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NUMERICAL AND ANALYTICAL STUDY ON THE DEFLECTION OF THE V-BEAM THERMAL SENSOR

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Abstract: *The purpose of this paper is to present some results on the determination of the V-beam thermal sensor's deflection. The influence of its geometrical parameters on the output deflection was tested while also taking into account the deformation of the substrate. The finite element (FE) studies that were performed on virtual replicas of the V-beam thermal sensors led to the validation of the theoretical model.* ***Key words:*** *V-beam thermal sensor, thermal load, substrate deformation*

1. INTRODUCTION

V-beam thermal sensors (also called Chevron geometry sensors) are a common type of thermal sensors due to their linear trajectory of the output displacement. Another popular geometry for thermal sensors and actuators is the pseudo bimorph geometry but this type of actuator is less used due to its angular output deflection.

Que, L., et al. [1], present some analytical modeling aspects related to cascaded bent beam thermal actuators for motion amplification.

Sinclair, M.J. [2], presents an analytical model for a V-beam geometrical type thermal actuator, but in this case the actuation elements are heated to a level at which buckling occurs.

Another suggested functionality of this geometry type is presented by Guo, J., et al., in [3]. They are using a differential assembly scheme in order to amplify the output capacitance signal. This time there is no thermal load present, only one of the anchors is fixed and the other one is in contact with the input strain.

All of the theoretical models presented in the above mentioned references are based on the assumption that the anchors are fixed, but in the case of thermal sensing this in not always true. Due to the fact that these sensors are mounted on a substrate that is impossible to insulate it from the heat source, a displacement of the anchors appears which leads to an altered output response.

This paper presents an analytical model that takes into account the displacement described above and thus provides more accurate results.

2. PROBLEM DEFINITION

The geometrical parameters of the V-beam thermal sensor are presented in Figure 1.

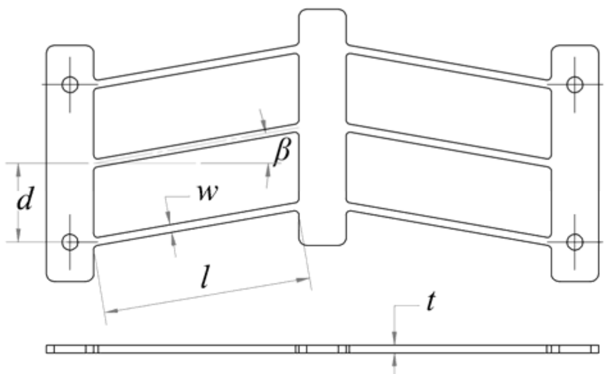

Fig.1. Geometrical parameters of the V-beam thermal sensor

In order to assess the influence of their variation and the accuracy of the analytical model, several geometrical types were taken into account and compared to a reference sample defined by parameters: beam inclination angle – β = 10deg; beam length – l = 26.4mm;

beam width – w = 1mm; distance between consecutive beams – d = 10mm; number of beam pairs – n = 3. The other geometrical types tested are: β 15, β 20; L 21.3, L 31.5; d 15, d 20; w 1.5, w 2; n 4, n 5 and n 6, where the coding represents the parameter that changes in regard with the parameters of the reference beam (e.g. β 15 means that the sample has an inclination angle of 15deg while all the other parameters have the same value as those of the reference beam). All the samples, including the reference one, have the same thickness – t = 1mm.

3. ANALYTICAL MODEL

The development of the analytical model starts from the one proposed by N. Lobontiu, et al., in [4]. Figure 2 illustrates the theoretical model of the sensor with boundary conditions.

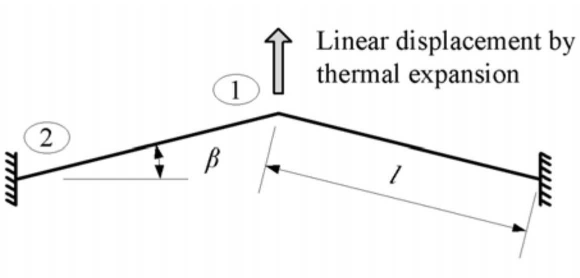

Fig.2. Theoretical model with boundary conditions

Due to the symmetry of the geometry and the load of the sensor, the problem can be solved for the half model by introducing the effects of the half removed.

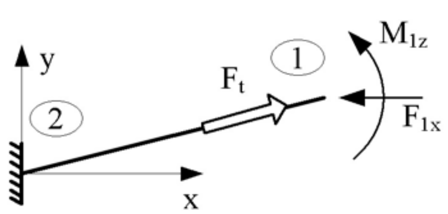

Fig.3. Half model with thermal load

The equation given by [4] for the model shown in Figure 3 is:

$$u_{1y}^{(Fig\_3)} = \frac{A\alpha\Delta T l^3 \sin(\beta)}{12 I_z \cos^2(\beta) + A l^2 \sin^2(\beta)} \quad (1)$$

in which the terms that weren't previously defined are: ΔT – temperature difference between the initial and final deformed states of the sensor; A – cross-sectional area of the beam; I_z – moment of inertia of the cross-section of the beam.

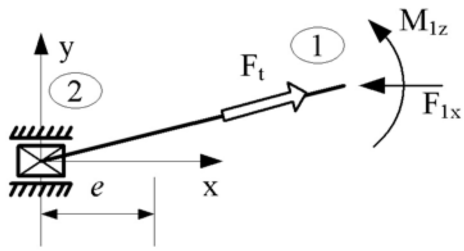

Fig.4. Half model with thermal load and imposed displacement along the x-axis

In Figure 4 the theoretical model with imposed displacement is presented. This displacement takes into account the expansion of the substrate of the sensor as well as the elongation along the x-axis of the central shaft.

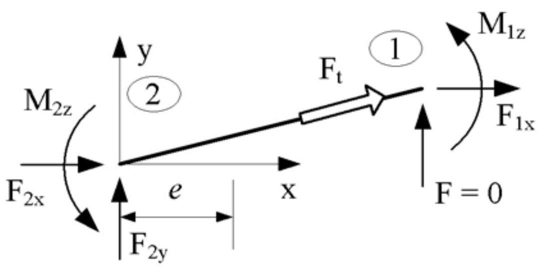

Fig.5. Free body under equilibrium of forces

Figure 5 is derived from Figure 4 by replacing the supports with the reactive forces. Also, a null force along the y-axis is added at node 1 in order to determine the deflection along its direction by the means of the 2nd Castigliano theorem.

The expressions of the reactive forces are:

$$F_{2x} = \frac{12eEAI_z}{12lI_z\cos^2(\beta) + Al^3\sin^2(\beta)} + \frac{EA\alpha\Delta T l \sin^2(\beta)\cos(\beta)(Al^2 - 12I_z)}{12lI_z\cos^2(\beta) + Al^3\sin^2(\beta)} \quad (2)$$

$$F_{2y} = -EA\alpha\Delta T \sin(\beta) \quad (3)$$

$$M_{2z} = \frac{6EAI_z \sin(\beta)(\alpha \Delta Tl \cos(\beta) - e)}{12I_z \cos^2(\beta) + Al^2 \sin^2(\beta)} \quad (4)$$

$$F_{1x} = -\frac{12EAI_z (e + \alpha \Delta Tl \sin(\beta)\cos(\beta))}{12lI_z \cos^2(\beta) + Al^3 \sin^2(\beta)} \quad (5)$$

$$M_{1z} = -\frac{6EAI_z \sin(\beta)(\alpha \Delta Tl \cos(\beta) + e)}{12I_z \cos^2(\beta) + Al^2 \sin^2(\beta)} \quad (6)$$

The equation for the displacement of the sensor with the imposed displacement *e* is:

$$u_{1y}^{(Fig\_5)} = \alpha \Delta Tl \sin(\beta) + (e + \alpha \Delta Tl \cos(\beta)) \cdot \cdot \frac{\sin(\beta)\cos(\beta)(Al^2 - 12I_z)}{12I_z \cos^2(\beta) + Al^2 \sin^2(\beta)} \quad (7)$$

This equation expresses the deformation of the middle point of the central shaft along the y-axis. The deformation of this shaft has to be taken into account if the transducer is situated at the tip of the sensor.

4. NUMERICAL SIMULATION

In order to test the derived formula, a FE analysis using ANSYS v12 FEA Software was conducted for all the geometrical types described earlier.

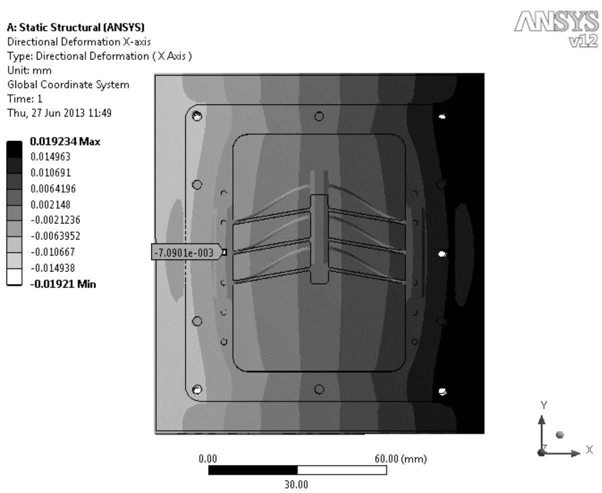

Fig.6. Directional deformation of the assembly along x-axis

Figure 6 illustrates the deformation along the x-axis of the entire assembly.

The deformed shape and the output deflection at the tip of the sensor can be seen in Figure 7.

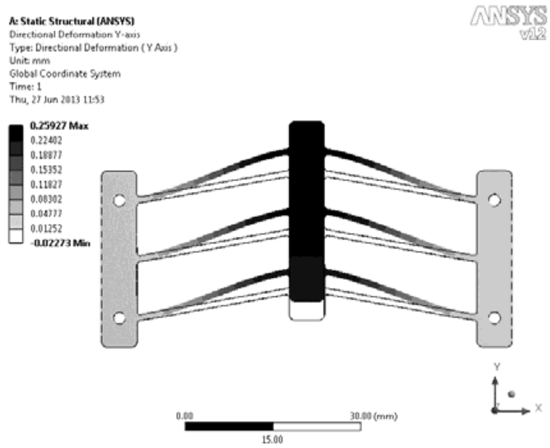

Fig.7. Directional deformation of the sensor along y-axis

5. RESULTS AND CONCLUSIONS

The analytical and FE analysis results are presented in Table 1.

Table 1

Comparative analytical and numerical results

Sample	Analytical deflection [mm]	Numerical deflection [mm]
Reference	0.254	0.259
n 4	0.260	0.264
n 5	0.268	0.273
n 6	0.276	0.277
β 15	0.181	0.172
β 20	0.147	0.140
d 15	0.261	0.263
d 20	0.267	0.272
w 1.5	0.240	0.246
w 2	0.226	0.231
L 21.3	0.209	0.206
L 31.5	0.295	0.280

The analytical results are in good agreement with the numerical calculations which validates the theoretical model proposed. The relative deviation between the two sets of results is less than 6%. The same geometrical types of sensors have been ordered and the experimental results will be compared with data as soon as the samples will be available.

From the data above we can conclude that an increase of the length of the beams produces larger deflection. The same effect is achieved by decreasing the inclination angle and the width of the beam.

6. REFERENCES

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STUDIUL NUMERIC ŞI ANALITIC AL DEFORMAŢIEI SENSORILOR TERMICI DE TIP V-BEAM

Abstract: *Scopul acestei lucrări este de a prezenta nişte rezultate privind determinarea deformaţiilor senzorilor termici de tip V-beam. A fost studiată influenţa parametrilor geometrici asupra deformaţiei senzorilor luând, de asemenea, în considerare dilatarea substratului. Studiul deformaţiilor prin metoda elementelor finite, efectuat pe modele virtuale ale senzorilor, a validat modelul teoretic.*

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