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# DYNAMICS EQUATIONS OF TRTRR ROBOT USING LAGRANGE FORMALITY 

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#### Abstract

Consider the case of a robot with five degrees of mobility, consisting of two modules of translation and three modules of rotation, for which, investigating dynamic action under active forces and linkage, by using Lagrange's equations. The study shall be carried out with a view to in order to optimize energy consumption, by placing modules in mechanical structure of the robot


Keywords: TRTTR robot, dynamics equations, Lagrange formality.

## 1. GENERAL KNOWLEDGES

The robot industrial modular serial TRTRR cinematic diagram of which is shown in figure 1 consists of the following modules, based on robot: The vertical translation module (MTV), module of robot arm rotation (MRB), the module for translation in the horizontal direction (MT), module for the horizontal rotation (MR), the guidance module (MO) of the prehensive tool.

Confronted by [Isp04], [Isp09a], and [Isp09b], were introduced in the figure 1 the following notations:

- $1_{\mathrm{i}}, \mathrm{i}=1 \div 7$, the constructive parameters of robot;
- $\Delta_{\mathrm{i}}, \mathrm{i}=1 \div 5$, motion axes;
- $\Delta_{6}$, parallel axis to the axis $\Delta_{2}$ of rotation of the arm, which passes through the center $\mathrm{C}_{6}$ of the prehensiune device;
- $\mathrm{k}_{\mathrm{i}}, \mathrm{i}=1 \div 5$, number of freedom degrees;
- $\mathrm{q}_{\mathrm{k}}, \dot{\mathrm{q}}_{\mathrm{k}}, \ddot{\mathrm{q}}_{\mathrm{k}}$, generalized: coordinates, velocities and accelerations;
- $\mathrm{O}_{\mathrm{i}}, \mathrm{i}=1 \div 5$, reference systems origins $\mathrm{O}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}$, which coincide with the centers of gravity of robot modules;
- $\mathrm{O}_{0}$, measuring base (zero point);
- $\mathrm{O}_{0} \mathrm{X}_{0} \mathrm{y}_{0} \mathrm{z}_{0}$, fixed reference Cartesian system;
- $\mathrm{O}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}, \mathrm{i}=1 \div 5$, mobile reference Cartesian systems, connected to the moving parts of robot modules;
- $\overline{\mathrm{G}}_{\mathrm{i}}, \mathrm{i}=1 \div 6$, weight forces modules related, respectively, the prehensive tool;
- $\bar{F}_{1}, \overline{\mathrm{~F}}_{3}$, motor forces;
- $\overline{\mathrm{M}}_{2}, \overline{\mathrm{M}}_{4}, \overline{\mathrm{M}}_{5}$, motor moments;
- $\mathrm{m}_{\mathrm{i}}$, $\mathrm{i}=1 \div 6$, masses modules related, respectively, the prehensive tool;
- $\mathrm{J}_{\Delta_{2}}^{(2)}$, mechanical axial moment of inertia for mobile crew rotation module 2 , determined in relation to the robot arm axis of rotation $\Delta_{2}$;
- $J_{z_{3}}^{(3)}$, the axial mechanical inertia of the module 3 in relation to the axis $\mathrm{O}_{3} \mathrm{Z}_{3}$;
- $J_{z_{4}}^{(4)}$, the axial mechanical inertia of the module 4 in relation to the axis $\mathrm{O}_{4} \mathrm{Z}_{4}$;
- $J_{\mathrm{z}_{5}}^{(5)}$, the axial mechanical inertia of the module 5 in relation to the axis $\mathrm{O}_{5} \mathrm{z}_{5}$;
- $J_{y_{5}}^{(5)}$, the axial mechanical inertia of the module 5 in relation to the axis $\mathrm{O}_{5} \mathrm{y}_{5}$;
- $J_{y_{6}}^{(6)}$, the axial mechanical inertia of the module 6 in relation to the axis $\mathrm{O}_{6} \mathrm{y}_{6}$;
- $J_{\Delta_{4}}^{(4)}$, the axial mechanical inertia of the module equipment 4 in relation to the axis $\Delta_{4}$;


Fig.4.11. Kinematics Structure of the TRTRR Robot for the
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- $\mathrm{J}_{\mathrm{x}_{6}}^{(6)}$, the axial mechanical inertia of the module 6 in relation to the axis $\mathrm{O}_{6} \mathrm{x}_{6}$;
- $J_{\Delta_{5}}^{(5)}$ the axial mechanical inertia of the mobile equipment 5 in relation to the axis $\Delta_{5}$;
- $J_{\Delta_{6}}^{(6)}$ the axial mechanical inertia of the prehensive device 6 in relation to the axis $\Delta_{6}$.


## 2. DINAMICS EQUATIONS OF THE TRTRR INDUSTRIAL ROBOT

Dynamic equations of industrial robot TRTRR shown in figure 1 is determined
using the Lagrange's equations II (second order), written, in accordance with [Isp04] and [Rip77], in the form of:

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{Ec}}{\partial \dot{\mathrm{q}}_{\mathrm{k}}}\right)-\frac{\partial \mathrm{Ec}}{\partial \mathrm{q}_{\mathrm{k}}}=\mathrm{Q}_{\mathrm{k}}, \mathrm{k}=1 \div 5 . \tag{1}
\end{equation*}
$$

In relations (1) shall be made the following points: Ec - represents kinetic energy of the entire robot; $\mathrm{q}_{\mathrm{k}}, \dot{\mathrm{q}}_{\mathrm{k}}$ - generalized coordinates and velocities, $\mathrm{Q}_{\mathrm{k}}$ - generalized forces; k - the number of freedom degrees of robot.

In the case of a material system, kinetic energy of the system is equal to the sum of kinetic energies its constituent elements. In accordance with [Voi83], the kinetic energy of a component " i " considered body rigid is determined by the relationship:

$$
\begin{equation*}
E_{c_{i}}=\frac{1}{2} M_{i}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right)_{i}+\frac{1}{2}\left(J_{x} \omega_{x}^{2}+J_{y} \omega_{y}^{2}+J_{z} \omega_{z}^{2}\right)_{i}, i=1 \div 6, \tag{2}
\end{equation*}
$$

If in figure 1 shall be chosen origins sheep of Cartesian reference systems, $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}$, i $=1 \div 5$, in the centers of gravity of modules, so that $\mathrm{x}_{\mathrm{c}}=\mathrm{y}_{\mathrm{c}}=\mathrm{z}_{\mathrm{c}}=0$. By choosing, also mobile reference systems so that their axes are aligned with the main directions corresponding inertia origin of these systems, the moments of mechanical inertia centrifugal are zero, i.e. $\mathrm{J}_{\mathrm{xy}}$ $=\mathrm{J}_{\mathrm{yz}}=\mathrm{J}_{\mathrm{zx}}=0$.

Having regard to similarity up to module 3 of robotics TRTR and TRTRR, in accordance with Figure 1, kinetic energies corresponding modules MTV, MRB and MT, watching relationship established in [Isp09a], have expressions:
$\mathrm{E}_{\mathrm{c}_{1}}=\frac{1}{2} \mathrm{~m}_{1} \dot{\mathrm{q}}_{1}^{2} ; \mathrm{E}_{\mathrm{c}_{2}}=\frac{1}{2} \mathrm{~m}_{2} \dot{\mathrm{q}}_{1}^{2}+\frac{1}{2} \mathrm{~J}_{\Delta_{2}}^{(2)} \dot{\mathrm{q}}_{2}^{2} ;$
$E_{\mathrm{c}_{3}}=\frac{1}{2} \mathrm{~m}_{3}\left(\dot{\mathrm{q}}_{1}^{2}+\dot{\mathrm{q}}_{3}^{2}\right)+\frac{1}{2}\left[\mathrm{~J}_{z_{3}}^{(3)}+\mathrm{m}_{3}\left(1_{4}+\mathrm{q}_{3}\right)^{2}\right]_{\dot{q}_{2}^{2}} ;$
For the guidance module MR of the composition robot arm is made the following points:

$$
\begin{align*}
& \omega_{x_{4}}=0, \omega_{y_{4}}=\dot{q}_{4}, \omega_{z_{4}}=\dot{q}_{2}, \\
& \bar{v}_{0_{4}}=\dot{\bar{q}}_{1}+\dot{\bar{q}}_{2} \times \bar{r}_{4}+\dot{\bar{q}}_{3}, J_{y_{4}}=J_{\Delta_{4}}^{(4)}, \quad J_{z_{4}}=J_{z_{4}}^{(4)} \tag{4}
\end{align*}
$$



Kinetic energy of the module MR, having regarded to figures 1 and 2 and the relations (2), (3), has the expression:
$\mathrm{E}_{\mathrm{c}_{4}}=\frac{1}{2} \mathrm{~m}_{4}\left(\dot{\mathrm{q}}_{1}^{2}+\dot{\mathrm{q}}_{3}^{2}\right)+\frac{1}{2}\left[\mathrm{~J}_{z_{4}}^{(4)}+\mathrm{m}_{4}\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{q}_{3}\right)^{2} \dot{\mathrm{q}}_{2}^{2}+\frac{1}{2} \mathrm{~J}_{4_{4}}^{(4)} \dot{\mathrm{q}}_{4}^{2} ;\right.$
For the guidance module MO of the prehensive device shall be reported following kinematics parameters:
$\omega_{x_{5}}=-\dot{q}_{5}, \omega_{y_{5}}=\dot{q}_{4}, \omega_{z_{5}}=\dot{q}_{2}, \bar{v}_{O_{5}}=\dot{\bar{q}}_{1}+\dot{\bar{q}}_{2} \times \bar{r}_{5}+\dot{\bar{q}}_{3}$,
$J_{x_{5}}=J_{\Lambda_{5}}^{(5)}, J_{y_{5}}=J_{y_{5}}^{(5)}, J_{z_{5}}=J_{z_{5}}^{(5)}$.
$\overline{\mathrm{r}}_{5}=\overline{\mathrm{I}}_{3}+\overline{\mathrm{l}}_{4}+\overline{\mathrm{q}}_{3}+\overline{\mathrm{l}}_{5}+\overline{\mathrm{I}}_{6}, \dot{\overline{\mathrm{q}}}_{2} \times \overline{\mathrm{r}}_{5}=\dot{\overline{\mathrm{q}}}_{2} \times\left(\overline{\mathrm{I}}_{4}+\overline{\mathrm{q}}_{3}+\overline{\mathrm{l}}_{5}+\overline{\mathrm{I}}_{6}\right)$,
$\overline{\mathrm{v}}_{\mathrm{O}_{5}}=\dot{\overline{\mathrm{q}}}_{1}+\dot{\overline{\mathrm{q}}}_{2} \times\left(\overline{\mathrm{I}}_{4}+\overline{\mathrm{q}}_{3}+\overline{\mathrm{I}}_{5}+\overline{\mathrm{I}}_{6}\right)+\dot{\overline{\mathrm{q}}}_{3}$,
$\mathrm{v}_{\mathrm{O}_{5}}^{2}=\dot{\mathrm{q}}_{1}^{2}+\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{l}_{6}+\mathrm{q}_{3}\right)^{2} \dot{\mathrm{q}}_{2}^{2}+\dot{\mathrm{q}}_{3}^{2}$,
Since
$\dot{\bar{q}}_{1} \perp \dot{\bar{q}}_{2} \times\left(\overline{\overline{1}}_{4}+\overline{\mathrm{I}}_{5}+\overline{\mathrm{I}}_{6}+\overline{\mathrm{q}}_{3}\right), \dot{\overline{\mathrm{q}}}_{1} \perp \dot{\bar{q}}_{3}, \dot{\overline{\mathrm{q}}}_{3} \perp \dot{\overline{\mathrm{q}}} 2 \times\left(\overline{\overline{1}}_{4}+\overline{\mathrm{l}}_{5}+\overline{\mathrm{I}}_{6}+\overline{\mathrm{q}}_{3}\right)$.
With these details, having regard to (2), the kinetic energy of guidance module MO has the expression:

$$
\begin{align*}
& E_{c_{5}}=\frac{1}{2} m_{5}\left(\dot{q}_{1}^{2}+\dot{q}_{3}^{2}\right)+\frac{1}{2}\left[J_{z_{5}}^{(5)}+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)^{2}\right] \dot{q}_{2}^{2}+ \\
& +\frac{1}{2} J_{v_{s}}^{(5)} \dot{q}_{4}^{2}+\frac{1}{2} J_{\Delta_{s}}^{(5)} \dot{q}_{5}^{2} ; \tag{8}
\end{align*}
$$

For prehensive DP device, watching figures 1 and 3 , it is necessary to make the following comments:

$$
\begin{align*}
& \omega_{v 5}=-\dot{q}_{5}, \omega_{y 5}=\dot{q}_{4}, \omega_{25}=\dot{q}_{2}, \bar{v}_{C_{6}}=\dot{\bar{q}}_{1}+\dot{\bar{q}}_{2} \times \bar{r}_{6}+\dot{\bar{q}}_{3}+\dot{\bar{q}}_{5} \times \bar{l}_{7}, \\
& \bar{r}_{6}=\bar{l}_{3}+\bar{l}_{4}+\bar{q}_{3}+\bar{l}_{5}+\bar{l}_{6}+\bar{l}_{7}, \dot{\bar{q}}_{2} \times \bar{r}_{6}=\dot{\bar{q}}_{2} \times\left(\bar{l}_{4}+\bar{l}_{5}+\bar{l}_{6}+\bar{l}_{7}+\bar{q}_{3}\right), \\
& \overline{\mathrm{v}}_{\mathrm{C}_{6}}=\dot{\bar{q}}_{1}+\dot{\bar{q}}_{2} \times\left(\overline{\mathrm{I}}_{4}+\overline{\mathrm{l}}_{5}+\overline{\mathrm{I}}_{6}+\overline{\mathrm{l}}_{7}+\overline{\mathrm{q}}_{3}\right)+\dot{\bar{q}}_{3}+\dot{\bar{q}}_{5} \times \overline{\mathrm{I}}_{7}, \\
& \mathrm{v}_{\mathrm{C}_{6}}=\dot{\mathrm{q}}_{1}^{2}+\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{l}_{6}+\mathrm{l}_{7}+\mathrm{q}_{3}\right)^{2} \dot{\mathrm{q}}_{2}^{2}+\dot{\mathrm{q}}_{3}^{2}+\mathrm{l}_{7}^{2} \dot{\mathrm{q}}_{5}^{2}+21_{7} \dot{\dot{q}}_{1} \dot{\mathrm{q}}_{5}, \tag{9}
\end{align*}
$$

Whereas, in accordance with Figure 3,

$$
\begin{align*}
& \dot{\bar{q}}_{1} \perp \dot{\bar{q}}_{2} \times\left(\bar{l}_{4}+\bar{l}_{5}+\bar{l}_{6}+\bar{l}_{7}+\bar{q}_{3}\right), \dot{\bar{q}} \perp \dot{\bar{q}}_{3}, \\
& \dot{\bar{q}}_{3} \perp \dot{\bar{q}}_{2} \times\left(\bar{l}_{4}+\bar{l}_{5}+\bar{l}_{6}+\bar{l}_{7}+\bar{q}_{3}\right), \\
& \dot{\overline{\mathrm{q}}}_{5} \times \overline{\mathrm{l}}_{7} \perp \dot{\overline{\mathrm{q}}}_{2} \times\left(\overline{\bar{l}}_{4}+\overline{\mathrm{l}}_{5}+\overline{\mathrm{l}}_{6}+\overline{\mathrm{l}}_{7}+\overline{\mathrm{q}}_{3}\right), \dot{\overline{\mathrm{q}}}_{3} \perp \dot{\overline{\mathrm{q}}}_{5} \times \overline{\mathrm{1}}_{7}, \tag{10}
\end{align*}
$$

And $\dot{\bar{q}}_{1}, \dot{\bar{q}}_{5} \times \overline{\mathrm{l}}_{7}$ are collinear, respectively

$$
\begin{equation*}
\mathrm{J}_{\mathrm{x}_{6}}=\mathrm{J}_{\mathrm{x}_{6}}^{(6)}, \mathrm{J}_{\mathrm{y}_{6}}=\mathrm{J}_{\mathrm{y}_{6}}^{(6)}, \mathrm{J}_{\mathrm{z}_{6}}=\mathrm{J}_{\Delta_{6}}^{(6)} . \tag{11}
\end{equation*}
$$

The kinetic energy of the prehensive, having regard to (2) and specifications (9), has the expression:
$E_{c_{6}}=\frac{1}{2} m_{6}\left(\dot{q}_{1}^{2}+\dot{q}_{3}^{2}\right)+\frac{1}{2}\left[J_{\Lambda_{6}}^{(6)}+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)^{2}\right] \dot{q}_{2}^{2}+$ $+\frac{1}{2} J_{y_{6}}^{(6)} \dot{q}_{4}^{2}+\frac{1}{2}\left[J_{x_{6}}^{(6)}+\frac{1}{2} m_{6} l_{7}^{2}\right] \dot{q}_{5}^{2}+m_{6} l_{7} \dot{q}_{1} \dot{q}_{5}$.

By adding kinetic energy of modules, expressed by relations (3), (4), (8) and prehensiune, expressed by the relationship (12), expression can be obtained kinetic energy of the robot:


Fig.3. $\mathrm{C}_{6}$ Gravity Force Point Velocity

$$
\begin{align*}
\mathrm{E}_{\mathrm{c}} & =\frac{1}{2}\left(\sum_{\mathrm{i}=1}^{6} \mathrm{~m}_{\mathrm{i}}\right) \dot{\mathrm{q}}_{1}^{2}+\frac{1}{2}\left[\mathrm{~J}_{\Delta_{2}}^{(2)}+\sum_{\mathrm{i}=3}^{5} \mathrm{~J}_{\mathrm{z}_{\mathrm{i}}}^{(\mathrm{i})}+\mathrm{J}_{\Delta_{6}}^{(6)}+\mathrm{m}_{3}\left(\mathrm{l}_{4}+\mathrm{q}_{3}\right)^{2}+\mathrm{m}_{4}\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{q}_{3}\right)^{2}+\right. \\
& \left.+\mathrm{m}_{5}\left(1_{4}+\mathrm{l}_{5}+\mathrm{l}_{6}+\mathrm{q}_{3}\right)^{2}+\mathrm{m}_{6}\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{l}_{6}+\mathrm{l}_{7}+\mathrm{q}_{3}\right)^{2}\right]_{\dot{q}_{2}^{2}}^{2}+\frac{1}{2}\left(\sum_{\mathrm{i}=3}^{6} \mathrm{~m}_{\mathrm{i}}\right) \dot{\mathrm{q}}_{3}^{2}+ \\
& +\frac{1}{2}\left[\mathrm{~J}_{4_{4}}^{(4)}+\sum_{\mathrm{i}=5}^{6} \mathrm{~J}_{\mathrm{y}_{\mathrm{i}}}^{(\mathrm{i}}\right] \dot{\mathrm{a}}_{4}^{2}+\frac{1}{2}\left[\mathrm{~J}_{\Delta_{5}}^{(5)}+\mathrm{J}_{\mathrm{x}_{6}}^{(6)}+\mathrm{m}_{6} 1_{7}^{2}\right] \dot{\mathrm{q}}_{5}^{2}+\mathrm{m}_{6} \mathrm{l}_{7} \dot{\mathrm{q}}_{1} \dot{q}_{5} . \tag{13}
\end{align*}
$$

In the expression (13) is made the following notations:
$\frac{1}{2}\left(\sum_{i=1}^{6} m\right)=c_{1}, \frac{1}{2}\left[\mathrm{~J}_{\Delta_{2}}^{(2)}+\sum_{\mathrm{i}=3}^{5} \mathrm{j}_{\mathrm{z}_{\mathrm{i}}}^{(\mathrm{i})}+\mathrm{J}_{\Delta_{6}}^{(6)}+\mathrm{m}_{3}\left(\mathrm{l}_{4}+\mathrm{q}_{\mathrm{B}}\right)^{2}+\mathrm{m}_{4}\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{q}_{3}\right)^{2}+\right.$ $\left.+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)^{2}+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)^{2}\right]=c_{2}, \frac{1}{2}\left(\sum_{i=3}^{6} m\right)=c_{3}$,
$\frac{1}{2}\left[\mathrm{~J}_{\Delta_{4}}^{(4)}+\sum_{\mathrm{i}=5}^{6} \mathrm{y}_{\mathrm{y}_{\mathrm{i}}}^{(\mathrm{i})}\right]=\mathrm{c}_{4}, \frac{1}{2}\left[\mathrm{~J}_{\Delta_{5}}^{(5)}+\mathrm{J}_{\mathrm{x}_{6}}^{(6)}+\mathrm{m}_{6} \mathrm{l}_{7}\right]=\mathrm{c}_{5}, \quad \mathrm{~m}_{6} \mathrm{l}_{7}=\mathrm{c}_{15}$

Thus, the kinetic energy of robot becomes:
$\mathrm{E}_{\mathrm{c}}=\mathrm{c}_{1} \dot{\mathrm{q}}_{1}^{2}+\mathrm{c}_{2} \dot{\mathrm{q}}_{2}^{2}+\mathrm{c}_{3} \dot{\mathrm{q}}_{3}^{2}+\mathrm{c}_{4} \dot{\mathrm{q}}_{4}^{2}+\mathrm{c}_{5} \dot{\mathrm{q}}_{5}^{2}+\mathrm{c}_{15} \dot{\mathrm{q}}_{1} \dot{\mathrm{q}}_{5}$.
Watching relations (1), made the following points:
$\frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \mathrm{q}_{1}}=0, \frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \mathrm{q}_{2}}=0, \frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \mathrm{q}_{4}}=0, \frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \mathrm{q}_{5}}=0 ;$
$\frac{\partial E_{c}}{\partial q_{3}}=m_{3}\left(l_{4}+q_{3}\right)+m_{4}\left(l_{4}+l_{5}+q_{3}\right)+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)+$
$+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)$,
$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \dot{\mathrm{q}}_{1}}\right)=2 \mathrm{c}_{1} \ddot{\mathrm{q}}_{1}+\mathrm{c}_{15} \ddot{\mathrm{q}}_{5}=\left(\sum_{\mathrm{i}=1}^{6} \mathrm{~m}_{\mathrm{i}}\right) \ddot{\mathrm{q}}_{1}+\mathrm{m}_{6} 1_{7} \ddot{\mathrm{q}}_{5}$, $\frac{d}{d}\left(\frac{\partial E_{c}}{\partial \dot{q}_{2}}\right)=2 c_{2} \ddot{q}_{2}+2 \frac{d c_{1}}{d t} \dot{q}_{2}=\left[J_{\Delta_{2}}^{2)}+\sum_{i=3}^{5} j_{z_{i}}^{(i)}+J_{A_{0}}^{(6)}+m_{3}\left(l_{4}+q_{3}\right)^{2}+m_{4}\left(l_{4}+l_{5}+q_{3}\right)^{2}+\right.$ $\left.+m_{3}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)^{2}+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)^{2}\right]_{q_{2}}+2\left[m_{3}\left(l_{4}+q_{3}\right)+\right.$ $+m_{4}\left(l_{4}+l_{5}+q_{3}\right)+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right) \dot{q}_{2} \dot{q}_{3}$, $\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \dot{\mathrm{q}}_{3}}\right)=2 \mathrm{c}_{3} \ddot{\mathrm{q}}_{3}=\left(\sum_{\mathrm{i}=3}^{6} \mathrm{~m}_{\mathrm{i}}\right) \ddot{\mathrm{q}}_{3}$,
$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \dot{\mathrm{q}}_{4}}\right)=2 \mathrm{c}_{4} \ddot{\mathrm{q}}_{4}=\left[\mathrm{J}_{\Delta_{4}}^{(4)}+\sum_{\mathrm{i}=5}^{6} \mathrm{~J}_{\mathrm{y}_{\mathrm{i}}}^{(\mathrm{i})}\right] \ddot{\mathrm{q}}_{4}$,
$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{E}_{\mathrm{c}}}{\partial \dot{\mathrm{q}}_{5}}\right)=2 \mathrm{c}_{5} \ddot{\mathrm{q}}_{5}+\mathrm{c}_{15} \ddot{\mathrm{q}}_{1}=\left[\mathrm{J}_{\Delta_{5}}^{(5)}+\mathrm{J}_{\mathrm{x}_{6}}^{(6)}+\mathrm{m}_{6} 1_{7}^{2}\right] \ddot{\mathrm{q}}_{5}+\mathrm{m}_{6} 1_{7} \ddot{\mathrm{q}}_{1}$.

Generalized forces corresponding to the five modules of robot are obtained by giving the placing of incremental virtual in such a way as to vary per row parameters $\mathrm{q}_{\mathrm{k}}$ respectively with $\delta q_{1}, \delta q_{2}, \delta q_{3}, \delta q_{4}, \delta q_{5}$. Motor forces and moments acting pairs as follows: Force $\overline{\mathrm{F}}_{1}$ acts on the mobile part of the module MTV and $-\overline{\mathrm{F}}_{1}$ acts on the fixed part of the translation module vertical; The $\overline{\mathrm{M}}_{2}$ acts on mobile part of the module MRB and $-\overline{\mathrm{M}}_{2}$ acts upon this module joined to the element of the module MTV; The force $\overline{\mathrm{F}}_{3}$ acts on mobile part of the module MT on displacement direction of $\mathrm{q}_{3}$ and $-\overline{\mathrm{F}}_{3}$ acts on part of the same translation module jointly and severally liable with the movable section of the rotation module MRB ; The moment $\overline{\mathrm{M}}_{4}$ acts on mobile part of the module MR of rotation and $-\overline{\mathrm{M}}_{4}$ acts on the rotation module party jointly and severally liable with the movable section of the translation module MT; The moment $\overline{\mathrm{M}}_{5}$ acts on mobile part of guidance mode device MO prehensive device and $-\overline{\mathrm{M}}_{5}$ acts on mode section MO joined to the element of the way MR of rotation.

For the purpose of determining generalized forces is to be determined virtual elementary mechanical work. Virtual mechanical work properly external forces and
moments and movements of virtual robot compatible with connections, as shown in figure 1 and [Isp04], is the expression:
$\delta=\left(F_{1}-G_{1}-G_{2}\right) \dot{d}_{1}+M_{2} \delta q_{2}+\left(\bar{F}_{3}+\overline{G_{3}}\right) \cdot \delta \overline{O_{0} O_{3}}+M_{4} \delta \dot{q}_{4}+$
$+\bar{G}_{4} \cdot \delta \overline{O_{0} O_{4}}+M_{5} \delta q_{5}+\bar{G}_{5} \cdot \delta \overline{O_{0} O_{5}}+\bar{G}_{6} \cdot \delta \overline{\delta O_{0}}$.
Having regard to that
$\overline{\mathrm{O}_{0} \mathrm{O}_{3}}=\overline{1}_{1}+\overline{\mathrm{q}}_{1}+\overline{1}_{2}+\overline{1}_{3}+\overline{1}_{4}+\overline{\mathrm{q}}_{3}, \quad \delta \overline{\mathrm{O}_{0} \mathrm{O}_{3}}=\delta \overline{\mathrm{q}}_{1}+\delta \overline{\mathrm{q}}_{3}$,
$\overline{\mathrm{O}_{0} \mathrm{O}_{4}}=\overline{\mathrm{l}}_{1}+\overline{\mathrm{q}}_{1}+\overline{\mathrm{I}}_{2}+\overline{\mathrm{I}}_{3}+\overline{\mathrm{I}}_{4}+\overline{\mathrm{q}}_{3}+\overline{1}_{5}, \quad \delta \overline{\mathrm{O}_{0} \mathrm{O}_{4}}=\delta \overline{\mathrm{q}}_{1}+\delta \overline{\mathrm{q}}_{3}$, $\overline{O_{0} O_{5}}=\bar{l}_{1}+\bar{q}_{1}+\bar{l}_{2}+\bar{l}_{3}+\bar{l}_{4}+\bar{q}_{3}+\bar{l}_{5}+\bar{l}_{6}, \delta \overline{O_{0} O_{5}}=\delta \bar{q}_{1}+\delta \bar{q}_{3}$, $\overline{O_{0} C_{6}}=\bar{l}_{1}+\bar{q}_{1}+\bar{l}_{2}+\bar{l}_{3}+\bar{l}_{4}+\bar{q}_{3}+\bar{l}_{5}+\bar{l}_{6}+\bar{l}_{7}$, $\delta \overline{O_{0} C_{6}}=\delta \bar{q}_{1}+\delta \bar{q}_{3}$,

The expression (17) mechanical work virtual reach form:
$\delta L=\left(\mathrm{F}_{1}-\sum_{\mathrm{i}=1}^{6} \mathrm{G}_{\mathrm{i}}\right) \delta \mathrm{q}_{1}+\mathrm{M}_{2} \delta \mathrm{q}_{2}+\mathrm{F}_{3} \delta \mathrm{q}_{3}+\mathrm{M}_{4} \delta \mathrm{q}_{4}+\mathrm{M}_{5} \delta \mathrm{q}_{5}$.
Using (19) it can observe that the $F_{1}$ and $G_{i}, i=1 \div 6$ forces produce the mechanical work for the virtual displacement $\delta q_{1}$, and the moment $\mathrm{M}_{2}$ produces the mechanical work for the virtual displacement $\delta \mathrm{q}_{2}$, and so one.

În conformitate cu [Isp04] şi [Isp06], forțele generalizate se obțin din relațiile:
$\mathrm{Q}_{\mathrm{k}}=\frac{\partial \mathrm{L}}{\partial \mathrm{q}_{\mathrm{k}}}, \mathrm{k}=1 \div 5$.
Considering connections olonome of the robot, they do not depend on explicit general speeds support that request and, accordingly, no virtual movements $\delta q_{k}$. Thus, incremental movements are independent this is why they can be considered null and void, with the exception of one. For example, if $\delta q_{1} \neq 0$, then $\delta q_{2}=\delta q_{3}=\delta q_{4}=\delta q_{5}=0 ; \quad$ If $\quad \delta q_{1}=0$, $\delta \mathrm{q}_{2} \neq 0, \quad \delta \mathrm{q}_{3}=\delta \mathrm{q}_{4}=\delta \mathrm{q}_{5}=0$, and so on. With these details, from (19) and (20) results generalized forces, whose expressions are:
$Q_{1}=F_{1}-\sum_{i=1}^{6} G_{i}, Q_{2}=M_{2}, Q_{3}=F_{3}, Q_{4}=M_{4}, Q_{5}=M_{5}$.
By entering into the equations (1) of the II Lagrange's expressions (16) and (19), the system of dynamic equations robot modular serial TRTRR, written in the form of:

$$
\left(\sum_{\mathrm{i}=1}^{6} \mathrm{~m}_{\mathrm{i}}\right) \ddot{\mathrm{q}}_{1}+\mathrm{m}_{6} \mathrm{l}_{7} \ddot{\mathrm{q}}_{5}=\mathrm{F}_{1}-\sum_{\mathrm{i}=1}^{6} \mathrm{G}_{\mathrm{i}}
$$

$$
\begin{align*}
& {\left[J_{\Delta_{2}}^{(2)}+\sum_{i=3}^{5} J_{z_{i}}^{(i)}+J_{\Lambda_{6}}^{(6)}+m_{3}\left(l_{4}+q_{3}\right)^{2}+m_{4}\left(l_{4}+l_{5}+q_{3}\right)^{2}+\right.} \\
& \left.+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)^{2}+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)^{2}\right] \ddot{q}_{2}+ \\
& +2\left[m_{3}\left(l_{4}+q_{3}\right)+m_{4}\left(l_{4}+l_{5}+q_{3}\right)+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)+\right. \\
& \left.+m_{6}\left(l_{4}+l_{5}+l_{6}+l_{7}+q_{3}\right)\right] \dot{q}_{2} \dot{q}_{3}=M_{2} \\
& \left(\sum_{i=3}^{6} m_{1} \ddot{q}_{3}-\left[m_{3}\left(l_{4}+q_{3}\right)+m_{4}\left(l_{4}+l_{5}+q_{3}\right)+m_{5}\left(l_{4}+l_{5}+l_{6}+q_{3}\right)+\right.\right. \\
& +\mathrm{m}_{( }\left(\mathrm{l}_{4}+\mathrm{l}_{5}+\mathrm{l}_{6}+\mathrm{l}_{7}+\mathrm{q}_{3}\right) \dot{q}^{2}=\mathrm{F}_{3} \\
& \mathrm{~m}_{5} 1_{6} \ddot{\mathrm{q}}_{1}+\left[\mathrm{J}_{\Delta_{4}}^{(4)}+\sum_{\mathrm{i}=5}^{6} \mathrm{~J}_{\mathrm{y}_{\mathrm{i}}}^{(\mathrm{i}}\right] \ddot{\mathrm{q}}_{4}=\mathrm{M}_{4} \text {, } \\
& \mathrm{m}_{6} 1_{7} \ddot{\mathrm{q}}_{1}+\left[\mathrm{J}_{\Delta_{5}}^{(5)}+\mathrm{J}_{\mathrm{x}_{6}}^{(6)}+\mathrm{m}_{6} \mathrm{l}_{7}^{2}\right] \ddot{\mathrm{q}}_{5}=\mathrm{M}_{5} \text {. } \tag{22}
\end{align*}
$$

## 3. CONCLUSIONS

Differential equations (22) of the robot have been established on the assumption that all movements were taking place at the same time. With the system of differential equations obtained should be able to fix the two fundamental questions mutual, straight forward and reverse, robot. In the case direct problem is determined robot, if known forces and moments acting on him. In the case inverse problem is assumed known and robot motion is required to determine the variation of laws and forces moments engines. This problem allows you to choose the drive motors taking into account organologia each module and robot dynamics; Allows, also, choice of law of movement on each axis and an optimal variants of the windowing modules in mechanical structure of the robot, in such a way that energy consumption are minimal.

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## ECUAȚIILE DINAMICE ALE ROBOTULUI TRTRR UTILIZÂND FORMALISMUL LUI LAGRANGE

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[^0]:    Rezumat: Se consideră un robot cu cinci grade de mobilitate, format din două module de translație şi trei module de rotație, pentru care, se studiază dinamica sub acțiune forțelor active şi de legătură, prin utilizarea ecuațiilor lui Lagrange. Studiul se realizează în vederea în vederea optimizării consumurilor energetice, prin aşezarea modulelor în structura mecanică a robotului.

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