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THE DYNAMIC EQUATIONS OF THE TRTTRR1 ROBOT USING THE LAGRANGE FORMALISM

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Abstract: *The dynamic study of the industrial robots allows solving the following issues: the optimal choice of arrangement of modules in a modular robot structure so that the energy consumptions to be minimal; the choice of a law motion on the robot axes so that the energy consumptions to be minimal; the choice of robot actuators, taking into account the organologia of each module and the whole robot dynamics. For this, the structural kinematic scheme of the robot is presented, then, using the second type of Lagrange's equations the kinetic energy and the generalized forces of the engines are determined. Finally, the dynamic equations of the TRTTRR1 robot are obtained, which are essential for solving the direct and inverse problems of the robot's dynamic.* ***Key words:*** *robot, kinetic energy, generalized forces, Lagrange's equations, engine moments, orientation module.*

1. INTRODUCTION

The dynamic study of serial industrial robots can be achieved using different methods, according to [1]. Thus, if using Lagrange's equations of second type or the virtual displacements principle as dynamic form, the differential equations of motion can be derived without being taken into account the reactions of torque and friction.

An original method of choosing the actuators is one that takes into account the whole dynamic of the robot and the mechanical structure of each module separately. Based on these considerations and using the Lagrange's equations of second type, the authors establish in this paper the dynamic equations of the robot.

2. THE DYNAMIC EQUATIONS USING THE LAGRANGE FORMALISM

In figure 1 is presented the kinematic scheme of the serial modular TRTTRR1 robot. Starting from the base of the robot, it can be mentioned the component parts of this: the module 1,called MTB, is the translation module from the robot's base; the module 2, called MRB, is the rotation module of the robot's arm; the module 3, MTV the vertical translation module; the module 4, MT the translation module of the clamping device; the module 5, MR is the rotation module from the arm composition; MO the orientation module of the clamping device.

In the structural kinematic scheme of the robot, according to [2] is noted:

$\Delta_i, i = 1 \div 6,$ - the movement axes;

Δ_7 - the parallel axis to Δ_2 of arm rotation, passing through the center C_7 of gravity of the clamping device;

$k_i, i = 1 \div 6$ - degrees of freedom; $q_k, \dot{q}_k, \ddot{q}_k$ - the generalized coordinates, velocities and accelerations; $l_0, l_i, i = 1 \div 7,$ - the design parameters of the robot; $m_i, i = 1 \div 7$ - masses;

$O_i, i = 1 \div 6$ - mobile reference systems origins that concided with the centers of gravity of the modules;

$\overline{G}_i, i = 1 \div 7$ - the gravity forces of the modules and of the clamping device;

$\overline{F}_1, \overline{F}_3, \overline{F}_4$ - the engine forces;

$\overline{M}_2, \overline{M}_5, \overline{M}_6$ - the engine moments;

O_0 - the basic measurement (zero point);

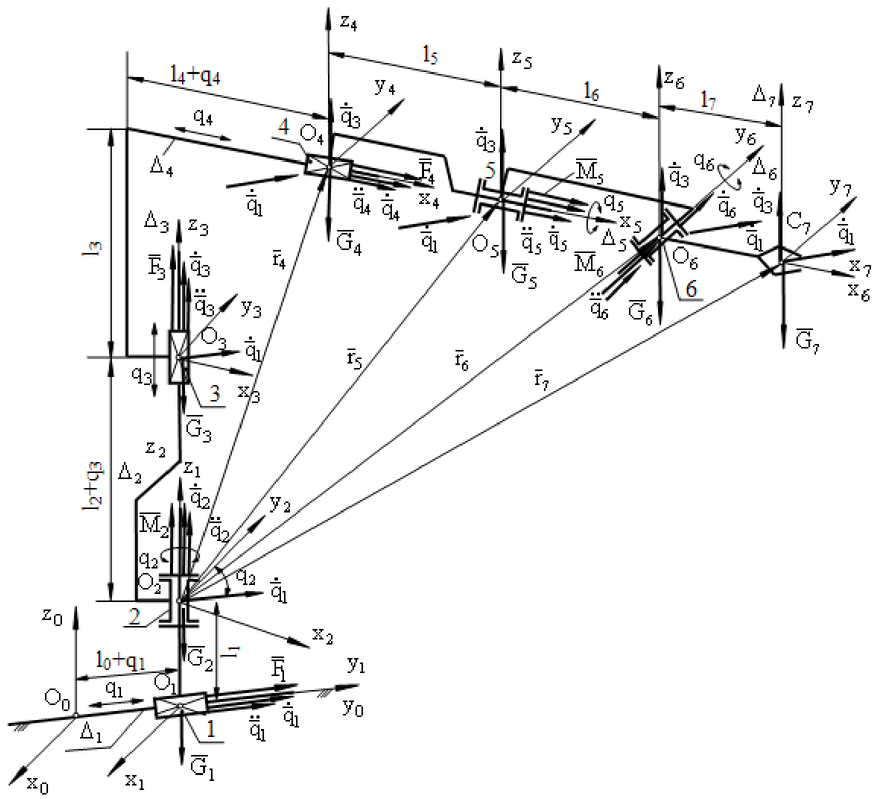

Fig. 1. The kinematic scheme of the TRTTRR1 robot

$O_0x_0y_0z_0$ - the fixed cartesian system;

$O_ix_iy_iz_i, i = 1 \div 6$ - the mobile reference systems, jointly with the mobile parts of the robot's modules;

$C_7x_7y_7z_7$ - the mobile reference system, jointly with the clamping device, having the origin C_7 in the center of gravity of the clamping device;

$J_{\Delta_2}^{(2)}$ - the mechanical moment of inertia for the mobile crew of the MRB module, determined in relation with the Δ_2 axis;

$J_{\Delta_2}^{(3)}$ - the mechanical moment of inertia of the vertical translation module MTV, determined in relation with the Δ_2 axis;

$J_{z_4}^{(4)}$ - the mechanical moment of inertia for the translation module in relation with the O_4z_4 axis;

$J_{\Delta_5}^{(5)}$ - the mechanical moment of inertia for the mobile crew of the orientation module MO, determined in relation with the rotation axis Δ_5.

$J_{z_5}^{(5)}$ - the mechanical moment of inertia for the module MO, determine in relation with the O_5z_5 axis;

$J_{z_6}^{(6)}$ - the mechanical moment of inertia for the module MO, determine in relation with the O_6z_6 axis;

$J_{x_6}^{(6)}$ - the mechanical moment of inertia for the module MO, determine in relation with the O_6x_6 axis;

$J_{\Delta_6}^{(6)}$ - the mechanical moment of inertia for the orientation module, determine in relation with the axis Δ_6;

$J_{x_7}^{(7)}$ - the mechanical moment for the clamping device in relation with the C_7x_7 axis.

$J_{y_7}^{(7)}$ - the mechanical moment for the clamping device, determine in relation with the C_7y_7 axis.

The motion differential equations of the robot, can be determined according to [3] and [4], using the Lagrange's equations of the second type:

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_k}\right) - \frac{\partial Ec}{\partial q_k} = Q_k, \; k = 1 \div 6. \qquad (1)$$

In the equations (1) are defined the following:

Ec - represents the kinetic energy of the robot;

$q_k, \dot{q}_k$ - represents the generalized coordinates and velocities;

Q_k - are the generalized forces;

k -represents the number of degrees of freedom.

In accordance with [3], the kinetic energy of an item „i" from the mechanical structure of the robot has the expression:

$$Ec_i = \frac{1}{2} M_i \left(v_x^2 + v_y^2 + v_z^2 \right)_i + \frac{1}{2} \left(J_x \omega_x^2 + J_y \omega_y^2 + J_z \omega_z^2 \right)_i . \quad (2)$$

where: ω_x, ω_y, ω_z are the cartesian components of the angular velocity, v_x, v_y, v_z are the cartesian components of the velocity center of gravity of the element of order "i", and J_x, J_y, J_z are the mechanical axial moments of inertia of the item "i".

To express the kinetic energy of an element with the relation (2), it is necessary that its center of gravity is the origin of the reference system $Oxyz$, such that $x_c = y_c = z_c = 0$, and the system $Oxyz$, must be main system of inertia, that $J_{xy} = J_{yz} = J_{zx} = 0$.

Starting from the base of the robot and following the figure 1 and the relation (2), the kinetic energies of the robot's modules can be determined, respectively, of the gripper and namely for:

- for the modules MTB, MRB, MTV and MT, the expression of kinetic energies are the same from the TRTTR1, because there are similarities up to MT module. These kinetic energies have the expressions:

$$Ec_1 = \frac{1}{2} m_1 \dot{q}_1^2 , \;\; Ec_2 = \frac{1}{2} m_2 \dot{q}_1^2 + \frac{1}{2} J_{\Delta_2}^{(2)} \dot{q}_2^2 ; \quad (3)$$

$$Ec_3 = \frac{1}{2} m_3 \left(\dot{q}_1^2 + \dot{q}_3^2 \right) + \frac{1}{2} J_{\Delta_2}^{(3)} \dot{q}_2^2 ; \quad (4)$$

$$\begin{aligned} E_{c_4} &= \frac{1}{2} m_4 \left(\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2 \right) + \frac{1}{2} \Big[J_{z_4}^{(4)} + \\ &+ m_4 (l_4 + q_4)^2 \Big] \dot{q}_2^2 + m_4 \big[(l_4 + q_4) c q_2 \dot{q}_1 \dot{q}_2 + \\ &+ s q_2 \dot{q}_1 \dot{q}_4 \big] . \end{aligned} \quad (5)$$

- for rotation module MR, the kinematic parameters which characterized the movement are:

$$\begin{aligned} &\omega_{x_5} = \dot{q}_5 , \;\; \omega_{y_5} = 0 , \;\; \omega_{z_5} = \dot{q}_2 ; \\ &\bar{v}_{O_5} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \bar{r}_5 + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 , \\ &\bar{r}_5 = \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 , \\ &\dot{\bar{q}}_2 \times \bar{r}_5 = \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{q}_4 + \bar{l}_5 \right), \qquad (6) \\ &\bar{v}_{O_5} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{q}_4 + \bar{l}_5 \right) + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 , \end{aligned}$$

$$\begin{aligned} v_{O_5}^2 &= \dot{q}_1^2 + (l_4 + l_5 + q_4)^2 \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_4^2 + \\ &+ 2 (l_4 + l_5 + q_4) c q_2 \dot{q}_1 \dot{q}_2 + 2 s q_2 \dot{q}_1 \dot{q}_4 , \end{aligned}$$

the other terms of the development are void, because they are inner products performed with perpendicular vectors two by two, respectively the main mechanical moments of inertia are:

$$J_{x_5} = J_{\Delta_5}^{(5)} , \quad J_{y_5} = J_{y_5}^{(5)} , \quad J_{z_5} = J_{z_5}^{(5)} . \quad (7)$$

The kinetic energy of the rotation module, in wiew of (2), (6) and (7) is expressed as:

$$\begin{aligned} E_{c_5} &= \frac{1}{2} m_5 \left(\dot{q}_1^2 + \dot{q}_3^2 + \dot{q}_4^2 \right) + \frac{1}{2} \left[J_{z_5}^{(5)} + m_5 (l_4 + l_5 + q_4)^2 \right] \dot{q}_2^2 + \\ &+ \frac{1}{2} J_{\Delta_5}^{(5)} \dot{q}_5^2 + m_5 (l_4 + l_5 + q_4) c q_2 \dot{q}_1 \dot{q}_2 + m_5 s q_2 \dot{q}_1 \dot{q}_2 . \end{aligned} \quad (8)$$

- for the orientation module MO, the kinematic parameters which characterized the movement are:

$$\begin{aligned} &\omega_{x_6} = \dot{q}_5 , \;\; \omega_{y_6} = \dot{q}_6 , \;\; \omega_{z_6} = \dot{q}_2 , \\ &\bar{v}_{O_6} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \bar{r}_6 + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 . \end{aligned} \quad (9)$$

Following the figures 1 and 2, the relations can be written:

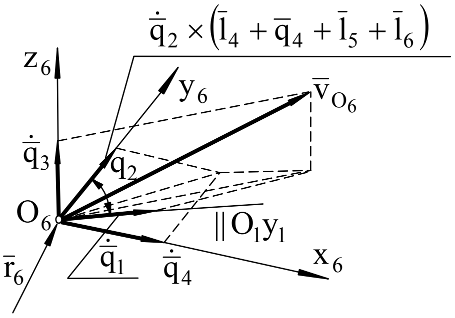

Fig. 2. The velocity of the O_6 point

$$\begin{aligned} &\bar{r}_6 = \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 + \bar{l}_6 , \\ &\dot{\bar{q}}_2 \times \bar{r}_6 = \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4 \right), \qquad (10) \\ &\bar{v}_{O_6} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4 \right) + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 , \\ &v_{O_6}^2 = \dot{q}_1^2 + (l_4 + l_5 + l_6 + q_4)^2 \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_4^2 + \\ &\quad + 2 (l_4 + l_5 + l_6 + q_4) c q_2 \dot{q}_1 \dot{q}_2 + 2 s q_2 \dot{q}_1 \dot{q}_4 , \end{aligned}$$

because

$$\begin{aligned} &\dot{\bar{q}}_3 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4 \right) \right] = 0 , \;\; \dot{\bar{q}}_1 \cdot \dot{\bar{q}}_3 = 0 , \\ &\dot{\bar{q}}_3 \cdot \dot{\bar{q}}_4 = 0 , \; \dot{\bar{q}}_4 \cdot \left[\dot{\bar{q}}_2 \times \left(\bar{l}_4 + \bar{l}_5 + \bar{l}_6 + \bar{q}_4 \right) \right] = 0 , \end{aligned}$$

those vectors are perpendiculars (figure 2).

The main mechanical moments of inertia of this module are:

$$J_{x_6} = J_{x_6}^{(6)} , \quad J_{y_6} = J_{\Delta_6}^{(6)} , \quad J_{z_6} = J_{z_6}^{(6)} . \quad (11)$$

The kinetic energy of the orientation module MO is obtained by inserting in (2), the details made byrelations (10) and (11). Thus,

$$E_{c_6}=\frac{1}{2}m_6\left(\dot q_1^2+\dot q_3^2+\dot q_4^2\right)+\frac{1}{2}\Big[J_{z_6}^{(6)}+ +m_6\left(l_4+l_5+l_6+q_4\right)^2\Big]\dot q_2^2+\frac{1}{2}J_{x_6}^{(6)}\dot q_5^2+ +\frac{1}{2}J_{\Delta_6}^{(6)}\dot q_6^2+m_6\left(l_4+l_5+l_6+q_4\right)cq_2\dot q_1\dot q_2+ +m_6 sq_2\dot q_1\dot q_4\,; \qquad (12)$$

- for the clamping device, the kinematic parameters that influence the movement are:

$$\omega_{x_7}=\dot q_5\,,\ \omega_{y_7}=\dot q_6\,,\ \omega_{z_7}=\dot q_2\,,$$
$$\bar v_{C_7}=\dot{\bar q}_1+\dot{\bar q}_2\times\bar r_7+\dot{\bar q}_3+\dot{\bar q}_4+\dot{\bar q}_6\times\bar l_7\,,$$
$$\bar r_7=\bar l_2+\bar q_3+\bar l_3+\bar l_4+\bar q_4+\bar l_5+\bar l_6+\bar l_7\,,$$
$$\dot{\bar q}_2\times\bar r_7=\dot{\bar q}_2\times\left(\bar l_4+\bar l_5+\bar l_6+\bar l_7+\bar q_4\right),$$
$$\bar v_{C_7}=\dot{\bar q}_1+\dot{\bar q}_2\times\left(\bar l_4+\bar l_5+\bar l_6+\bar l_7+\bar q_4\right)+\dot{\bar q}_3+ +\dot{\bar q}_4+\dot{\bar q}_6\times\bar l_7\,, \qquad (13)$$
$$v_{c_7}^2=\dot q_1^2+\left(l_4+l_5+l_6+l_7+q_4\right)^2\dot q_2^2+\dot q_3^2+ +\dot q_4^2+l_7^2\dot q_6^2+2\left(l_4+l_5+l_6+l_7+q_4\right)\dot q_1\dot q_2 cq_2+ +2sq_2\dot q_1\dot q_4-2l_7\dot q_3\dot q_6\,,$$

because the scalar products $\dot{\bar q}_1\cdot\dot{\bar q}_3$, $\dot{\bar q}_1\cdot\left(\dot{\bar q}_5\times\bar l_7\right)$, $\dot{\bar q}_3\cdot\left[\dot{\bar q}_2\times\left(\bar l_4+\bar l_5+\bar l_6+\bar l_7+\bar q_4\right)\right]$, $\dot{\bar q}_4\cdot\left[\dot{\bar q}_2\times\left(\bar l_4+\bar l_5+\bar l_6+\bar l_7+\bar q_4\right)\right]$, $\left[\dot{\bar q}_2\times\left(\bar l_4+\bar l_5+\bar l_6+\bar l_7+\bar q_4\right)\right]\cdot\left(\dot{\bar q}_5\times\bar l_7\right)$, $\dot{\bar q}_3\cdot\dot{\bar q}_4$ and $\dot{\bar q}_4\cdot\left(\dot{\bar q}_5\times\bar l_7\right)$ are null, because those vectors are perpendicular (figure 3).

The main moments of inertia of the clamping device are:

$$J_{x_7}=J_{x_7}^{(7)},\ J_{y_7}=J_{y_7}^{(7)},\ J_{z_7}=J_{\Delta_7}^{(7)}. \qquad (14)$$

Having in wiew the relations (2), (13) and (14), the kinetic energy of the clamping device, has the value:

$$E_{c_7}=\frac{1}{2}m_7\left(\dot q_1^2+\dot q_3^2+\dot q_4^2\right)+ +\frac{1}{2}\left[J_{\Delta_7}^{(7)}+m_7\left(l_4+l_5+l_6+l_7+q_4\right)^2\right]\dot q_2^2+ +\frac{1}{2}J_{x_7}^{(7)}\dot q_5^2+\frac{1}{2}\left[J_{y_7}^{(7)}+m_7 l_7^2\right]\dot q_6^2+ +m_7\begin{bmatrix}\left(l_4+l_5+l_6+l_7+q_4\right)cq_2\dot q_1\dot q_2+\\ +sq_2\dot q_1\dot q_4-l_7\dot q_3\dot q_3\end{bmatrix}. \qquad (15)$$

By summing the kinetic energies of the component modules, is obtained the kinetic energy of the TRTTRR1 robot. Thus, considering (3), (4), (5), (8), (12) and (15) we obtain the expression:

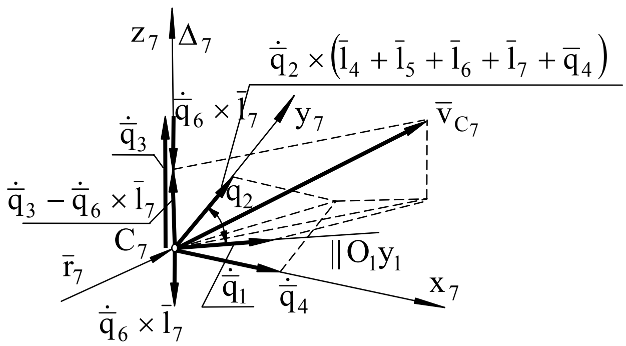

Fig. 3. The velocity of the center of gravity C_6

$$E_c=\frac{1}{2}\left(\sum_{i=1}^{7}m_i\right)\dot q_1^2+\frac{1}{2}\Bigg[\sum_{i=2}^{3}J_{\Delta_2}^{(i)}+\sum_{i=4}^{6}J_{z_i}^{(i)}+J_{\Delta_7}^{(7)}+ +m_4\left(l_4+q_4\right)^2+m_5\left(l_4+l_5+q_4\right)^2+ +m_6\left(l_4+l_5+l_6+q_4\right)^2+ +m_7\left(l_4+l_5+l_6+l_7+q_4\right)^2\Bigg]\dot q_2^2+\frac{1}{2}\left(\sum_{i=3}^{7}m_i\right)\dot q_3^2+ +\frac{1}{2}\left(\sum_{i=4}^{7}m_i\right)\dot q_4^2+\frac{1}{2}\left[J_{\Delta_5}^{(5)}+\sum_{i=6}^{7}J_{x_i}^{(i)}\right]\dot q_5^2+ +\frac{1}{2}\left[J_{\Delta_6}^{(6)}+J_{y_7}^{(7)}+m_7 l_7^2\right]\dot q_6^2+\big[m_4\left(l_4+q_4\right)+ +m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right) +m_7\left(l_4+l_5+l_6+l_7+q_4\right)\big]cq_2\dot q_1\dot q_2+ +\left(\sum_{i=4}^{7}m_i\right)sq_2\dot q_1\dot q_4-m_7 l_7\dot q_3\dot q_6\,. \qquad (16)$$

In relation (16) are made are made these notations:

$$\frac{1}{2}\left(\sum_{i=1}^{7}m_i\right)=c_1,\ \frac{1}{2}\Bigg[\sum_{i=2}^{3}J_{\Delta_2}^{(i)}+\sum_{i=4}^{6}J_{z_i}^{(i)}+J_{\Delta_7}^{(7)}+m_4\left(l_4+q_4\right)^2+ +m_5\left(l_4+l_5+q_4\right)^2+m_6\left(l_4+l_5+l_6+q_4\right)^2+ +m_7\left(l_4+l_5+l_6+l_7+q_4\right)^2\Bigg]=c_2,\ \frac{1}{2}\left(\sum_{i=3}^{7}m_i\right)=c_3, \qquad (17)$$
$$\frac{1}{2}\left(\sum_{i=4}^{7}m_i\right)=c_4,\ \frac{1}{2}\left[J_{\Delta_5}^{(5)}+\sum_{i=6}^{7}J_{x_i}^{(i)}\right]=c_5,$$
$$\frac{1}{2}\left[J_{\Delta_6}^{(6)}+J_{y_7}^{(7)}+m_7 l_7^2\right]=c_6,$$
$$\big[m_4\left(l_4+q_4\right)+m_5\left(l_4+l_5+q_4\right)+m_6\left(l_4+l_5+l_6+q_4\right)+ +m_7\left(l_4+l_5+l_6+l_7+q_4\right)\big]cq_2=c_{12}$$
$$\left(\sum_{i=4}^{7}m_i\right)sq_2=c_{14}\,,\ -m_7 l_7=c_{36}\,.$$

that the kinetic energy of the robot reaches the form:

$$E_c = c_1\dot{q}_1^2 + c_2\dot{q}_2^2 + c_3\dot{q}_3^2 + c_4\dot{q}_4^2 + c_5\dot{q}_5^2 + \\ + c_6\dot{q}_6^2 + c_{12}\dot{q}_1\dot{q}_2 + c_{14}\dot{q}_1\dot{q}_4 + c_{36}\dot{q}_3\dot{q}_6 . \qquad (18)$$

Following the relations (1), (17) and (18), are made the following precisions:

$$\frac{\partial E_c}{\partial q_1} = 0,\ \frac{\partial E_c}{\partial q_2} = \frac{\partial c_{12}}{\partial q_2}\dot{q}_1\dot{q}_2 + \frac{\partial c_{14}}{\partial q_2}\dot{q}_1\dot{q}_4 = -[m_4(l_4+q_4)+ \\ + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4)+ \\ + m_7(l_4+l_5+l_6+l_7+q_4)]\cdot sq_2\dot{q}_1\dot{q}_2 + \left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_1\dot{q}_4 ,$$

$$\frac{\partial E_c}{\partial q_3} = 0,\ \frac{\partial E_c}{\partial q_5} = 0,\ \frac{\partial E_c}{\partial q_6} = 0,$$

$$\frac{\partial E_c}{\partial q_4} = \frac{\partial c_2}{\partial q_4}\dot{q}_2^2 + \frac{\partial c_{12}}{\partial q_4}\dot{q}_1\dot{q}_2 = [m_4(l_4+q_4) + m_5(l_4+l_5+q_4)+ \\ + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)]\dot{q}_2^2 + \qquad (19) \\ + \left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_1\dot{q}_2 ;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_1}\right) = 2c_1\ddot{q}_1 + \frac{dc_{12}}{dt}\dot{q}_2 + c_{12}\ddot{q}_2 + c_{14}\ddot{q}_4 + \frac{dc_{14}}{dt}\dot{q}_4 = \\ = \left(\sum_{i=1}^{7} m_i\right)\ddot{q}_1 + [m_4(l_4+q_4) + m_5(l_4+l_5+q_4)+ \\ + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)]cq_2\ddot{q}_2 + \\ + \left(\sum_{i=4}^{7} m_i\right)sq_2\ddot{q}_4 - [m_4(l_4+q_4) + m_5(l_4+l_5+q_4)+ \\ + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)]sq_2\dot{q}_2^2 + \\ + 2\left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_2\dot{q}_4 ;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_2}\right) = [m_4(l_4+q_4) + m_5(l_4+l_5+q_4)+ \\ + m_6(l_4+l_5+l_6+q_4) + m_7(l_4+l_5+l_6+l_7+q_4)]cq_2\ddot{q}_1 + \\ + \left[\sum_{i=2}^{3} J_{\Delta_2}^{(i)} + \sum_{i=4}^{6} J_{z_i}^{(i)} + J_{\Delta_7}^{(7)} + m_4(l_4+q_4)^2 + \\ + m_5(l_4+l_5+q_4)^2 + m_6(l_4+l_5+l_6+q_4)^2 + \\ + m_7(l_4+l_5+l_6+l_7+q_4)^2\right]\ddot{q}_2 - [m_4(l_4+q_4)+ \\ + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4)+ \\ + m_7(l_4+l_5+l_6+l_7+q_4)]sq_2\dot{q}_1\dot{q}_2 + \left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_1\dot{q}_4 + \\ + 2[m_4(l_4+q_4) + m_5(l_4+l_5+q_4) + m_6(l_4+l_5+l_6+q_4)+ \\ + m_7(l_4+l_5+l_6+l_7+q_4)]\dot{q}_2\dot{q}_4;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_3}\right) = 2c_3\ddot{q}_3 + c_{36}\ddot{q}_6 = \left(\sum_{i=3}^{7} m_i\right)\ddot{q}_3 - m_7 l_7\ddot{q}_6 ; \qquad (20)$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_4}\right) = c_{14}\ddot{q}_1 + 2c_4\ddot{q}_4 + \frac{dc_{14}}{dt}\dot{q}_1 = \\ = \left(\sum_{i=4}^{7} m_i\right)(sq_2\ddot{q}_1 + \ddot{q}_4 + cq_2\dot{q}_1\dot{q}_2);$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_5}\right) = \left[J_{\Delta_5}^{(5)} + \sum_{i=6}^{7} J_{x_i}^{(i)}\right]\ddot{q}_5;$$

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_6}\right) = \left[J_{\Delta_6}^{(6)} + J_{y_7}^{(7)} + m_7 l_7^2\right]\ddot{q}_6 - m_7 l_7\ddot{q}_3 .$$

To deduce the generalized forces corresponding to the six modules of the robot, are given the basic virtual displacement movements so that all the parameters q_k will vary with $\delta q_1, \delta q_2, \ldots, \delta q_5, \delta q_6$, after that the virtual mechanical work is determined. According to figure 1, the virtual mechanical work suitable for the external forces and moments and for some virtual displacement movements compatible with the robot's links, has the expression:

$$\delta L = F_1\delta q_1 + M_2\delta q_2 + (F_3 - G_3)\delta q_3 + (\overline{F}_4 + \overline{G}_4)\cdot \delta\overline{O_0O_4} + \\ + \overline{G}_5\cdot\delta\overline{O_0O_5} + M_5\delta q_5 + \overline{G}_6\cdot\delta\overline{O_0O_6} + M_6\delta q_6 + \qquad (21) \\ + \overline{G}_7\cdot\delta\overline{O_0C_7}.$$

Considering that

$$\overline{O_0O_4} = \bar{l}_0 + \bar{q}_1 + \bar{l}_1 + \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 , \\ \delta\overline{O_0O_4} = \delta\bar{q}_1 + \delta\bar{q}_3 + \delta\bar{q}_4 \\ \overline{O_0O_5} = \bar{l}_0 + \bar{q}_1 + \bar{l}_1 + \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 , \\ \delta\overline{O_0O_5} = \delta\bar{q}_1 + \delta\bar{q}_3 + \delta\bar{q}_4 \\ \overline{O_0C_6} = \bar{l}_0 + \bar{q}_1 + \bar{l}_1 + \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 + \bar{l}_6 , \qquad (22) \\ \delta\overline{O_0C_6} = \delta\bar{q}_1 + \delta\bar{q}_3 + \delta\bar{q}_4 , \\ \overline{O_0C_7} = \bar{l}_0 + \bar{q}_1 + \bar{l}_1 + \bar{l}_2 + \bar{q}_3 + \bar{l}_3 + \bar{l}_4 + \bar{q}_4 + \bar{l}_5 + \bar{l}_6 + \bar{l}_7 , \\ \delta\overline{O_0C_7} = \delta\bar{q}_1 + \delta\bar{q}_3 + \delta\bar{q}_4 ,$$

the expression (21) of the virtual mechanical work, reaches the form:

$$\delta L = (F_1 + F_4 sq_2)\delta q_1 + M_2\delta q_2 + \left(F_3 - \sum_{i=3}^{7} G_i\right)\delta q_3 + \\ + F_4\delta q_4 + M_5\delta q_5 + M_6\delta q_6 . \qquad (23)$$

The generalized forces, in accordance with [3], are obtained from relations:

$$Q_k = \frac{\partial L}{\partial q_k},\ k = 1 \div 6. \qquad (24)$$

that is $Q_1 = F_1 + F_4 sq_2,\ Q_2 = M_2,\ Q_3 = F_3 - \sum_{i=3}^{7} G_i,$

$$Q_4 = F_4,\ Q_5 = M_5, Q_6 = M_6. \qquad (25)$$

The differential equations of the robot are obtained from (1) in which are introduce the relations (19), (20) and (25). Thus,

$$\left(\sum_{i=1}^{7} m_i\right)\ddot{q}_1 + [m_4(l_4+q_4)+m_5(l_4+l_5+q_4)+$$
$$+m_6(l_4+l_5+l_6+q_4)+m_7(l_4+l_5+l_6+l_7+q_4)]cq_2\ddot{q}_2 +$$
$$+\left(\sum_{i=4}^{7} m_i\right)sq_2\ddot{q}_4 - [m_4(l_4+q_4)+m_5(l_4+l_5+q_4)+$$
$$+m_6(l_4+l_5+l_6+q_4)+m_7(l_4+l_5+l_6+l_7+q_4)]sq_2\dot{q}_2^2 +$$
$$+2\left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_2\dot{q}_4 = F_1 + F_4 sq_2$$
$$[m_4(l_4+q_4)+m_5(l_4+l_5+q_4)+m_6(l_4+l_5+l_6+q_4)+$$
$$+m_7(l_4+l_5+l_6+l_7+q_4)]cq_2\ddot{q}_1 + \left[\sum_{i=2}^{3} J_{\Delta_2}^{(i)} + \sum_{i=4}^{6} J_{z_i}^{(i)} +\right.$$
$$+m_4(l_4+q_4)^2 + m_5(l_4+l_5+q_4)^2 + m_6(l_4+l_5+l_6+q_4)^2 +$$
$$\left.+m_7(l_4+l_5+l_6+l_7+q_4)^2\right]\ddot{q}_2 + 2[m_4(l_4+q_4)+$$
$$+m_5(l_4+l_5+q_4)+m_6(l_4+l_5+l_6+q_4)+$$
$$+m_7(l_4+l_5+l_6+l_7+q_4)]\dot{q}_2\dot{q}_4 = M_2$$
$$\left(\sum_{i=3}^{7} m_i\right)\ddot{q}_3 - m_7 l_7 \ddot{q}_6 = F_3 - \sum_{i=3}^{7} G_i$$
$$\left(\sum_{i=4}^{7} m_i\right)(sq_1\ddot{q}_1 + \ddot{q}_4 + cq_2\dot{q}_1\dot{q}_2) - [m_4(l_4+q_4)+m_5(l_4+l_5+q_4)+$$
$$+m_6(l_4+l_5+l_6+q_4)+m_7(l_4+l_5+l_6+l_7+q_4)]\dot{q}_2^2 -$$
$$-\left(\sum_{i=4}^{7} m_i\right)cq_2\dot{q}_1\dot{q}_2 = F_4$$
$$\left[J_{\Delta_5}^{(5)} + \sum_{i=6}^{7} J_{x_i}^{(i)}\right]\ddot{q}_5 = M_5$$
$$-m_7 l_7 \ddot{q}_3 + \left[J_{\Delta_6}^{(6)} + J_{y_7}^{(7)} + m_7 l_7^2\right]\ddot{q}_6 = M_6 . \qquad (26)$$

The differential equations system (26) of the TRTTRR1 robot were determined on the assumption that all movements take place simultaneously. By solving the inverse problem of the dynamics of the robot result: optimal modules arrangement in a robot structure such as the energy consumption to be minimal; an optimal variant for choosing the laws of motion of the robot's axes in such way that the energy consumption to be minimal; choosing the actuators of the modules, considering their arrangement and the whole dynamics of the robot.

3. CONCLUSION

The dynamic study of the industrial robots is necessary for solving the direct problem, respectively, the inverse of their dynamics.

In the case of the direct problem, the robot's movement is determined, if are known the forces and moments that are acting on it.

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Ecuațiile dinamice ale robotului TRTTRR1 utilizând formalismul lui Lagrange

Rezumat: Studiul dinamic al roboților industriali permite rezolvarea următoarelor probleme: alegerea variantei optime de aranjare a modulelor într-o structură de robot modular, astfel încât consumurile energetice să fie minime; alegerea unor legi de mişcare pe axele robotului în aşa fel încât consumurile energetice să fie minime; alegerea motoarelor de acționare a modulelor robotului luând în considerare organologia fiecărui modul în parte şi dinamica întregului robot. Pentru acesta, schema cinematică a robotului, apoi, utilizând ecuațiile lui Lagrange de speța a II-a, se determină energia cinetică a robotului şi forțele generalizate motoare. În final, se obțin ecuațiile dinamice ale robotului TRTTRR1, care sunt necesare pentru rezolvarea celor doua probleme: directă şi inversă a dinamicii robotului.

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