \\ 0 \end{bmatrix} +
$$

$$
+\begin{bmatrix} 0 & \dot{q}_4 sq_5 - \dot{q}_2 cq_5 cq_4 & \dot{q}_4 cq_5 + \dot{q}_2 sq_5 cq_4 \\ -\dot{q}_4 sq_5 + \dot{q}_2 cq_5 cq_4 & 0 & \dot{q}_2 sq_4 - \dot{q}_5 \\ -\dot{q}_4 cq_5 - \dot{q}_2 sq_5 cq_4 & -\dot{q}_2 sq_4 + \dot{q}_5 & 0 \end{bmatrix} \cdot
$$

$$
\left. \cdot \begin{bmatrix} 0 & \dot{q}_4 sq_5 - \dot{q}_2 cq_5 cq_4 & \dot{q}_4 cq_5 + \dot{q}_2 sq_5 cq_4 \\ -\dot{q}_4 sq_5 + \dot{q}_2 cq_5 cq_4 & 0 & \dot{q}_2 sq_4 - \dot{q}_5 \\ -\dot{q}_4 cq_5 - \dot{q}_2 sq_5 cq_4 & -\dot{q}_2 sq_4 + \dot{q}_5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_7 \\ 0 \end{bmatrix} \right\} =
$$

$$
= \left[\begin{array}{l}
cq_4\left[-\ddot{q}_2\left(l_4 + l_5 + q_3\right) - 2\dot{q}_2\dot{q}_3\right] - sq_4\left(\ddot{q}_1 + g\right) + \left(\ddot{q}_2 cq_4 + \dot{q}_2\dot{q}_4 sq_4\right)l_6 - \dot{q}_2\dot{q}_4 l_6 sq_4 + \\
\quad + \left[\ddot{q}_4 sq_5 - cq_5\left(\ddot{q}_2 cq_4 - \dot{q}_2\dot{q}_4 sq_4\right) - \dot{q}_5\left(-\dot{q}_2 sq_5 cq_4 - \dot{q}_4 cq_5\right)\right]l_7 + \\
\quad + \left(\dot{q}_4 cq_5 + \dot{q}_2 sq_5 cq_4\right)\cdot\left(-\dot{q}_2 sq_4 + \dot{q}_5\right)l_7 \\ \hline
cq_5\left[-\dot{q}_2^2\left(l_4 + l_5 + q_3\right) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2\right)l_6\right] + \\
\quad + sq_5\left\{sq_4\left[-\ddot{q}_2\left(l_4 + l_5 + q_3\right) - 2\dot{q}_2\dot{q}_3\right] + cq_4\left(\ddot{q}_1 + g\right) + \right. \\
\quad \left. + \left(-\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4\right)l_6 + \dot{q}_2\dot{q}_4 l_6 cq_4\right\} + \left[\left(-\dot{q}_2 cq_4 cq_5 + \dot{q}_4 sq_5\right)\cdot\right. \\
\quad \left. \cdot\left(\dot{q}_2 cq_4 cq_5 - \dot{q}_4 sq_5\right) + \left(\dot{q}_2 sq_4 - \dot{q}_5\right)\cdot\left(-\dot{q}_2 sq_4 + \dot{q}_5\right)l_7\right] \\ \hline
-sq_5\left[-\dot{q}_2^2\left(l_4 + l_5 + q_3\right) + \ddot{q}_3 + \left(-c^2 q_4 \dot{q}_2^2 - s^2 q_4 \dot{q}_2^2\right)l_6\right] + \\
\quad + cq_5\left\{sq_4\left[-\ddot{q}_2\left(l_4 + l_5 + q_3\right) - 2\dot{q}_2\dot{q}_3\right] + cq_4\left(\ddot{q}_1 + g\right) + \right. \\
\quad \left. + \left(-\ddot{q}_2 sq_4 - \dot{q}_2\dot{q}_4 cq_4\right)l_6 + \dot{q}_2\dot{q}_4 l_6 cq_4\right\} + \left(-\ddot{q}_2 sq_4 - \dot{q}_4\dot{q}_2 cq_4 + \ddot{q}_5\right)l_7 + \\
\quad + \left[\left(-\dot{q}_2 cq_4 sq_5 - \dot{q}_4 cq_5\right)\cdot\left(-\dot{q}_2 cq_4 cq_5 + \dot{q}_4 sq_5\right)\right]l_7
\end{array}\right] \tag{37}
$$

Kinematics operational parameters in the system $O_6x_6y_6z_6$ may be written, in accordance with (ISP04), following form:

$$
\left[\dot{X}\right]^6 = \left[\begin{array}{c}
\left[-\dot{q}_2\left(l_4 + l_5 + q_3\right)\right]cq_4 - \dot{q}_1 sq_4 - \dot{q}_2 l_6 cq_4 + \left(\dot{q}_4 sq_5 - \dot{q}_2 cq_5 cq_4\right)l_7 \\
\dot{q}_3 cq_5 + sq_5\left\{\left[-\dot{q}_2\left(l_4 + l_5 + q_3\right)\right]sq_4 + \dot{q}_1 cq_4 - \dot{q}_2 l_6 sq_4\right\} \\
-\dot{q}_3 sq_5 + cq_5\left\{\left[-\dot{q}_2\left(l_4 + l_5 + q_3\right)\right]sq_4 + \dot{q}_1 cq_4 - \dot{q}_2 l_6 sq_4\right\} + \left(-\dot{q}_2 sq_4 + \dot{q}_5\right)l_7 \\ \hline
-sq_4\dot{q}_2 + \dot{q}_5 \\
\dot{q}_4 cq_5 + \dot{q}_2 sq_5 cq_4 \\
-\dot{q}_4 sq_5 + \dot{q}_2 cq_5 cq_4
\end{array}\right]; \tag{38}
$$

$$
\left[\ddot{\bar{X}}\right]^{6}=\left[\begin{array}{c}
cq_4\left[-\ddot{q}_2\left(l_4+l_5+q_3\right)-2\dot{q}_2\dot{q}_3\right]-sq_4\left(\ddot{q}_1+g\right)+\left(\ddot{q}_2cq_4+\dot{q}_2\dot{q}_4sq_4\right)l_6-\dot{q}_2\dot{q}_4l_6sq_4+\\
+\left[\ddot{q}_4sq_5-cq_5\left(\ddot{q}_2cq_4-\dot{q}_2\dot{q}_4sq_4\right)-\dot{q}_5\left(-\dot{q}_2sq_5cq_4-\dot{q}_4cq_5\right)\right]l_7+\\
+\left(\dot{q}_4cq_5+\dot{q}_2sq_5cq_4\right)\cdot\left(-\dot{q}_2sq_4+\dot{q}_5\right)l_7\\
\hdashline
cq_5\left[-\dot{q}_2^2\left(l_4+l_5+q_3\right)+\ddot{q}_3+\left(-c^2q_4\dot{q}_2^2-s^2q_4\dot{q}_2^2\right)l_6\right]+\\
+sq_5\left\{sq_4\left[-\ddot{q}_2\left(l_4+l_5+q_3\right)-2\dot{q}_2\dot{q}_3\right]+cq_4\left(\ddot{q}_1+g\right)+\right.\\
\left.+\left(-\ddot{q}_2sq_4-\dot{q}_2\dot{q}_4cq_4\right)l_6+\dot{q}_2\dot{q}_4l_6cq_4\right\}+\left[\left(-\dot{q}_2cq_4cq_5+\dot{q}_4sq_5\right)\cdot\right.\\
\left.\cdot\left(\dot{q}_2cq_4cq_5-\dot{q}_4sq_5\right)+\left(\dot{q}_2sq_4-\dot{q}_5\right)\cdot\left(-\dot{q}_2sq_4+\dot{q}_5\right)l_7\right]\\
\hdashline
-sq_5\left[-\dot{q}_2^2\left(l_4+l_5+q_3\right)+\ddot{q}_3+\left(-c^2q_4\dot{q}_2^2-s^2q_4\dot{q}_2^2\right)l_6\right]+\\
+cq_5\left\{sq_4\left[-\ddot{q}_2\left(l_4+l_5+q_3\right)-2\dot{q}_2\dot{q}_3\right]+cq_4\left(\ddot{q}_1+g\right)+\right.\\
\left.+\left(-\ddot{q}_2sq_4-\dot{q}_2\dot{q}_4cq_4\right)l_6+\dot{q}_2\dot{q}_4l_6cq_4\right\}+\left(-\ddot{q}_2sq_4-\dot{q}_4\dot{q}_2cq_4+\ddot{q}_5\right)l_7+\\
+\left[\left(-\dot{q}_2cq_4sq_5-\dot{q}_4cq_5\right)\cdot\left(-\dot{q}_2cq_4cq_5+\dot{q}_4sq_5\right)\right]l_7\\
\hline
-\ddot{q}_2sq_4-\dot{q}_2\dot{q}_4cq_4+\ddot{q}_5\\
\hdashline
\ddot{q}_4cq_5+sq_5\left(\ddot{q}_2cq_4-\dot{q}_2\dot{q}_4sq_4\right)+\dot{q}_5\left(\dot{q}_2cq_5cq_4-\dot{q}_4sq_5\right)\\
\hdashline
-\ddot{q}_4sq_5+cq_5\left(\ddot{q}_2cq_4-\dot{q}_2\dot{q}_4sq_4\right)+\dot{q}_5\left(-\dot{q}_2sq_5cq_4-\dot{q}_4cq_5\right)
\end{array}\right].
\tag{39}
$$

Using relations of processing, the kinematics parameters can be determined of the fixed system operational $O_0x_0y_0z_0$ at the base robot. Those relations are:

$$
\left[\bar{v}\right]_6^0=\left[R\right]_6^0\cdot\left[\bar{v}\right]_6^6,
\tag{40}
$$

$$
\left[\bar{v}\right]_6^0=\begin{bmatrix}
cq_2cq_4 & -sq_2cq_5+cq_2sq_4sq_5 & sq_2sq_5+cq_2sq_4cq_5\\
sq_2cq_4 & cq_2cq_5+sq_2sq_4sq_5 & -cq_2sq_5+sq_2sq_4cq_5\\
-sq_4 & cq_4sq_5 & cq_4cq_5
\end{bmatrix}\cdot
$$
$$
\cdot\begin{bmatrix}
\left[-\dot{q}_2\left(l_4+l_5+q_3\right)\right]cq_4-\dot{q}_1sq_4-\dot{q}_2l_6cq_4+\left(\dot{q}_4sq_5-\dot{q}_2cq_5cq_4\right)l_7\\
\dot{q}_3cq_5+sq_5\left\{\left[-\dot{q}_2\left(l_4+l_5+q_3\right)\right]sq_4+\dot{q}_1cq_4-\dot{q}_2l_6sq_4\right\}\\
-\dot{q}_3sq_5+cq_5\left\{\left[-\dot{q}_2\left(l_4+l_5+q_3\right)\right]sq_4+\dot{q}_1cq_4-\dot{q}_2l_6sq_4\right\}+\left(-\dot{q}_2sq_4+\dot{q}_5\right)l_7
\end{bmatrix}=
$$
$$
=\left[\begin{array}{c}
\dot{q}_4l_7cq_2cq_4sq_5-\dot{q}_3sq_2-\dot{q}_2l_7sq_2sq_4sq_5+\dot{q}_5l_7sq_2sq_5-\dot{q}_2q_3cq_2-l_4\dot{q}_2cq_2-l_5\dot{q}_2cq_2-\\
-l_6\dot{q}_2cq_2+\dot{q}_5l_7cq_2sq_4cq_5-\dot{q}_2l_7cq_2cq_5\\
\hdashline
\dot{q}_3cq_2-\dot{q}_5l_7cq_2sq_5-\dot{q}_2q_3sq_2-l_4\dot{q}_2sq_2-l_5\dot{q}_2sq_2-l_6\dot{q}_2sq_2+\dot{q}_4l_7sq_2cq_4sq_5+\\
+\dot{q}_2l_7cq_2sq_4sq_5+\dot{q}_5l_7sq_2sq_4cq_5-\dot{q}_2l_7sq_2cq_5\\
\hdashline
\dot{q}_1-\dot{q}_4l_7sq_4sq_5+\dot{q}_5l_7cq_5cq_4
\end{array}\right];
\tag{41}
$$

$$
\left[\bar{\omega}\right]_6^0=\left[R\right]_6^0\cdot\left[\bar{\omega}\right]_6^6,\quad
\left[\bar{\omega}\right]_6^0=\begin{bmatrix}
\dot{q}_5cq_2cq_4-\dot{q}_4sq_2\\
\dot{q}_5sq_2cq_4+\dot{q}_4cq_2\\
\dot{q}_2-\dot{q}_5sq_4
\end{bmatrix};
\tag{42}
$$

$$[\bar{a}]_6^0 = [R]_6^0 \cdot [\bar{a}]_6^6 ,$$

$$[\bar{a}]_6^0 = \begin{bmatrix} \begin{gathered} -\ddot{q}_3 sq_2 - \ddot{q}_2 l_7 sq_2 sq_5 sq_4 - 2\dot{q}_2\dot{q}_4 l_7 sq_2 sq_5 cq_4 + \ddot{q}_5 l_7 cq_2 sq_4 cq_5 + \ddot{q}_5 l_7 sq_2 sq_5 + \\ + 2\dot{q}_2\dot{q}_5 l_7 cq_2 sq_5 + \dot{q}_5^2 l_7 sq_2 cq_5 + \dot{q}_2^2 l_5 sq_2 + q_3\dot{q}_2^2 sq_2 + \dot{q}_2^2 l_4 sq_2 - \\ - \ddot{q}_2 l_6 cq_2 - \ddot{q}_2 l_7 cq_2 cq_5 - q_3\ddot{q}_2 cq_2 - \ddot{q}_2 l_4 cq_2 - \dot{q}_4^2 l_7 cq_2 sq_4 sq_5 - \\ - \dot{q}_2^2 l_7 cq_2 sq_4 sq_5 - 2\dot{q}_2\dot{q}_3 cq_2 - \ddot{q}_5 l_5 cq_2 + \dot{q}_2^2 l_7 sq_2 cq_5 + \dot{q}_2^2 l_6 sq_2 - \\ - 2\dot{q}_2\dot{q}_5 l_7 sq_2 cq_5 sq_4 + \ddot{q}_4 l_7 cq_2 cq_4 sq_5 + 2\dot{q}_4\dot{q}_5 l_7 cq_2 cq_4 cq_5 - \dot{q}_5^2 l_7 cq_2 sq_4 sq_5 \end{gathered} \\ \begin{gathered} -\ddot{q}_2 l_6 sq_2 - \dot{q}_2^2 l_7 cq_2 cq_5 - q_3\ddot{q}_2 sq_2 - \dot{q}_4^2 l_7 sq_2 sq_4 sq_5 + 2\dot{q}_2\dot{q}_5 l_7 sq_2 sq_5 - \\ - \dot{q}_2^2 l_7 sq_2 sq_4 sq_5 - \dot{q}_2^2 l_6 cq_2 - 2\dot{q}_2\dot{q}_3 sq_2 - \ddot{q}_2 l_4 sq_2 - \dot{q}_2^2 l_4 cq_2 - \ddot{q}_5 l_5 sq_2 - \\ - \ddot{q}_2 l_7 sq_2 cq_5 - \dot{q}_2^2 l_5 cq_2 - q_3\dot{q}_2^2 cq_2 + \ddot{q}_3 cq_2 - \dot{q}_5^2 l_7 cq_2 cq_5 - \ddot{q}_5 l_7 cq_2 sq_5 + \\ + \ddot{q}_4 l_7 sq_2 cq_4 sq_5 + 2\dot{q}_4\dot{q}_5 l_7 sq_2 cq_4 cq_5 + 2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_5 sq_4 - \dot{q}_5^2 l_7 sq_2 sq_4 sq_5 + \\ + \ddot{q}_2 l_7 cq_2 sq_5 sq_4 + 2\dot{q}_2\dot{q}_4 l_7 cq_2 sq_5 cq_4 + \ddot{q}_5 l_7 sq_2 sq_4 cq_5 \end{gathered} \\ \ddot{q}_1 + g - \dot{q}_4^2 l_7 cq_4 sq_5 - \ddot{q}_4 l_7 sq_4 sq_5 - \dot{q}_5^2 l_7 cq_4 sq_5 - 2\dot{q}_4\dot{q}_5 l_7 sq_4 cq_5 + \ddot{q}_5 l_7 cq_4 cq_5 \end{bmatrix}; \quad (43)$$

$$[\bar{\varepsilon}]_6^0 = [R]_6^0 \cdot [\bar{\varepsilon}]_6^6 , \; [\bar{\varepsilon}]_6^0 = \begin{bmatrix} \ddot{q}_5 cq_2 cq_4 - \dot{q}_2\dot{q}_4 cq_2 - \dot{q}_4\dot{q}_5 cq_2 sq_4 - \ddot{q}_4 sq_2 - \dot{q}_2\dot{q}_5 sq_2 cq_4 \\ \ddot{q}_5 sq_2 cq_4 - \dot{q}_2\dot{q}_4 sq_2 - \dot{q}_4\dot{q}_5 sq_2 sq_4 + \ddot{q}_4 cq_2 + \dot{q}_2\dot{q}_5 cq_2 cq_4 \\ \ddot{q}_2 - \ddot{q}_5 sq_4 - \dot{q}_4\dot{q}_5 cq_4 \end{bmatrix}. \quad (44)$$

In accordance with [Isp04], relations (41), (42), (43) and (44), may be expressed operational speed and acceleration in fixed system $O_0x_0y_0z_0$. So:

$$[\dot{\bar{X}}]^0 = \begin{bmatrix} \begin{gathered} \dot{q}_4 l_7 cq_2 cq_4 sq_5 - \dot{q}_3 sq_2 - \dot{q}_2 l_7 sq_2 sq_4 sq_5 + \dot{q}_5 l_7 sq_2 sq_5 - \dot{q}_2 q_3 cq_2 - l_4\dot{q}_2 cq_2 - l_5\dot{q}_2 cq_2 - \\ - l_6\dot{q}_2 cq_2 + \dot{q}_5 l_7 cq_2 sq_4 cq_5 - \dot{q}_2 l_7 cq_2 cq_5 \end{gathered} \\ \begin{gathered} \dot{q}_3 cq_2 - \dot{q}_5 l_7 cq_2 sq_5 - \dot{q}_2 q_3 sq_2 - l_4\dot{q}_2 sq_2 - l_5\dot{q}_2 sq_2 - l_6\dot{q}_2 sq_2 + \dot{q}_4 l_7 sq_2 cq_4 sq_5 + \\ + \dot{q}_2 l_7 cq_2 sq_4 sq_5 + \dot{q}_5 l_7 sq_2 sq_4 cq_5 - \dot{q}_2 l_7 sq_2 cq_5 \end{gathered} \\ \dot{q}_1 - \dot{q}_4 l_7 sq_4 sq_5 + \dot{q}_5 l_7 cq_5 cq_4 \\ \hline \dot{q}_5 cq_2 cq_4 - \dot{q}_4 sq_2 \\ \dot{q}_5 sq_2 cq_4 + \dot{q}_4 cq_2 \\ \dot{q}_2 - \dot{q}_5 sq_4 \end{bmatrix}; \quad (45)$$

$$
\left[\ddot{X}\right]^0 = \begin{bmatrix}
\begin{array}{l}
-\ddot{q}_3 sq_2 - \ddot{q}_2 l_7 sq_2 sq_5 sq_4 - 2\dot{q}_2\dot{q}_4 l_7 sq_2 sq_5 cq_4 + \ddot{q}_5 l_7 cq_2 sq_4 cq_5 + \ddot{q}_5 l_7 sq_2 sq_5 + \\
+2\dot{q}_2\dot{q}_5 l_7 cq_2 sq_5 + \dot{q}_5^2 l_7 sq_2 cq_5 + \dot{q}_2^2 l_5 sq_2 + q_3\dot{q}_2^2 sq_2 + \dot{q}_2^2 l_4 sq_2 - \\
-\ddot{q}_2 l_6 cq_2 - \ddot{q}_2 l_7 cq_2 cq_5 - q_3\ddot{q}_2 cq_2 - \ddot{q}_2 l_4 cq_2 - \dot{q}_4^2 l_7 cq_2 sq_4 sq_5 - \\
-\dot{q}_2^2 l_7 cq_2 sq_4 sq_5 - 2\dot{q}_2\dot{q}_3 cq_2 - \ddot{q}_5 l_5 cq_2 + \dot{q}_2^2 l_7 sq_2 cq_5 + \dot{q}_2^2 l_6 sq_2 - \\
-2\dot{q}_2\dot{q}_5 l_7 sq_2 cq_5 sq_4 + \ddot{q}_4 l_7 cq_2 cq_4 sq_5 + 2\dot{q}_4\dot{q}_5 l_7 cq_2 cq_4 cq_5 - \dot{q}_5^2 l_7 cq_2 sq_4 sq_5
\end{array} \\
\begin{array}{l}
-\ddot{q}_2 l_6 sq_2 - \dot{q}_2^2 l_7 cq_2 cq_5 - q_3\ddot{q}_2 sq_2 - \dot{q}_4^2 l_7 sq_2 sq_4 sq_5 + 2\dot{q}_2\dot{q}_5 l_7 sq_2 sq_5 - \\
-\dot{q}_2^2 l_7 sq_2 sq_4 sq_5 - \dot{q}_2^2 l_6 cq_2 - 2\dot{q}_2\dot{q}_3 sq_2 - \ddot{q}_2 l_4 sq_2 - \dot{q}_2^2 l_4 cq_2 - \ddot{q}_5 l_5 sq_2 - \\
-\ddot{q}_2 l_7 sq_2 cq_5 - \dot{q}_2^2 l_5 cq_2 - q_3\dot{q}_2^2 cq_2 + \ddot{q}_3 cq_2 - \dot{q}_5^2 l_7 cq_2 cq_5 - \ddot{q}_5 l_7 cq_2 sq_5 + \\
+\ddot{q}_4 l_7 sq_2 cq_4 sq_5 + 2\dot{q}_4\dot{q}_5 l_7 sq_2 cq_4 cq_5 + 2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_5 sq_4 - \dot{q}_5^2 l_7 sq_2 sq_4 sq_5 + \\
+\ddot{q}_2 l_7 cq_2 sq_5 sq_4 + 2\dot{q}_2\dot{q}_4 l_7 cq_2 sq_5 cq_4 + \ddot{q}_5 l_7 sq_2 sq_4 cq_5
\end{array} \\
\ddot{q}_1 + g - \dot{q}_4^2 l_7 cq_4 sq_5 - \ddot{q}_4 l_7 sq_4 sq_5 - \dot{q}_5^2 l_7 cq_4 sq_5 - 2\dot{q}_4\dot{q}_5 l_7 sq_4 cq_5 + \ddot{q}_5 l_7 cq_4 cq_5 \\
\hline
\ddot{q}_5 cq_2 cq_4 - \dot{q}_2\dot{q}_4 cq_2 - \dot{q}_4\dot{q}_5 cq_2 sq_4 - \ddot{q}_4 sq_2 - \dot{q}_2\dot{q}_5 sq_2 cq_4 \\
\ddot{q}_5 sq_2 cq_4 - \dot{q}_2\dot{q}_4 sq_2 - \dot{q}_4\dot{q}_5 sq_2 sq_4 + \ddot{q}_4 cq_2 + \dot{q}_2\dot{q}_5 cq_2 cq_4 \\
\ddot{q}_2 - \ddot{q}_5 sq_4 - \dot{q}_4\dot{q}_5 cq_4
\end{bmatrix}. \qquad (46)
$$

3. CONCLUSIONS

Equations (39), (40), (45) and (46) are direct kinematics model equations with which can be determined the kinematics operational parameters of the prehensiune device in relation to the reference systems (T_6) and (T_0).

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MODELUL CINEMATIC DIRECT AL ROBOTULUI SERIAL MODULAR TRTRR

Rezumat: Lucrarea prezintă modelarea cinematică a unui robot serial modular cu cinci grade de mobilitate, care este compus din trei module de rotație şi două de translație. Robotul, astfel conceput, face parte dintr-o celulă de fabricație, pentru care structura este impusă de mişcările necesare din punct de vedere funcțional, pentru încadrarea în structura propusă. În această lucrare se propune modelarea cinematică a robotului TRTRR, care este continuarea firească a modelării geometrice.

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